PRACTICE PAPER - 1

MATHEMATICS

CLASS XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. Construct a 3×2 matrix whose elements are given by $a_{ij}=|2i-j|$.

2. If A is a square matrix of order 3 such that $|adj\ A|=144$, write the value of $|A|$.

3. If $0<x<\pi$ and the matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular, write the value(s) of x.

4. Let * be a binary operation on the set Q of non-zero rational numbers defined as $a*b=\frac{ab}{3}$. Write the identity element for * , if any.

5. Find x, if $\tan^{-1}5+\cot^{-1}x=\frac{\pi}{2}$.

6. Write the value of $\int 3^x\,dx$.

7. If $f(x)=\sin x^0$, write the value of $\frac{dy}{dx}$.

8. Write the position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ externally in the ratio 2:1 , where $\vec{a}=\hat{i}+2\hat{j}-k$ and $\vec{b}=-\hat{i}+\hat{j}+k$.

9. Write the value of $\left|\vec{a}-\vec{b}\right|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $\left|\vec{a}\right|=2$, $\left|\vec{b}\right|=3$ and $\vec{a}.\vec{b}=4$.

10. If a line makes α,β,γ with the x-axis, y-axis and z-axis respectively, then write the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$.

SECTION – B

11. Consider $f : \mathrm{R}_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y - 4}$, where R_+ is the set of all non-negative real numbers.

OR

Prove that the relation R on the set Z of all integers defined by $(a,b)\in R \Leftrightarrow a-b$ is divisible by 4 is an equivalence relation.

12. Prove that: $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$.

13. Using properties of determinants, show that:

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca).$$

14. Show that the function $f(x)=|x+2|$ is continuous at every $x\in R$ but fails to be differentiable at $x=-2$.

15. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

OR

Find $\frac{dy}{dx}$, if $y=(x)^{\cos x}+(\log x)^x$.

16. Find the equation of the normals to the curve $y=x^3+2x+6$ which are parallel to the line $x+14y+4=0$.

17. Evaluate the following: $\int \frac{\sin x + \cos x}{\sqrt{\sin x . \cos x}} dx$

OR

Evaluate the following: $\int \frac{2+\sin 2x}{1+\cos 2x} e^x \, dx$

18. Using the properties of definite integral, evaluate the following: $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$.

19. Solve the following differential equation:

$$\frac{dy}{dx} + \frac{2x}{x^2+1} y = \frac{1}{\left(x^2+1\right)^2} \; ; \; y(0) = 0$$

OR

Solve the following differential equation:

$$\left(x^3 + y^3\right) dy - x^2 y \, dx = 0 .$$

20. Show that the four points whose poition vectors are $6\hat{i} - 7\hat{j}$, $16\hat{i} - 19\hat{j} - 4\hat{k}$, $3\hat{j} - 6\hat{k}$ and $2\hat{i} - 5\hat{j} + 10\hat{k}$ are coplanar.

21. Find the shortest distance between the following pair of parallel lines:

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-3}{1} \text{ and } \frac{x-2}{-1} = \frac{y+1}{1} = \frac{z+1}{-1}$$

22. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

SECTION – C

23. Using matrix method, solve the following system of linear equations:

$$x + 2y + z = 7 \; ; \; x + 3z = 11 \; ; \; 2x - 3y = 1$$

24. Evaluate the following integral as limit of sums: $\int_0^4 \left(x + e^{2x}\right) dx$

25. Find the area of the region enclosed between the circles $x^2 + y^2 = 16$ and $\left(x+4\right)^2 + y^2 = 16$.

26. A wire of length 28 m is to be cut into two pieces. One of the pieces will be bent into shape of a square and the other into shape of an equilateral triangle. Where the wire should be cut so thet the sum of the areas of the square and triangle is minimum?

OR

Show that the right circular cylinder of given volume open at the top has minimum total surface area, provided its height is equal to the radius of its base.

27. Find the equation of the plane which contains line of intersection of planes $\vec{r}.\left(\hat{i}+2\hat{j}+3k\right)-4=0$, $\vec{r}.\left(2\hat{i}+\hat{j}-k\right)+5=0$ and which is perpendicular to the plane $\vec{r}.\left(5\hat{i}+3\hat{j}-6k\right)+8=0$.

OR

Find the image of the point $(1,2,3)$ in the plane $x+2y+4z=38$. Also find the distance of the given point from the plane.

28. A pair of dice is thrown 8 times If getting a total of 7 is considered as success, Find the probability of:

(i) no success (ii) 6 successes (iii) atleast 6 successes (iv) atmost 6 successes

29. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 5760 to invest and has space for at most 20 items. A fan costs him Rs 360 and a sewing machine costs him Rs 240. He expects to sell a fan a profit of Rs 22 and a sewing machine at a profit of Rs 18. Assuming that he can sell all the items that he buys, how should he invest his money to maximize the profit? What is the maximum profit?

ANSWERS:

1. $\begin{bmatrix}1&0\\3&2\\5&4\end{bmatrix}$ 2. 12 3. $\frac{\pi}{3}$ or $\frac{2\pi}{3}$ 4. 3 5. 5 6. $\frac{3^x}{\log 3}+c$ 7. $\frac{\pi}{180}\cos x^0$ 8. $-3\hat{i}+3\hat{k}$ 9. $\sqrt{5}$

10. 2 15. $x^{\cos x}\left(\frac{\cos x}{x}-\sin x.\log x\right)+(\log x)^x\left(\frac{1}{\log x}+\log(\log x)\right)$ 16. $x+14y=254, x+14y+86=0$

17. $\sqrt{2}\sin^{-1}(\sin x-\cos x)+c$ or $\tan x.e^x+c$ 18. $\frac{\pi}{12}$ 19. $x^3=-3y^3.\log\left|\frac{c}{y}\right|$ 21. $\sqrt{26}$ unit 22. $\frac{1}{4}$

23. $x=2, y=1, z=3$ 24. $\frac{15+e^8}{2}$ 25. $8\left(\frac{7\pi}{3}-\sqrt{3}\right)$ sq. units 27. $33x+45y+50z-41=0$ or $(3,6,11)$, $\sqrt{21}$

28 (i) $\left(\frac{5}{6}\right)^8$ (ii) ${}^8C_6\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)^2$ (iii) ${}^8C_6\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)^2+{}^8C_7\left(\frac{1}{6}\right)^7\left(\frac{5}{6}\right)+{}^8C_8\left(\frac{1}{6}\right)^8$ (iv) $1-\left[{}^8C_7\left(\frac{1}{6}\right)^7\left(\frac{5}{6}\right)+{}^8C_8\left(\frac{1}{6}\right)^8\right]$

29. 8 fans, 12 machines, Max Profit Rs 392