# Supplement Assignment

## Class XII Mathematics (Based on Book 1)

**(Questions not covered in the Assignment Booklet already given)**

### Inverse Trigonometry

**Q1.** Find value of : (a) $\sin^{-1}\left(\cos\left(\frac{43\pi}{5}\right)\right)$ (b) $\sin^{-1}\left(\sin\left(-600^0\right)\right)$

Ans: (a) $-\frac{\pi}{10}$ (b) $\frac{\pi}{3}$

**Q2.** (a) If $x+\frac{1}{x}=2$, then find the principal value of $\sin^{-1} x$

(b) If $3\tan^{-1} x+\cot^{-1} x = \pi$, What is the value of $x$?

(c) If $a \le 2\sin^{-1} x+\cos^{-1} x \le b$, then find value of $a$ and $b$

(d) If $\tan^{-1} x+\tan^{-1} y = \frac{4\pi}{5}$, then what is value of $\cot^{-1} x+\cot^{-1} y$ ?

(e) If $\tan^{-1} x+\tan^{-1} y=\frac{\pi}{4}$, what is the value of $x+y+xy$.

(f) Evaluate $\cos\left(2\cos^{-1} x+\sin^{-1} x\right)$ at $x=\frac{1}{5}$, where $0 \le \cos^{-1} x \le \pi$ and $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$.

Ans: (a) $\frac{\pi}{2}$ (b) 1 (c) $a=0, b=\pi$ (d) $\frac{\pi}{5}$ (e) 1 (f) $-\sqrt{\frac{24}{25}}$

**Q3.** Write the following in the simplest form:

(a) $\sin^{-1}\left(x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right)$ (b) $\sin^{-1}\left(\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right)$

Ans: (a) $\sin^{-1} x-\sin^{-1}\sqrt{x}$ (b) $\frac{\pi}{4}+\sin^{-1} x$

**Q4.** Prove that: (a) $\sin^{-1}\frac{1}{\sqrt{5}}+\sin^{-1}\frac{2}{\sqrt{5}}=\frac{\pi}{2}$ (b) $2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$

(c) $\cos\left(2\tan^{-1}\frac{1}{7}\right)=\sin\left(4\tan^{-1}\frac{1}{3}\right)$ (d) $\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)=7$

(e) $\cos\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)=\frac{6}{5\sqrt{13}}$ (f) $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\frac{1}{2}\cos^{-1}\frac{3}{5}=\frac{1}{2}\sin^{-1}\frac{4}{5}$

(g) $2\left(\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}\right)=\tan^{-1}\frac{4}{3}$

**Q5.** Solve the following equations:

(a) $\tan^{-1}(2+x)+\tan^{-1}(2-x)=\tan^{-1}\frac{2}{3}$ (b) $\tan^{-1}\left(\frac{x+1}{x-1}\right)+\tan^{-1}\left(\frac{x-1}{x}\right)=\tan^{-1}(-7)$

(c) $\tan\left(\cos^{-1}x\right)=\sin\left(\cot^{-1}\frac{1}{2}\right)$ (d) $\sin\left(2\cos^{-1}\left(\cot\left(2\tan^{-1}x\right)\right)\right)=0$

(e) $3\sin^{-1}\left(\frac{2x}{1+x^2}\right)-4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)+2\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{\pi}{3}$

(f) $\tan^{-1}\frac{1}{1+2x}+\tan^{-1}\frac{1}{4x+1}=\tan^{-1}\frac{2}{x^2}$

Ans: (a) $x=\pm 3$ (b) 2 (c) $\frac{\sqrt{5}}{3}$ (d) $x=\pm 1$ (e) $\frac{1}{\sqrt{3}}$ (f) $3,\ -\frac{2}{3}$



**Q6.** Simplify: $\cos^{-1}\left(\frac{3\cos x+4\sin x}{5}\right)$ Ans: $x-\tan^{-1}\left(\frac{4}{3}\right)$

**Q7.** Evaluate: (a) $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$ (b) $\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$ Ans: (a) $\frac{1}{\sqrt{10}}$ (b) $\frac{3-\sqrt{5}}{2}$

