MATHEMATICS FORMULA LIST

FOR CLASS XII (For I Term Paper)

Factoring Formulas

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$$a^2 - b^2 = (a-b)(a+b)$$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Area and Volume

Circle: $C = 2\pi r = \pi D$, where C is circumference, r is radius and D is diameter

$A = \pi r^2$, where A is the area

Triangle: $A = \frac{1}{2} bh$, where b is the base and h is the perpendicular height

$A = \sqrt{s(s-a)(s-b)(s-c)}$; where $s = \frac{a+b+c}{2}$ (Heron's Formula)

Equilateral Triangle $A = \frac{\sqrt{3}}{4}(\text{side})^2$

Parallelogram: $A = \text{base} \times \text{corresponding height}$

Square $A = (\text{side})^2$; Perimeter = 4 x side

Rectangle $A = lb$; Perimeter = $2(l+b)$

Rhombus $A = \frac{1}{2} d_1 d_2$

Trapezium: $A = \frac{1}{2}(a+b)h$, where a and b are the lengths of the parallel sides and h is the perpendicular height

Cuboid $(\text{length} = l, \text{breadth} = b, \text{height} = h)$

$(i) V = lbh$ $\quad (ii) CSA = 2h(l+b)$ $\quad (iii) TSA = 2(lb+bh+lh)$ $\quad (iv) Diagonal = \sqrt{l^2+b^2+h^2}$

Cube $(\text{side} = a)$

$(i) V = a^3$ $\quad (ii) CSA = 4a^2$ $\quad (iii) TSA = 6a^2$ $\quad (iv) Diagonal = \sqrt{3}\,a$

Cylinder $(\text{radius} = r, \text{height} = h)$

$(i) V = \pi r^2 h$ $\quad (ii) CSA = 2\pi rh$ $\quad (iii) TSA = 2\pi r(r+h)$

Cone $(\text{radius} = r, \text{height} = h, \text{ slant height} = l)$

$(i) V = \frac{1}{3}\pi r^2 h$ $\quad (ii) CSA = \pi rl$ $\quad (iii) TSA = \pi r(r+l)$ $\quad (iv) l = \sqrt{r^2+h^2}$

Sphere $(\text{radius} = r)$

$(i) V = \frac{4}{3}\pi r^3$ $\quad (ii) A = 4\pi r^2$

Hemi-Sphere $(\text{radius} = r)$

$(i) V = \frac{2}{3}\pi r^3$ $\quad (ii) CSA = 3\pi r^2$ $\quad (iii) TSA = 4\pi r^2$

Polygon

Sum of all the angles in a n-sided polygon : $180^0 \times (n-2)$

Each angles of a n-sided regular polygon : $\frac{180^0 \times (n-2)}{n}$

Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of roots $= -\frac{b}{a}$; Product of roots $= \frac{c}{a}$

Logarithmic Function

$\log_a x = y \Leftrightarrow x = a^y$; $x > 0, a > 0, a \neq 1$

(i) $\log_a 1 = 0$ $\quad (ii) \log_a a = 1$

$(iii) \log_a(xy) = \log_a x + \log_a y$ $\quad (iv) \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$(v) \log_a x^n = n\log_a x$ $\quad (vi) \log_{a^n} x^m = \frac{m}{n}\log_a x$

$(vii) \log_a x = \frac{1}{\log_x a}$ $\quad (viii) \log_b a = \frac{\log_c a}{\log_c b}$

(ix) If $a > 1$ then $x < y \Leftrightarrow \log_a x < \log_a y$

(x) If $0 < a < 1$ then $x < y \Leftrightarrow \log_a x > \log_a y$

Trigonometry

$180^0 = \pi$ radians ; $\theta = \dfrac{l}{r}$, θ is measured in radians

Trigonometric Ratios of Special Angles

$\cos 45° = \dfrac{1}{\sqrt{2}}$ $\cos 60° = \dfrac{1}{2}$ $\cos 30° = \dfrac{\sqrt{3}}{2}$

$\sin 45° = \dfrac{1}{\sqrt{2}}$ $\sin 60° = \dfrac{\sqrt{3}}{2}$ $\sin 30° = \dfrac{1}{2}$

