PRE BOARD EXAMINATION 2012-13

MATHEMATICS

Class XII SET - B

Time : 3 Hours **Max. Marks : 100**

General Instructions:

1. All questions are compulsory.

2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.

3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

SECTION – A

1. If $f:R \to R$ is defined by $f(x)=3x+4$, find $f(f(x))$.

2. If $\sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1} x = \frac{\pi}{2}$, then find x.

3. Find the value of x, if $\cos\left(\sin^{-1}\frac{1}{3} + \cos^{-1} x\right) = 0$

4. If A is a square matrix of order 3 such that $|adj\ A| = 49$, find $|A|$.

5. For what value of k, the matrix $A = \begin{bmatrix} 3-2k & k+1 \\ 2 & 4 \end{bmatrix}$ is singular?

6. write the value of anti-derivative of $\tan^{-1}(\cot x)$.

7. What is the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.

8. Find the projection of the vector $\vec{a} = \hat{i} + \hat{j}$ on $\vec{b} = \hat{i} + \hat{k}$.

9. If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=15$, then what is the value of $|\vec{x}|$.

10. Write the Cartesian equation of the line through the point (5, 2, – 4) and which is parallel to the vector $3\hat{i}+2\hat{j}-4\hat{k}$.

SECTION B

11. Using properties of determinants, show that:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3.$$

12. Consider $f: R_+ \to [4,\infty)$ given by $f(x)=x^2+4$. Show that f is invertiblw with the inverse of f given by $f^{-1}(x)=\sqrt{x-4}$, where R_+ is set of all non-negative real numbers.

13. Solve for x, if $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\left(\frac{8}{31}\right)$

14. The function f is defined as $f(x)=\begin{cases} x^2+ax+b & ,0\le x<2 \\ 3x+2 & ,2\le x\le 4 \\ 2ax+5b & ,4<x\le 8 \end{cases}$. If $f(x)$ is continous on $[0,8]$, find the values of a and b.

15. If $x=\sqrt{a^{\sin^{-1}t}}$ and $y=\sqrt{a^{\cos^{-1}t}}$, then show that $\frac{dy}{dx}=-\frac{y}{x}$.

OR

If $y=(\tan^{-1}x)^2$, prove that $(1+x^2)^2\frac{d^2y}{dx^2}+2x(1+x^2)\frac{dy}{dx}=2$.

16. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 m/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

OR

Find the intervals in which the function f given by $f(x)=\sin x+\cos x$; $0\le x\le 2\pi$ is strictly increasing or strictly decreasing.

17. Two dice are thrown simultaneously. Let X denotes the number of sixes. Find the probability distribution of X.

A dice is normally used in gambling. Is gambling is a good way to earn money?

18. Find x, such that points $A(3,2,1)$, $B(4,x,5)$, $C(4,2,-2)$ and $D(6,5,-1)$ are coplanar.

OR

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 18$.

19. Evaluate the following: $\int x\sqrt{1 + x - x^2}\, dx$

OR

Evaluate the following: $\int \frac{3x + 5}{(x+1)(x-1)^2}\, dx$

20. Using the properties of definite integral, evaluate the following: $\int_0^{\pi/4} \log(1 + \tan x)\, dx$

21. Evaluate $\int_0^2 (x^2 + 3x)\, dx$ as the limit of a sum.

22. Find the shortest distance between the following pair of lines:

$\vec{r} = (2\hat{i} - 3\hat{j} + 5\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (-\hat{i} - \hat{j} + 5\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 3\hat{k})$

SECTION C

23. Using elementary transformations, find the inverse matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$.

OR

If $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$, find AB and use this result to solve the following equations: $x - y + 2z = 1$; $2y - 3z = 1$ and $3x - 2y + 4z = 2$.

24. The sum of the perimeter of a circle and square is *k*, where *k* is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

25. Find the particular solution of the differential equation $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x \quad ; x \neq 0$ given that $y = 0$ when $x = \frac{\pi}{2}$.

OR

Show that the differential equation $x \cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$ is homogeneous and hence solve it.

26. Find the equation of the plane passing through the point $(1, 1, -1)$ and perpendicular to the planes $x + 2y + 3z - 7 = 0$ and $2x - 3y + 4z = 0$.

27. Using integration, find area of region bounded by the following lines:
$5x - 2y - 10 = 0$; $x + y - 9 = 0$ and $2x - 5y - 4 = 0$.

28. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostlier?

"Grading system is better than numerical marking." Comment (in brief) on this statement.

29. A manufacturer produces two types of steel trunks. He has two machines A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type of trunk requires 3 hours on machine A and 2 hours on machine B. Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of Rs 30 and Rs 25 per trunk of the first type and second type respectively. How many trunks of each type must he make each day to make the maximum profit? Formulate the problem as L.P.P. and solve it graphically.

What should be the qualities of good machine?