**Q8.** Evaluate the following:

(a) $\sin^{-1}(\sin 10)$ (b) $\sin^{-1}(\sin 5)$ (c) $\cos^{-1}(\cos 10)$ (d) $\tan^{-1}(\tan(-6))$

Ans: (a) $3\pi-10$ (b) $5-2\pi$ (c) $4\pi-10$ (d) $2\pi-6$

**Q9.** Prove that:

(a) $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$

(b) $\sec^2\left(\tan^{-1}2\right)+\operatorname{cosec}^2\left(\cot^{-1}3\right)=15$

**Q10.** Prove the following: $\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right)=0$.

**Q11.** If $y=\cot^{-1}\left(\sqrt{\cos x}\right)-\tan^{-1}\left(\sqrt{\cos x}\right)$, then prove that $\sin y=\tan^2\left(\frac{x}{2}\right)$.

## Matrices



**Q1.** Prove that the product of the matrices $\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$ and $\begin{bmatrix}\cos^2\phi & \cos\phi\sin\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$ is a null matrix, when $\theta$ and $\phi$ differ by an odd multiple of $\frac{\pi}{2}$.

**Q2.** Find the matrix $A$ such that $\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}A=\begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$. Ans: $\begin{bmatrix}1 & -2 & -5\\ 3 & 4 & 0\end{bmatrix}$

**Q3.** If $A=\begin{bmatrix}1 & 2 & x\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2 & y\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ and $AB=I_3$, what is the value of $x+y$? Ans: 0

**Q4.** If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$, $B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A+B)^2=A^2+B^2$, what is the value of $a$ and $b$?

Ans: $a=1, b=4$

**Q5.** If $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$, prove that $(A+B)(A-B)\neq A^2-B^2$

**Q6.** If $A=\begin{bmatrix}3 & -4\\ 1 & 1\\ 2 & 0\end{bmatrix}$ and $B=\begin{bmatrix}2 & 1 & 2\\ 1 & 2 & 4\end{bmatrix}$, then verify $(BA)^2\neq B^2A^2$.

**Q7.** Find the integral values of $x$, $\begin{bmatrix} x & 4 & -1 \end{bmatrix}\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 0 \\ 2 & 2 & 4 \end{bmatrix}\begin{bmatrix} x & 4 & -1 \end{bmatrix}^t = 0$. Ans: $x = -4$

**Q8.** If $\begin{bmatrix} 2x & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 8 \end{bmatrix} = 0$, find the value of $x$. Ans: $x = 0$, $-\frac{23}{2}$

**Q9.** If $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$, show that $B^T AB$ is a diagonal matrix.

**Q10.** For what value of $x$ the matrix $A = \begin{bmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{bmatrix}$ is singular? Ans: $-1$ and $2$

**Q11.** If $A^2 - A + I = 0$, then what is the inverse of $A$? Ans: $I - A$

**Q12.** If $A$ is a square matrix such that $A^2 = I$, then what is $A^{-1}$? Ans: $A$

**Q13.** If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then what is the value of $k$? Ans: $\frac{1}{9}$

**Q14.** If $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, find the value(s) of $x$ for which $A^2 = B$. Ans: no value

**Q15.** If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, find the matrix $B$ such that $A^2 = BA - 3I$ Ans: $B = \begin{bmatrix} 8 & -6 \\ -2 & 8 \end{bmatrix}$

## Determinants

**Q1.** Using properties of determinants prove that:

(a) $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

(b) $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ac)^3$

(c) $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

**Q2.** If $a+b+c\neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then prove that $a=b=c$

**Q3.** Given $A=\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, $B=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$' find $BA$ and use this to solve the system of equations

$$\begin{aligned} y+2z&=7 \\ x-y&=3 \\ 2x+3y+2z&=17 \end{aligned}.$$

Ans: $BA=6I$ ; $x=2, y=-1, z=4$

**Q4.** Solve the given system of equations:

$$\frac{2}{x}-\frac{3}{y}+\frac{3}{z}=10 \;;\; \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=10 \;;\; \frac{3}{x}-\frac{1}{y}+\frac{2}{z}=13$$