$\tan 45° = 1$ $\tan 60° = \sqrt{3}$ $\tan 30° = \dfrac{1}{\sqrt{3}}$

$\sin(90° - \theta) = \cos\theta$; $\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$; $\cot(90^0 - \theta) = \tan\theta$

$\sec(90° - \theta) = \cos ec\theta$; $\text{cosec}(90° - \theta) = \sec\theta$

$\cos(-\theta) = \cos\theta$; $\sin(-\theta) = -\sin\theta$; $\tan(-\theta) = -\tan\theta$

$\sec\theta = \dfrac{1}{\cos\theta}$; $\text{cosec}\theta = \dfrac{1}{\sin\theta}$; $\cot\theta = \dfrac{1}{\tan\theta}$; $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$; $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$; $1 + \tan^2\theta = \sec^2\theta$; $1 + \cot^2\theta = \cos ec^2\theta$

$-1 \le \sin x \le 1$; $-1 \le \sin x \le 1$; $-\infty < \tan x < \infty$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$; $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$; $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$; $\tan(A-B) = \dfrac{\tan A + \tan B}{1 + \tan A\tan B}$

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$; $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$; $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$; $\sin A - \sin B = 2\cos\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$

$\cos A + \cos B = 2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$; $\cos A - \cos B = -2\sin\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$

$$\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$$

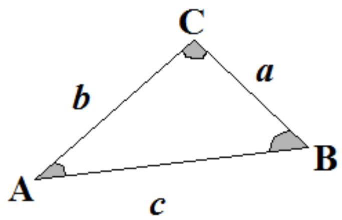
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$\sin A = \pm\sqrt{\dfrac{1-\cos 2A}{2}}$; $\cos A = \pm\sqrt{\dfrac{1+\cos 2A}{2}}$

$\sin 3A = 3\sin A - 4\sin^3 A$; $\cos 3A = 4\cos^3 A - 3\cos A$; $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

$$\cos A \cos 2A \cos 2^2 A \cos 2^3 A \ldots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$$



Area of triangle formula:

$$= \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B$$

	Trigonometric Equation	General Solution
(i)	$\sin\theta = 0$	$\theta = n\pi$, $n \in Z$
(ii)	$\cos\theta = 0$	$\theta = (2n+1)\dfrac{\pi}{2}$, $n \in Z$
(iii)	$\tan\theta = 0$	$\theta = n\pi$, $n \in Z$
(iv)	$\sin\theta = \sin\alpha$	$\theta = n\pi + (-1)^n \alpha$, $n \in Z$
(v)	$\cos\theta = \cos\alpha$	$\theta = 2n\pi \pm \alpha$, $n \in Z$
(vi)	$\tan\theta = \tan\alpha$	$\theta = n\pi + \alpha$, $n \in Z$
(vii)	$\left.\begin{array}{l}\sin^2\theta = \sin^2\alpha \\ \cos^2\theta = \cos^2\alpha \\ \tan^2\theta = \tan^2\alpha\end{array}\right\}$	$\theta = n\pi \pm \alpha$, $n \in Z$

$\sin n\pi = 0$; $\cos n\pi = (-1)^n$; $\tan n\pi = 0$

Inverse Trigonometry

Inverse Function	**Domain**	**Range**
$\sin^{-1}$	$[-1, 1]$	$\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$
$\cos^{-1}$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}$	R	$\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$
$\operatorname{cosec}^{-1}$	$R-(-1, 1)$	$\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right] - \{0\}$
$\sec^{-1}$	$R-(-1, 1)$	$[0, \pi] - \left\{\dfrac{\pi}{2}\right\}$
$\cot^{-1}$	R	$(0, \pi)$

$\sin^{-1}(-x) = -\sin^{-1}(x)$ for all $x \in [-1, 1]$

$\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ for all $x \in [-1, 1]$

$\tan^{-1}(-x) = -\tan^{-1}(x)$ for all $x \in R$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x)$ for all $x \in [-\infty, -1] \cup [1, \infty)$

$\sec^{-1}(-x) = \pi - \sec^{-1}(x)$ for all $x \in [-\infty, -1] \cup [1, \infty)$

