

Assignment Class XI

Complex Numbers and Quadratic Equations

Q1. Prove that: (i) $i^{104} + i^{109} + i^{114} + i^{119} = 0$ (ii) $(1+i)^4 (1+\frac{1}{i})^4 = 16$
(iii) $6i^{54} + 5i^{37} - 2i^{11} + 6i^{68} = 7i$ (iv) $\frac{1}{i} - \frac{1}{i^2} + \frac{1}{i^3} - \frac{1}{i^4} = 0$

Q2. Solve for x and y

(i) $(3+i)x + (1-2i)y + 7i = 0$ (ii) $\frac{(x-1)}{(3+i)} + \frac{(y-1)}{(3-i)} = i$ (iii) $(1+i)y^2 + (6+i) = (2+i)x$

Q3. Express the following in the form of $x+iy$:

(i) $\frac{(2+3i)^2}{1+i}$ (ii) $\frac{(1-i)^3}{1-i^3}$ (iii) $\frac{3-i}{2+i} + \frac{3+i}{2-i}$ (iv) $\frac{3+2i}{2-3i} + \frac{3-2i}{2+3i}$
(v) $\frac{3}{1+i} - \frac{2}{2-i} + \frac{2}{1-i}$ (vi) $(-1+\sqrt{3}i)^{-1}$ (vii) $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$ (viii) $\left(\frac{1}{1-2i} + \frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$

Q4. Find the multiplicative inverse of the following:

(i) $\frac{2+3i}{3-2i}$ (ii) $(2-5i)^2$ (iii) $(6+5i)^2$ (iv) $\frac{(i+1)(i+2)}{(i-1)(i-2)}$

Q5. Express the following numbers in the polar form:

(i) $1+\sqrt{3}i$ (ii) $-1-\sqrt{3}i$ (iii) $-4+4\sqrt{3}i$ (iv) $\frac{5-i}{2-3i}$
(v) $\frac{1-3i}{1+2i}$ (vi) $\frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$ (vii) $-3\sqrt{2}+3\sqrt{2}i$

Q 6. Find the square roots of the following complex numbers:

(i) $3+4i$ (ii) $5+12i$ (iii) $7+24i$ (iv) $12-5i$
(v) $8-15i$ (vi) $16-30i$ (vii) $-48-14i$ (viii) $-11-60\sqrt{-1}$

Q7. (i) If $x=3+2i$, find the value of $x^4 - 4x^3 + 4x^2 + 8x + 44$.

(ii) If $x = \frac{1+i}{\sqrt{2}}$, find the value of $x^6 + x^4 + x^2 + 1$.

(iii) If $x=4+\sqrt{7}i$ find value of $x^3 - 4x^2 - 9x + 97$

Q8. Show that if $\left| \frac{z-5i}{z+5i} \right| = 1$, then z is a real number.

Q9. If $a+ib = \frac{c+i}{c-i}$, where a and b are real, prove that $a^2 + b^2 = 1$ and $\frac{b}{a} = \frac{2c}{c^2 - 1}$.

Q10. For complex values of z , solve $|z| + z = 2 + i$

Q11. Show that $\frac{\sqrt{7} + i\sqrt{3}}{\sqrt{7} - i\sqrt{3}} + \frac{\sqrt{7} - i\sqrt{3}}{\sqrt{7} + i\sqrt{3}}$ is real.

Q12. Show that: $\frac{(1+i)(3+i)}{(3-i)} - \frac{(1-i)(3-i)}{(3+i)} = \frac{14}{5}i$

Q13. Find the values of x and y for which the complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate of each other.

Q14. If z is a complex number such that $|z| = 1$, prove that $\left(\frac{z-1}{z+1} \right)$ is purely imaginary.

Q15. Find all non-zero complex numbers z satisfying $\bar{z} = iz^2$.

Q16. Show that the locus of a complex variable z , satisfying $\left| \frac{z-3}{z+3} \right| = 2$, is a circle.

Q17. If $x = -5 + 2\sqrt{-4}$, then find the value of $x^4 + 9x^3 + 35x^2 - x + 4$.

Q18. Find the value of $x^3 + 7x^2 - x + 16$, when $x = 1 + 2i$.

Q19. If $z = 2 - 3i$, show that $z^2 - 4z + 13 = 0$ and hence find the value of $4z^3 - 3z^2 + 169$.