Ans: $x=\frac{1}{2}, y=\frac{1}{3}, z=\frac{1}{5}$



**Q5.** An amount of Rs 5000 is put into three investments at 6%, 7% and 8% per annum respectively. The total annual income from these investments is Rs 358. If the total annual income from first two investments is Rs 70 more than the income from the third, find the amount of each investment by the matrix method. Ans: Rs 1000, Rs 2200, Rs 1800

**Q6.** A school wants to award its students for the values Honesty, Regularity and Hard Work with a total cash award of Rs 6,000. Three times the award money for Hard Work added to that given for Honesty amounts to Rs 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, suggest one more value which the school must include for awards. Ans: Rs 500, Rs 2000 and Rs 3500

**Q7.** 10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students and the third group consists vigilant and obedient students. Double the number of students of the first group added to the number in second group gives 13, while the combined strength of first and second group is four times that of the third group. Using matrix method, find the number of students in each group. Apart from the values described above, suggest one more value, which in your opinion, the school should consider for awards. Ans: 5, 3 and 2

# Continuity and Differentiability

**Q1.** Show that $f(x)=\begin{cases}\dfrac{\sin x}{x} & ,x>0\\ 2 & ,x=0\\ \dfrac{4\left(1-\sqrt{1-x}\right)}{x} & ,x<0\end{cases}$ is continuous at $x=0$

**Q2.** If the following function $f(x)$ is continuous at $x=0$, find the values of $a, b$ and $c$.

$$f(x)=\begin{cases}\dfrac{\sin(a+1)x+\sin x}{x} & ,x<0\\ c & ,x=0\\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}} & ,x>0\end{cases}$$

Ans: $a=-\dfrac{3}{2}, c=\dfrac{1}{2}$

**Q3.** Discuss the continuity of the function $f(x)$ at $x=\dfrac{1}{2}$, where

$$f(x)=\begin{cases}\dfrac{1}{2}-x & ;0\le x<\dfrac{1}{2}\\ 1 & ;x=\dfrac{1}{2}\\ \dfrac{3}{2}-x & ;\dfrac{1}{2}<x\le 1\end{cases}$$

Ans: discontinuous

**Q4.** If $y=\sqrt{x^2+1}-\log\left(\dfrac{1}{x}+\sqrt{1+\dfrac{1}{x^2}}\right)$, find $\dfrac{dy}{dx}$. Ans: $\dfrac{\sqrt{x^2+1}}{x}$

**Q5.** If $y=\sin^{-1}\left(x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right)$, find $\dfrac{dy}{dx}$. Ans: $\dfrac{1}{\sqrt{1-x^2}}-\dfrac{1}{2\sqrt{x}\sqrt{1-x}}$

**Q6.** Differentiate $x^{\tan x}+\sqrt{\dfrac{x^2+1}{x}}$ w.r.t. $x$. Ans: $x^{\tan x}\left\{\dfrac{\tan x}{x}+\log x\sec^2 x\right\}+\dfrac{1}{2}\sqrt{\dfrac{x^2+1}{x}}\left(\dfrac{x^2-1}{x\left(x^2+1\right)}\right)$

**Q7.** If $y=\log\sin\sqrt{x^2+1}$, prove that $\dfrac{dy}{dx}=\dfrac{x\cot\sqrt{x^2+1}}{\sqrt{x^2+1}}$.

**Q8.** If $y=\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $\left(1-x^2\right)\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}-y=0$.

**Q9.** If $y = \log\tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$, show that $\dfrac{dy}{dx} - \sec x = 0$.

**Q10.** Differentiate $y = \tan^{-1}\left(\dfrac{1-x}{1+x}\right) - \tan^{-1}\left(\dfrac{x+2}{1-2x}\right)$ w.r.t. $x$ Ans: $-\dfrac{2}{1+x^2}$

**Q11.** If $y = \operatorname{cosec}^{-1}x$ , $x > 1$, prove that $x\left(x^2-1\right)\dfrac{d^2y}{dx^2} + \left(2x^2-1\right)\dfrac{dy}{dx} = 0$

**Q12.** If $y = x\log\left(\dfrac{x}{a+bx}\right)$, prove that $\dfrac{d^2y}{dx^2} = \dfrac{1}{x}\left(\dfrac{a}{a+bx}\right)^2$.