$\cot^{-1}(-x) = \pi - \cot^{-1}(x)$ for all $x \in R$

$$\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1}(x) \quad ; \quad \cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1}(x) \quad ; \quad \tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1}(x)$$

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \quad ; \quad \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \quad ; \quad \sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$$

$$\sin^{-1}x \pm \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right)$$

$$\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\left(\dfrac{x+y}{1-xy}\right) & \text{, if } xy < 1 \\ \pi + \tan^{-1}\left(\dfrac{x+y}{1-xy}\right) & \text{, if } xy > 1 \end{cases}$$

$$\tan^{-1}x - \tan^{-1}y = \begin{cases} \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) & \text{, if } xy > -1 \\ \pi + \tan^{-1}\left(\dfrac{x-y}{1+xy}\right) & \text{, if } xy < -1 \end{cases}$$

$$2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right) \quad ; \quad 2\cos^{-1}x = \cos^{-1}\left(2x^2 - 1\right)$$

$$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Matrices

A matrix in which number of rows is equal to number of columns, say n is known as **square matrix** of order n.

Properties of **Transpose of a Matrix**:

$(i)\ \left(A^T\right)^T = A \qquad (ii)\ \left(A+B\right)^T = A^T + B^T \qquad (iii)\left(kA\right)^T = kA^T$

$(iv)\left(AB\right)^T = B^T A^T \qquad (v)\left(ABC\right)^T = C^T B^T A^T$

A square matrix $A=\left[a_{ij}\right]$ is called a **symmetric matrix**, if $a_{ij} = a_{ji}$ for all $i, j \iff A^T = A$.

A square matrix $A=\left[a_{ij}\right]$ is called a **skew-symmetric matrix**, if $a_{ij} = -a_{ji}$ for all $i, j \iff A^T = -A$.

All main diagonal elements of a skew-symmetric matrix are zero.

Every square matrix can be uniquely expressed as the sum of symmetric and skew-symmetric matrix.

Determinants

A square matrix A is a **singular matrix** if $|A| = 0$

For any square matrix A, the |A| satisfy following properties.

(a) |AB| = |A| B|

(b) If we interchange any two rows (or columns), then sign of determinant changes.

(c) If any two rows or any two columns are identical or proportional, then value of determinant is zero.

(d) If we multiply each element of a row or a column of a determinant by constant *k*, then value of determinant is multiplied by *k*.

(e) Multiplying a determinant by *k* means multiply elements of only one row (or one column) by *k*.

(f) If elements of a row or a column in a determinant can be expressed as sum of two or more elements, then the given determinant can be expressed as sum of two or more determinants.

(g) If to each element of a row or a column of a determinant the equimultiples of corresponding elements of other rows or columns are added, then value of determinant remains same.

Adjoint of a matrix A is the transpose of a cofactor matrix.

If A and B are square matrices of the same order n, then:

(a) $A(adj\ A) = |A| I_n = (adj\ A) A$

(b) $adj(AB) = (adj\ B)(adj\ A)$

(c) $adj(A)^T = (adj\ A)^T$

(d) $|adj\ A| = |A|^{n-1}$

(e) $adj(adj\ A) = |A|^{n-2} A$

(f) $|adj(adj\ A)| = |A|^{(n-1)^2}$

A square matrix A of order n is **invertible** if there exists a square matrix B of the same order such that $AB = I_n = BA$. We write, $A^{-1} = B$.

Properties of inverse of a matrix:

(a) Every invertible matrix possesses a unique inverse.

(a) $(A^{-1})^{-1} = A$

(c) $(AB)^{-1} = B^{-1}A^{-1}$

$|A^{-1}| = \frac{1}{|A|}$

(e) $(A^T)^{-1} = (A^{-1})^T$

(f) $A^{-1} = \frac{1}{|A|} adj(A)$

A system $AX = B$ of n linear equations has a unique solution given by $X = A^{-1}B$, if $|A| \neq 0$.

If $|A| = 0$ and $(adjA)B = 0$, then the system is consistent and has infinitely many solutions.

$|A| = 0$ and $(adjA)B \neq 0$, then the system is inconsistent.

