FIRST TERMINAL EXAMINATION 2014-15

MATHEMATICS

Class XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

1. All questions are compulsory.

2. The question paper consist of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, section B comprises of 13 questions of four marks each and section C comprises of 07 questions of six marks each.

3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

SECTION – A

1. If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$ and $B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A+B)^2 = A^2+B^2$, then find a.

2. Write the value of $\begin{vmatrix}a+ib & c+id\\ -c+id & a-ib\end{vmatrix}$.

3. If $y=\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$, then write the value of $\frac{dy}{dx}$

4. If $f(1)=4, f'(1)=2$, find the value of derivative of $\log f(e^x)$ w.r.t. x at $x=0$

5. What is the maximum value of $y=\sin x.\cos x$.

6. Write the value of $\int \frac{1}{x\cos^2(1+\log x)}dx$

SECTION – B

7. Solve the following equation:

$$\tan^{-1}\frac{x}{2}+\tan^{-1}\frac{x}{3}=\frac{\pi}{4}\quad ;\ 0<x<\sqrt{6}$$

8. Prove that: $2\left(\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}\right)=\tan^{-1}\frac{4}{3}$

9. Express the following matrices as the sum of symmetric and a skew-symmetric matrix:

$$\begin{bmatrix}6 & 1 & -5\\ -2 & -5 & 4\\ -3 & 3 & -1\end{bmatrix}$$

10. Using properties of determinants show that:

$$\begin{vmatrix}1 & a & a^2+bc\\ 1 & b & b^2+ca\\ 1 & c & c^2+ab\end{vmatrix}=2(a-b)(b-c)(c-a)$$

OR

Using properties of determinants, solve for x:

$$\begin{vmatrix}a+x & a-x & a-x\\ a-x & a+x & a-x\\ a-x & a-x & a+x\end{vmatrix}=0$$

11. Find $\frac{dy}{dx}$ if $x^y+y^x=a^b$

12. If $y=\sin(\sin x)$, prove that $\frac{d^2y}{dx^2}+\tan x\frac{dy}{dx}+y\cos^2 x=0$.

13. Find $\frac{d^2y}{dx^2}$ if $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$

14. Find the absolute maximum value and absolute minimum value of the following function:

$$f(x)=3x^4-8x^3+12x^2-48x+1 \text{ in } [1,4]$$

15. Using Mean Value Theorem ,find a point on the parabola $y=(x-3)^2$,where the tangent is parallel to chord joining $(3,0)$ and $(4,1)$.

OR

The two equal sides of an isosceles triangle with fixed base *b* are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

16. Evaluate the following: $\int \cos^4 2x \, dx$

17. Evaluate the following: $\int (3x-2)\sqrt{x^2+x+1} \, dx$

OR

Evaluate the following: $\int (\sin^{-1} x)^2 \, dx$

18. Evaluate the following: $\int \frac{x^2+1}{(x^2+4)(x^2+25)} \, dx$

19. Evaluate the following integrals as limit of sum: $\int_0^3 (x^2+x)\,dx$

SECTION – C

20. (a) Write the following in the simplest form: $\cot^{-1}\left(\sqrt{1+x^2}-x\right)$

(b) Evaluate the following: $\tan\left(2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right)$

21. Two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate of Rs. x, Rs. y and Rs.z respectively per person. The first institution decided to award respectively 4, 3 and 2 employees with a total prize money of Rs. 37000 and the second institution decided to award respectively 5, 3 and 4 employees with a total prize money of Rs. 47000. If all the three prizes per person together amount to Rs. 12000, then using matrix method find the value of x, y and z.

What values are described in this question?

22. Find the intervals in which the function f given by $f(x) = \sin x - \cos x$, $x \in [0 , 2\pi]$ is strictly increasing or strictly decreasing.

OR

Find equation of normal at a point on the curve $x^2 = 4y$ which passes through $(1, 2)$. Also find equation of tangent.

23. If length of three sides of a trapezium other than base are equal to 10 cm, then find the area of the trapezium when it is maximum.

OR

Show that the maximum volume of the cylinder which can be inscribed in the sphere of radius $5\sqrt{3}$ cm is 500π cm^3.

24. Evaluate the following: $I = \int_0^{\pi/4} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

25. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the line $\frac{x}{4} + \frac{y}{3} = 1$.

26. A manufacturer produces two types of steel trunks. He has two machines A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type of trunk requires 3 hours on machine A and 2 hours on machine B. Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of Rs 30 and Rs 25 per trunk of the first type and second type respectively. How many trunks of each type must he make each day to make the maximum profit?

What should be the qualities of good machine?