**Q13.** If $y = \log\left(\sqrt{\dfrac{1+\sin x}{1-\sin x}}\right)$, prove that $\dfrac{dy}{dx} = \sec x$

**Q14.** If $e^x + e^y = e^{x+y}$ , prove that $\dfrac{dy}{dx} + e^{y-x} = 0$

**Q15.** If $y = \left(x + \sqrt{x^2+1}\right)^m$ , show that $\left(x^2+1\right)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - m^2y = 0$

## Application of Derivatives

**Rate of change:**

**Q1.** The radius of a spherical soap bubble is increasing at the rate of 0.2 cm/sec. Find the rate of increase of its surface area, when the radius is 7 cm. Ans: $11.2\pi\ \text{cm}^2/\text{sec}$

**Q2.** A man 180 cm tall, walks at a rate of 2 m/sec away, from a source of light that is 9m above the ground. How fast is the length of his shadow increasing when he is 3m away from the base of light? Ans: 0.5 m/sec

**Q3.** The surface area of a spherical bubble is increasing at the rate of $2\,\text{cm}^2/\text{sec}$. When the radius of the bubble is 6 cm, at what rate is the volume of the bubble increasing? Ans: $6\,\text{cm}^3/\text{sec}$

**Q4.** Sand is being poured onto a conical pile at the constant rate of $50\,\text{cm}^3/\text{min}$ such that the height of the cone is always one half of the radius of its base. How fast is the height of the file increasing when the sand is 5 cm deep. Ans: $\dfrac{1}{2\pi}\text{cm}/\text{min}$

**Q5.** The volume of a spherical balloon is increasing at the rate of $25\,\text{cm}^3/\text{sec}$. Find the rate of change of its surface area at the instant when radius is 5 cm. Ans: $25\,\text{cm}^3/\text{sec}$

**Tangent and Normal**

**Q1.** Find the equation of tangent to the curve $y=(x^3-1)(x-2)$ at the points where the curve cuts the x-axis. Ans: $y+3x-3=0$ and $7x-y-14=0$

**Q2.** Find the coordinates of the points on the curve $y=x^2+3x+4$, the tangent at which pass through the origin. Ans: $(2,14)$ and $(-2,2)$

**Q3.** Find the equation of the tangent to the curve :

(a) $x=\sin 3t$ , $y=\cos 2t$ at $t=\frac{\pi}{4}$ Ans: $2\sqrt{2}\,x-3y-2=0$

(b) $4x^2+9y^2=36$ at $(3\cos\theta, 2\sin\theta)$ Ans: $2x\cos\theta+3y\sin\theta=6$

**Q4.** At what points will the tangents to the curve $y=2x^3-15x^2+36x-21$ be parallel to x-axis? Also, find the equations of tangents to the curve at these points. Ans: $(2,7)$ and $(3,6)$

**Q5.** Show that the curves $4x=y^2$ and $4xy=k$ cut at right angles if $k^2=512$.

**Increasing and decreasing function:**

Find the intervals on which the following functions are: (i) increasing (ii) decreasing:

**(a)** $f(x)=x^4-8x^3+22x^2-24x+21$ Ans: inc: $(1,2)\cup(3,\infty)$, dec: $(2,3)\cup(-\infty,1)$

**(b)** $f(x)=2\log(x-2)-x^2+4x+1$ Ans: inc: $(2,3)$, dec: $(3,\infty)$

**(c)** $f(x)=5x^3-15x^2-120x+3$ Ans: inc: $(-\infty,-2)\cup(4,\infty)$, dec: $(-2,4)$

**(d)** $f(x)=\frac{x^4}{4}+\frac{2}{3}x^3-\frac{5}{2}x^2-6x+7$ Ans:inc: $(-3,-1)\cup(2,\infty)$, dec: $(-\infty,-1)\cup(-1,2)$