Continuity and Differentiability

Limits

$\lim_{x\to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$; $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$; $\lim_{x\to 0} \dfrac{1 - \cos x}{x} = 0$

$\lim_{x\to 0} \dfrac{\tan x}{x} = 1$; $\lim_{x\to 0} \dfrac{e^x - 1}{x} = 1$; $\lim_{x\to 0} \dfrac{\log(1+x)}{x} = 1$

A function $f(x)$ is **continuous** at $x = a$ if $\lim_{x\to a} f(x) = f(a)$ i.e. $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$.

Following functions are continuous everywhere:

(a) Constant function (b) Identity function (c) Polynomial function

(d) Modulus function (e) Exponential function (f) Sine & Cosine functions

Following functions are continuous in their domains:

(a) Logarithmic function (b) Rational function

(c) Tan, Cot, Sec & Cosec functions (d) all inverse trigonometric functions

If f is continuous function then $|f|$ and $\dfrac{1}{f}$ are continuous in their domains.

A function $f(x)$ is **differentiable** at $x = a$ if $\lim_{x\to a} \dfrac{f(x) - f(a)}{x - a}$ exists finitely i.e.

$$\lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{f(a-h) - f(a)}{-h}.$$

Every differentiable function is continuous but, converse is not true.

Following functions are differentiable everywhere/their defined domain:

(a) Polynomial function (b) exponential function (c) constant function

(d) Logarithmic function (e) trigonometric & inverse trigonometric functions

The sum, difference, product, quotient and composition of two differentiable functions is differentiable.

Some Standard Derivatives:

(i) $\dfrac{d}{dx}\left(x^n\right) = n\,x^{n-1}$ (ii) $\dfrac{d}{dx}\left(\log_e x\right) = \dfrac{1}{x}$ (iii) $\dfrac{d}{dx}\left(e^x\right) = e^x$

(iv) $\dfrac{d}{dx}\left(a^x\right) = a^x \log_e a$ (v) $\dfrac{d}{dx}(\sin x) = \cos x$ (vii) $\dfrac{d}{dx}(\cos x) = \sin x$

(viii) $\dfrac{d}{dx}(\tan x) = \sec^2 x$ (ix) $\dfrac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ (x) $\dfrac{d}{dx}(\sec x) = \sec x \tan x$

(xi) $\dfrac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

$(xii)\ \frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}}$ $\quad (xiii)\ \frac{d}{dx}\left(\cos^{-1}x\right)=-\frac{1}{\sqrt{1-x^2}}$ $\quad (xiv)\ \frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$

$(xv)\ \frac{d}{dx}\left(\cot^{-1}x\right)=-\frac{1}{1+x^2}$ $\quad (xvi)\ \frac{d}{dx}\left(\sec^{-1}x\right)=\frac{1}{|x|\sqrt{x^2-1}}$ $\quad (xvii)\ \frac{d}{dx}\left(\operatorname{cosec}^{-1}x\right)=-\frac{1}{|x|\sqrt{x^2-1}}$

Following are some substitutions useful in finding derivatives:

Expression	Substitution
a^2+x^2	$x=a\tan\theta$ or $a\cot\theta$
$1+x^2$	$x=\tan\theta$ or $\cot\theta$
a^2-x^2	$x=a\sin\theta$ or $a\cos\theta$
$1-x^2$	$x=\sin\theta$ or $\cos\theta$
x^2-a^2	$x=a\sec\theta$ or $a\operatorname{cosec}\theta$
x^2-1	$x=\sec\theta$ or $\operatorname{cosec}\theta$
$\frac{a-x}{a+x}$; $\frac{a+x}{a-x}$; $\sqrt{\frac{a-x}{a+x}}$; $\sqrt{\frac{a+x}{a-x}}$	$x=a\cos 2\theta$

Chain rule:

If $z=f(y)$ and $y=g(x)$, then $\frac{dz}{dx}=\frac{dz}{dy}.\frac{dy}{dx}$

Product rule:

$y=uv$ then $\frac{dy}{dx}=\frac{du}{dx}v+u\frac{dv}{dx}$

Quotient rule:

