PRACTICE PAPER - 2

MATHEMATICS

CLASS XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

1. All questions are compulsory.

2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.

3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. If $A=\begin{bmatrix}1 & -3\\ 2 & 0\end{bmatrix}$ and $B=\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}$, find a matrix C such that $(A+B+C)$ is a zero matrix.

2. If A is a square matrix of order 3 and $|3A|=k|A|$, then write the value of k.

3. If A is a matrix of order 2×3 and B is a matrix of order 3×5, what is the order of matrix $(AB)^T$?

4. If $f:R-\{-1\}\to R-\{-1\}$ be defined as $f(x)=\dfrac{x}{x+1}$, find $f^{-1}(x)$.

5. What is the principal value of $\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$?

6. Find slope of the tangent to the curve $y=x^3-3x+2$ at the points whosce x-coordinate is 3.

7. Write the value of the integral $\int_{-\pi/2}^{\pi/2}\sin^5 x\,dx$.

8. Write a vector of magnitude of 6 units in the direction of the vector $\hat{i}-2\hat{j}+2\hat{k}$.

9. What is the cosine of the angle which the vector $\sqrt{2}\,\hat{i}+\hat{j}+\hat{k}$ makes with y-axis?

10. Find the value of λ such that the line $\dfrac{x-2}{9}=\dfrac{y-1}{\lambda}=\dfrac{z-3}{-6}$ is perpendicular to the plane $3x-y-2z=7$.

SECTION – B

11. Using properties of determinants, show that:

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc+bc+ca+ab$$

OR

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

12. Le N be the set of all natural numbers and let R be a relation in N, defined by $R = \{(a,b) : a \text{ is a factor of } b\}$. Show that R is reflexive and transitive but not symmetric.

13. Solve the following equation:

$$\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$$

OR

Solve the following equation:

$$\tan^{-1}\left(\frac{x-1}{x+1}\right) + \tan^{-1}\left(\frac{2x-1}{2x+1}\right) = \tan^{-1}\frac{23}{36}$$

14. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

If $y = \left[\log\left(x+\sqrt{x^2+1}\right)\right]^2$, show that $\left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2$.

15. If $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$, $y = a\sin t$, find $\frac{d^2y}{dx^2}$ **at** $t = \frac{\pi}{4}$.

16. Find the intervals on which the function $f(x) = 2x^3 - 12x^2 + 18x - 7$ is:

(i) increasing (ii) decreasing:

17. A and B throw a die alternately till one of them gets a '5' and wins the game. Find their respective probabilities of winning, if A starts the game.

18. Evaluate: $\int \frac{x^2-1}{x^4+1}dx$ **OR** $\int \frac{2\sin 2\theta - \cos\theta}{6-\cos^2\theta-4\sin\theta}\,d\theta$

19. Evaluate : $\int \left(\sin^{-1}x\right)^2 dx$

20. Using properties of definite integrals, evaluate $\int_0^{\pi/2} \frac{x\,dx}{\sin x + \cos x}$.

21. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

22. Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ intersect each other. Also, find point of their intersection.

SECTION – C

23. Use the product $\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$ to solve the equations $\begin{matrix}x-y+z=4\\ x-2y-2z=9\\ 2x+y+3z=1\end{matrix}$.

24. Using integration, find the area of the region $\{(x,y): x^2+y^2 \le 1 \le x+y\}$.

25. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also, find the maximum volume.

26. Solve the following differential equation:

$$\frac{dy}{dx}=-\frac{x+y\cos x}{1+\sin x}$$

OR

Show that the following differential equation is homogeneous, and then solve it:

$$y\,dx+x\log\left(\frac{y}{x}\right)dy-2x\,dy=0$$

27. Find the vector equation of the plane through the intersection of the planes $\vec{r}.\left(2\hat{i}+6\hat{j}\right)+12=0$ and $\vec{r}.\left(3\hat{i}-\hat{j}+4k\right)=0$, which is at a unit distance from the origin.

OR

Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar. Also, find the equation of the plane containing these two lines.

28. In a bolt factory machines, A, B and C manufacture respectively 25%, 35% and 40% of the total bolts. Of their output 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from product.

(i) What is the probability that the bolt drawn is defective ?

(ii) If the bolt is found to be defective find the probability that it is a product of machine B.

29. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximise the profit?

ANSWERS:

1. $\begin{pmatrix} -3 & 4 \\ -3 & 0 \end{pmatrix}$ 2. 27 3. 5×2 4. $\frac{x}{1-x}$; $x\neq 1$ 5. $-\frac{\pi}{3}$ 6. 24

7. 0 8. $2\hat{i}-4\hat{j}+4\hat{k}$ 9. $\frac{1}{2}$ 10. -3 13. $x=13$ or part $x=\frac{4}{3}$

15. $\sqrt{2}\,a$ 16. inc.in $(-\infty,1)\cup(3,\infty)$ and dec. in $(1,3)$ 17. $P(A)=\frac{6}{11}$, $P(B)=\frac{5}{11}$

18. $\frac{1}{2\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}\,x+1}{x^2+\sqrt{2}\,x+1}\right|+c$ or $2\log\left|\sin^2\theta-4\sin\theta+5\right|+7\tan^{-1}(\sin\theta-2)+c$

19. $x.\left(\sin^{-1}x\right)^2+2.\sin^{-1}x.\sqrt{1-x^2}-2x+c$ 20. $\frac{\pi}{4\sqrt{2}}\log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$ 21. 1

22. $(1,3,2)$ 23. $AB=8I$; $x=3$, $y=-2$, $z=-1$ 24. $\frac{1}{4}(\pi-2)$sq.units

26. $y=\frac{2c-x^2}{2(1+\sin x)}$ or $\log\left|\frac{y}{x}\right|-1=cy$ 27. $2x+y+2z+3=0$ and $x-2y+2z-3=0$

or part $x-2y+z=0$ 28. $\frac{28}{69}$ 29. $8,20$; Max Profit is Rs 160