**Word Problems (Based on Maxima and Minima)**

**1.** Find two positive numbers whose sum is 14 and the sum of whose squares is minimum. Ans: 7, 7

**2.** If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

**3.** Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

**4.** A wire of length 36 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum? Ans: $\frac{144}{\pi+4}m$ and $\frac{36\pi}{\pi+4}m$

**5.** Find the volume of the largest cylinder that can be inscribed in a sphere of radius r cm. Ans: $\frac{4\pi r^3}{3\sqrt{3}}$

**6.** Show that a cylinder of a given volume which is open at the top, has minimum total surface area, provided its height is equal to the radius of its base.

**7.** Find the point on the curve $y^2 = 2x$ which is at a minimum distance from point $(1,4)$. Ans: $(2,2)$

**8.** Show that the cone of the greatest volume which can be inscribed in a given sphere has an altitude equal to $\frac{2}{3}$ of the diameter of the sphere.

**9.** The sum of the surface areas of a sphere and a cube is given. Show that when the sum of their volumes is least, the diameter of the sphere is equal to the edge of the cube.

**10.** A given quantity of metal is to be cast into a half cylinder with a rectangular base and semi-circular ends. Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its semi-circular ends is $\frac{\pi}{\pi+2}$.

## Enrichment Problems (Not from CBSE point of view)

**Q1.** Prove that $2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\right)=\cos^{-1}\left(\frac{b+a\cos\theta}{a+b\cos\theta}\right)$.

**Q2.** If $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\theta$, prove that $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\theta+\frac{y^2}{b^2}=\sin^2\theta$.

**Q3.** If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$, prove that $x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}=2xyz$.

**Q4.** If $m\in N$ and $m\geq 2$, prove that $\begin{vmatrix} 1 & 1 & 1 \\ {}^{m}C_1 & {}^{m+1}C_1 & {}^{m+2}C_1 \\ {}^{m}C_2 & {}^{m+1}C_2 & {}^{m+2}C_2 \end{vmatrix}=1$.

**Q5.** Find the value of $\theta$ satisfying $\begin{vmatrix} 1 & 1 & \sin 3\theta \\ -4 & 3 & \cos 2\theta \\ 7 & -7 & -2 \end{vmatrix}=0$. Ans: $\theta=n\pi$ or $n\pi+(-1)^n\frac{\pi}{6}$; $n\in N$

**Q6.** If $f(x)=\begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} & if\ x<\frac{\pi}{2} \\ a & if\ x=\frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2} & if\ x>\frac{\pi}{2} \end{cases}$ is continous at $x=\frac{\pi}{2}$, find $a$ and $b$. Ans: $a=\frac{1}{2}$, $b=4$

**Q7.** If $y=\frac{ax^2}{(x-a)(x-b)(x-c)}+\frac{bx}{(x-b)(x-c)}+\frac{c}{x-c}+1$, prove that $\frac{y'}{y}=\frac{1}{x}\left(\frac{a}{a-x}+\frac{b}{b-x}+\frac{c}{c-x}\right)$.

**Q8.** It is given that for the function $f$ given by $f(x)=x^3+bx^2+ax$, $x\in[1,3]$ Rolle's Theorem holds with $c=2+\frac{1}{\sqrt{3}}$. Find the values of $a$ and $b$. Ans: $a=11$, $b=-6$

**Q9.** The curve $y=ax^3+bx^2+cx+5$ touches the x-axis at $P(-2,0)$ and cuts the y-axis at the point $Q$ where its gradient is 3. Find the equation of the curve. Ans: $y=-\frac{1}{2}x^3-\frac{3}{4}x^2+3x+5$

**Q10.** If $y=\frac{ax-b}{(x-1)(x-4)}$ has a turning point at $P(2,-1)$, find the values of $a$ and $b$ and show that $y$ is maximum at $P$. Ans: $a=1$, $b=0$