If $y=\frac{u}{v}$ then $\frac{dy}{dx}=\frac{\frac{du}{dx}v-u\frac{dv}{dx}}{v^2}$

If $x=f(t)$ and $y=g(t)$, then $\frac{d^2y}{dx^2}=\frac{d}{dt}\left\{\frac{g'(t)}{f'(t)}\right\}.\frac{dt}{dx}=\frac{f'(t)g''(t)-g'(t)f''(t)}{\{f'(t)\}^3}$

Applications of Derivatives

Rolle's Theorem:

Let f be a real valued function defined on $[a,b]$ such that:

(a) continuous on $[a,b]$ (b) differentiable on (a,b) (c) $f(a)=f(b)$

then, there exist a real number $c\in(a,b)$ such that $f'(c)=0$.

Mean Value Theorem:

Let f be a real valued function defined on $[a,b]$ such that:

(a) continuous on $[a,b]$ (b) differentiable on (a,b)

then, there exist a real number $c\in(a,b)$ such that $f'(c)=\frac{f(b)-f(a)}{b-a}$.

Tangents and Normals

If $y = f(x)$, then $\left(\frac{dy}{dx}\right)_P$ is slope of tangent to $y = f(x)$ at a point P.

If $y = f(x)$, then $-\frac{1}{\left(\frac{dy}{dx}\right)_P}$ is slope of normal to $y = f(x)$ at a point P.

If tangent is parallel to x-axis, then $\frac{dy}{dx} = 0$; If tangent is parallel to y-axis, then $\frac{dx}{dy} = 0$

If $P(x_1, y_1)$ is a point on the curve $y = f(x)$, then:

Equation of tangent at P is $y - y_1 = \left(\frac{dy}{dx}\right)_P (x - x_1)$

Equation of normal at P is $y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)_P}(x - x_1)$

The angle between the tangents to two given curves at their point of intersection is defined as the angle of intersection of two curves.

Approximations:

Let $y = f(x)$, Δx be a small increment in x and Δy be the increment in y corresponding to the increment in x, i.e., $\Delta y = f(x + \Delta x) - f(x)$. Then $\Delta y = \frac{dy}{dx}\Delta x$

Also, $f(x+\Delta x) = f(x) + f'(x)\Delta x$

Increasing and Decreasing Function

A function f is said to be:

(a) Increasing on (a,b) if $x_1 < x_2$ in $(a,b) \Rightarrow f(x_1) \le f(x_2)$ for all $x_1, x_2 \in (a,b)$.

Alternatively, $f'(x) \ge 0$ for each x in (a,b)

(b) Decreasing on (a,b) if $x_1 < x_2$ in $(a,b) \Rightarrow f(x_1) \ge f(x_2)$ for all $x_1, x_2 \in (a,b)$.

Alternatively, $f'(x) \le 0$ for each x in (a,b)

(c) Strictly increasing on (a,b) if

$x_1 < x_2$ in $(a,b) \Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in (a,b)$. Alternatively,

$f'(x) > 0$ for each x in (a,b).

(d) Strictly decreasing on (a, b) if

$x_1 < x_2$ in $(a, b) \Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in (a, b)$. Alternatively,

$f'(x) < 0$ for each x in (a, b)

A function f is **monotonic** on (a, b) if it is strictly increasing or strictly decreasing on (a, b).

A point c in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable is called a **critical point**.

Maxima and Minima

First Derivative Test:

Given a curve $y = \text{f}(x)$,

(a) For the stationary point at $x = a$,

(i) if $\frac{dy}{dx}$ changes sign from **negative** to **positive** as x increases through a, the point S is a minimum point,

(ii) if $\frac{dy}{dx}$ changes sign from **positive** to **negative** as x increases through a, the point S is a maximum point,

(iii) if $\frac{dy}{dx}$ does not change sign as x increase through a, the point S is a **point of inflexion**.

(b) A stationary point is called a **turning point** if it is either a maximum point or a minimum point.

Second Derivative Test

Given a curve $y = \text{f}(x)$,

(a) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} \neq 0$ at $x = a \Rightarrow S(a, \text{f}(a))$ is a turning point.