Q20. Evaluate: $\sqrt{4 + 3\sqrt{-20}} + \sqrt{4 - 3\sqrt{-20}}$

Quadratic Equations:

Q21. Solve for x :

(i) $9x^2 + 10x + 3 = 0$

(ii) $2x^2 - 4x + 3 = 0$

(iii) $25x^2 - 30x + 11 = 0$

(iv) $x^2 + 3ix + 10 = 0$

(v) $2x^2 + 3ix + 2 = 0$

(vi) $x^2 - (5-i)x + (18+i) = 0$

(vii) $x^2 - (5+i)x + (18-i) = 0$

(viii) $x^2 - (2+i)x + 1 - 7i = 0$

(ix) $2x^2 - (3+7i)x + (9i-3) = 0$

Q22. Solve for x :

(a) $3x^2 + 8ix + 3 = 0$

(b) $9x^2 - 12x + 20 = 0$

(c) $x^2 - 5ix - 6 = 0$

(d) $x^2 + 4ix - 4 = 0$

(e) $x^2 - \sqrt{2}ix + 12 = 0$

(f) $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

ANSWERS

(Complex Numbers and Quadratic Equations)

2. (i) $x=-1, y=3$ (ii) $x=-4, y=6$ (iii) $(x=5, y=2)$ or $(x=5, y=-2)$

3. (i) $\frac{7}{2} + \frac{17}{2}i$ (ii) $-2 + 0i$ (iii) $2 + 0i$ (iv) $0 + 0i$ (v) $\frac{17}{10} - \frac{9}{10}i$

(vi) $-\frac{1}{4} + \left(-\frac{\sqrt{3}}{4}\right)i$ (vii) $1 + 2\sqrt{2}i$ (viii) $\frac{1}{4} + \frac{9}{4}i$

4. (i) $-i$ (ii) $\frac{-21}{841} + \frac{20}{841}i$ (iii) $\frac{11}{3721} - \frac{60}{3721}i$ (iv) $\frac{-4}{5} - \frac{3}{5}i$

5. (i) $2 \left[\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right]$ (ii) $2 \left[\cos \left(\frac{-2\pi}{3} \right) + i \sin \left(\frac{-2\pi}{3} \right) \right]$ (iii) $8 \left[\cos \left(\frac{2\pi}{3} \right) + i \sin \left(\frac{2\pi}{3} \right) \right]$

(iv) $\sqrt{2} \left[\cos \left(\frac{\pi}{4} \right) + i \sin \left(\frac{\pi}{4} \right) \right]$ (v) $\sqrt{2} \left[\cos \left(\frac{3\pi}{4} \right) - i \sin \left(\frac{3\pi}{4} \right) \right]$ (vi) $2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$

(vii) $6 \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$

6. (i) $\pm(2+i)$ (ii) $\pm(3+2i)$ (iii) $\pm(4+3i)$ (iv) $\pm \frac{1}{\sqrt{2}}(5-i)$

(v) $\pm \frac{1}{\sqrt{2}}(5-3i)$ (vi) $\pm(5-3i)$ (vii) $\pm(1-7i)$ (viii) $\pm(5-6i)$

7. (i) 5 (ii) 0 10. $z = \frac{3}{4} + i$ 13. $x=1, y=-4$ or $x=-1, y=-4$

15. $0, i, \frac{\sqrt{3}}{2} - \frac{1}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}i$ 17. -160 18. $-17 + 24i$ 19. 0 20. ± 6

21. (i) $\frac{-5 + \sqrt{2}i}{9}, \frac{-5 - \sqrt{2}i}{9}$ (ii) $1 + \frac{1}{\sqrt{2}}i, 1 - \frac{1}{\sqrt{2}}i$ (iii) $\frac{3 + \sqrt{2}i}{5}, \frac{3 - \sqrt{2}i}{5}$ (iv) $2i, -5i$

(v) $\frac{1}{2}i, -2i$ (vi) $3-4i, 2+3i$ (vii) $3+4i, 2-3i$ (viii) $3-i, -1+2i$ (ix) $3i, \frac{3+i}{2}$

22. (a) $\frac{1}{3}i, -3i$ (b) $\frac{2 \pm 4i}{3}$ (c) $3i, 2i$ (d) $-2i$

(e) $-2\sqrt{2}i, 3\sqrt{2}i$ (f) $2i, 3\sqrt{2}$