FIRST TERMINAL EXAMINATION 2013-14

MATHEMATICS

Class XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

1. All questions are compulsory.

2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.

3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

SECTION – A

Q1. If A is a matrix given by $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$ and B is a matrix of order 4×3, write order of the matrix $(AB)^T$.

Q2. Find value(s) of x for which $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$.

Q3. Write the value of $\tan^{-1}\left(2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right)$.

Q4. If $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$, what is the value of $x+y+xy$.

Q5. If $y=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, write the value of $\frac{dy}{dx}$.

Q6. If $y=\log_5(\log x)$, what is the value of $\frac{dy}{dx}$.

Q7. Find the point(s) on the curve $y=3x^2-12x+5$ at which the tangent is parallel to x-axis.

Q8. What is the approximate change in the volume V of a cube of side *x* meters caused by increasing the side by 2%.

Q9. Evaluate: $\int \frac{\sin x}{1+\cos x}dx$

Q10. What is the value of $\int_{-\pi/4}^{\pi/4} \sin^3 x\, dx$

SECTION – B

Q11. If $A=\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$, show that $A^2-6A+17I = 0$. Hence find A^{-1}.

Q12. If $A=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, the prove that $A^n=\begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$; $n\in N$.

Q13. Using elementary transformation, find the inverse(if exist) of the matrix $A=\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$.

Q14. Using properties of determinants, prove the following:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

Q15. Write the following in the simplest form: $y=\cot^{-1}\left(\sqrt{1+x^2}-x\right)$.

Q16. Find $\frac{dy}{dx}$ if $(\cos x)^y=(\cos y)^x$.

Q17. If $x=a(\cos t+t\sin t)$ and $y=a(\sin t-t\cos t)$, find $\left(\frac{d^2y}{dx^2}\right)$.

Q18. If $y=\left(\cot^{-1}x\right)^2$, then show that $\left(x^2+1\right)^2\frac{d^2y}{dx^2}+2x\left(x^2+1\right)\frac{dy}{dx}=2$

OR

If $x\sqrt{1+y}+y\sqrt{1+x} = 0$, for $-1 < x < 1$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.

Q19. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is perpendicular to the line $5y - 15x = 13$.

OR

Discuss the applicability of Rolle's Theorem for the function $f(x) = x(x-3)^2$ on $[0\ ,\ 3]$.

Q20. Sand is being poured onto a conical pile at the constant rate of $50\,\text{cm}^3/\text{min}$ such that the height of the cone is always one half of the radius of its base. How fast is the height of the pile increasing when the sand is 5 cm deep.

Q21. Prove that $y = \dfrac{4\sin x}{2+\cos x} - x$ is an increasing function of x in $\left[0, \dfrac{\pi}{2}\right]$.

Q22. Evaluate the following: $\displaystyle\int \frac{2x+5}{\sqrt{7-6x-x^2}}\,dx$

OR

Evaluate the following: $\displaystyle\int \frac{dx}{(x-2)(x^2+1)}$

SECTION – C

Q23. Evaluate : $\displaystyle\int x(\log x)^2\,dx$

OR

Evaluate: $\displaystyle\int \frac{x^2-1}{x^4+1}\,dx$

Q24. If a, b, c are real numbers and $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$, show that either $a+b+c = 0$ or $a = b = c$.

Q25. Solve the following for x:

$$\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\frac{23}{36}$$

Q26. A school wants to award its students for the values Honesty, Regularity and Hard Work with a total cash award of Rs 6,000. Three times the award money for Hard Work added to that given for Honesty amounts to Rs 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, suggest one more value which the school must include for awards.

Q27. Using the properties of definite integral, evaluate the following: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$.

OR

Evaluate the following: $\int_0^{\pi/2} \left(\sqrt{\tan x}+\sqrt{\cot x}\right) dx$

Q28. Evaluate the following integral as limit of sums: $\int_0^{2} \left(3x^2-2x\right) dx$

Q29. Show that the right circular cylinder of given volume open at the top has minimum total surface area, provided its height is equal to the radius of its base.

OR

A window is in the form of an rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10m. Find the dimensions of the window to admit maximum light through the whole opening.