(i) If $\frac{d^2y}{dx^2} > 0$, then S is a **minimum** point. (ii) If $\frac{d^2y}{dx^2} < 0$, then S is a **maximum** point.

(b) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ at $x = a$, go back to First Derivative Test.

Working rule for finding **absolute maxima and/or absolute minima**:

Step1: Find all critical points of f in the given interval.

Step 2: Take end points of the interval.

Step 3: At all these points (listed in step 1 and 2), calculate the values of f.

Step 4: Identify the maximum and minimum values of f out of values calculated in step 3.

<u>Integrals</u>

$(i)\int x^n\,dx = \frac{x^{n+1}}{n+1} + c\ ;\ n \neq -1$ $\quad (ii)\int \frac{1}{x}dx = \log_e x + c$ $\quad (iii)\int a^x dx = \frac{a^x}{\log_e a} + c$

$(iv)\int \frac{1}{x^2}\,dx = -\frac{1}{x} + c$ $\quad (v)\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + c$ $\quad (vi)\int \sqrt{x}\,dx = \frac{2}{3}x^{3/2} + c$

$(vii)\int \sin x\,dx = -\cos x + c$ $\quad (viii)\int \cos x\,dx = \sin x + c$ $\quad (ix)\int \sec^2 x\,dx = \tan x + c$

$(x)\int \cos ec^2 x\,dx = -\cot x + c$ $\quad (x)\int \sec x \tan x\,dx = \sec x + c$ $\quad (xi)\int \cos ecx \cot x\,dx = -\cos ecx + c$

$(xii)\int \tan x\,dx = \log|\sec x| + c$ $\quad (xiii)\int \cot x\,dx = \log|\sin x| + c$

$(xiv)\int \sec x\,dx = \log|\sec x + \tan x| + c = \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$

$(xv)\int \cos ecx\,dx = \log|\text{cosec}\,x - \cot x| + c = \log\left|\tan\frac{x}{2}\right| + c$

$(xvi)\int \frac{1}{x^2 - a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$ $\quad (xvii)\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

$(xviii)\int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$ $\quad (xix)\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c$

$(xx)\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$ $\quad (xxi)\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c$

$(xxii)\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

$(xxiii)\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

$(xxiv)\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$

(xxv) If u and v are two functions of x, then $\int uv\,dx = u\int v dx - \int\left\{\frac{du}{dx}\int v dx\right\}dx$

i.e. (first function) x (integral of second function) – integral of {(derivative of first function) x (integral of second function)}

We can choose the first function as the function which comes first in the word **ILATE**, where I stands for inverse trigonometric functions, L for logarithmic functions, A for algebraic functions, T for trigonometric functions and E for exponential function.

$(xxvi)\int \left[f(x) + f'(x)\right]e^x\,dx = e^x f(x) + c$

Integration by **Partial fraction** of Rational Function of the form $\frac{P(x)}{Q(x)}$:

If degree of $P(x) \geq$ degree of $Q(x)$, then divide $P(x)$ by $Q(x)$

Form	Partial Fraction
$(i)\ \frac{px+q}{(x-a)(x-b)}$	$\frac{A}{(x-a)}+\frac{B}{(x-b)}$
$(ii)\ \frac{px+q}{(x-a)^2}$	$\frac{A}{(x-a)}+\frac{B}{(x-a)^2}$
$(iii)\ \frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{(x-a)}+\frac{B}{(x-b)}+\frac{C}{(x-c)}$
$(iv)\ \frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{(x-a)}+\frac{B}{(x-a)^2}+\frac{C}{(x-b)}$
$(v)\ \frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{(x-a)}+\frac{Bx+C}{x^2+bx+c}$ here, x^2+bx+c can't be factorised.

For Integrals of the form $\int \frac{dx}{ax^2+bx+c}$ or $\int \frac{dx}{\sqrt{ax^2+bx+c}}$ use completing the square method and then applying formulas xvi to xxi.

For Integrals of the form $\int \frac{px+q}{ax^2+bx+c}dx$ or $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$ or $\int (px+q)\sqrt{ax^2+bx+c}\,dx$,

write $px+q = A\frac{d}{dx}(ax^2+bx+c)+B$ where A and B are determined by comparing coefficients on both sides.

For Integrals of the form $\int \frac{1}{a+b\sin^2 x}dx$ or $\int \frac{1}{a+b\cos^2 x}dx$ or $\int \frac{1}{a\sin^2 x+b\cos^2 x}dx$ or $\int \frac{1}{(a\sin x+b\cos x)^2}dx$ or $\int \frac{1}{a+b\sin 2x}dx$ or $\int \frac{1}{a+b\cos 2x}dx$

Algorithm:

Step 1: Divide numerator and denominator by $\cos^2 x$

Step 2: Replace $\sec^2 x$, if any, in denominator by $1+\tan^2 x$

Step 3: Put $\tan x = t$ so that $\sec^2 x\,dx = dt$. This will reduce the integral in form $\int \frac{1}{at^2+bt+c}dx$

Step 4: Evaluate the integral now using completing the square method.

For Integrals of the form $\int \frac{1}{a+b\sin x}dx$ or $\int \frac{1}{a+b\cos x}dx$ or $\int \frac{1}{a\sin x+b\cos x}dx$

Algorithm:

Put $\sin x = \frac{2\tan x/2}{1+\tan^2 x/2}$, $\cos x = \frac{1-\tan^2 x/2}{1+\tan^2 x/2}$

For Integrals of the form $\int \frac{a\sin x+b\cos x}{c\sin x+d\cos x}dx$.

Algorithm:

Put $\text{Numerator} = A(\text{Denominator}) + B(\text{Derivative of Denominator})$

For Integrals of the form $\int \frac{x^2 \pm 1}{x^4+\lambda x^2+1}dx$ or $\int \frac{1}{x^4+\lambda x^2+1}dx$ or $\int \sqrt{\tan x}\, dx$ or $\int \sqrt{\cot x}\, dx$

Algorithm:

Step1: Divide numerator and denominator by x^2.

Step 2: Express the denominator in the form $\left(x+\frac{1}{x}\right)^2 \pm k^2$

Step 3: Introduce $d\left(x+\frac{1}{x}\right)$ or $d\left(x-\frac{1}{x}\right)$ in the numerator.

Step 4: Substitute $x+\frac{1}{x}=t$ or $x-\frac{1}{x}=t$ as the case may be.

First fundamental theorem of integral calculus:

Let the area function be defined by $A(x)=\int_a^x f(x)dx$ for all $x \geq a$ where the function f is assumed to be continuous on $[a,b]$. Then $A'(x)= f(x)$ for all $x\in[a,b]$

Second fundamental theorem of integral calculus:

Let f be a continuous function of x defined on the closed interval [a, b] and let F be another function such that $\frac{d}{dx}F(x)= f(x)$ for all x in the domain of f, then $\int_a^b f(x)\,dx = F(b)-F(a)$.

Properties of Definite Integral

$(i)\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$ $\qquad (ii)\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$(iii)\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ where $a<c<b$

$(iv)\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ $\qquad (v)\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$(vi)\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(-x)=f(x) \text{ i.e. } f \text{ is even function} \\ 0 & \text{if } f(-x)=-f(x) \text{ i.e. } f \text{ is odd function} \end{cases}$$

$$(vii)\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(2a-x)=f(x) \\ 0 & \text{if } f(2a-x)=-f(x) \end{cases}$$

Limit as a Sum

$$\int_a^b f(x)\,dx = \lim_{h\to0} h\left[f(a)+f(a+h)+f(a+2h)+\ldots+f(a+(n-1)h)\right] \quad \text{where } nh=b-a$$

Also,

$(i)\ 1+2+3+\ldots+(n-1)=\frac{n(n-1)}{2}$ $\qquad (ii)\ 1^2+2^2+3^2+\ldots+(n-1)^2=\frac{n(n-1)(2n-1)}{6}$

$(iii)\lim_{x\to0}\frac{e^x-1}{x}=1=\lim_{x\to0}\frac{x}{e^x-1}$ $\qquad (iv)\ a+ar+ar^2+\ldots ar^{n-1}=a\left(\frac{r^n-1}{r-1}\right)$