Class - XII Mathematics Pre-Board 2012-13 Solutions

Set - A

Ans 1 $fof = fof(x) = f(f(x)) = f((3-x^3)^{1/3})$

$= \left[3 - \left((3-x^3)^{1/3}\right)^3\right]^{1/3} = (3-3+x^3)^{1/3} = \underline{x}$ Ans

Ans 2 $\cos^{-1}\left(\cos\left(\pi + \frac{\pi}{6}\right)\right) + \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right)$

$= \cos^{-1}\left(-\cos\frac{\pi}{6}\right) + \sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$= \pi - \cos^{-1}\left(\cos\frac{\pi}{6}\right) + \frac{\pi}{3} = \pi - \frac{\pi}{6} + \frac{\pi}{3} = \underline{\frac{7\pi}{6}}$ Ans

Ans 3 $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \pi - \cos^{-1}\frac{\sqrt{3}}{2} = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\therefore$ Given expression $= \cos\left(\frac{5\pi}{6} + \frac{\pi}{6}\right) = \cos\pi = \underline{-1}$ Ans

Ans 4 $|A| = \cos^2\theta + \sin^2\theta = 1$

$A^{-1} = \frac{1}{|A|} \cdot \text{adj}\, A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ Ans

Ans 5 clearly $|A| = 0 \Rightarrow 2 - k + 20 = 0 \Rightarrow \underline{k = 22}$ Ans

Ans 6 clearly, $y = \sin^{-1}\frac{x^3}{2+x^3} + \cos^{-1}\frac{x^3}{2+x^3} = \frac{\pi}{2}$ $\therefore \frac{dy}{dx} = \underline{0}$ Ans

Ans 7 $y = mx$ — ① differentiating, $\frac{dy}{dx} = m$ — ②

from ① & ②, $y = \frac{dy}{dx} \times x \Rightarrow \underline{x\frac{dy}{dx} - y = 0}$ Ans.

Ans 8 here, $|\vec{a} \times \vec{b}| = 1 \Rightarrow ||\vec{a}|.|\vec{b}|.\sin\theta| = 1$

$\Rightarrow \sqrt{3} \times \frac{2}{3} \times \sin\theta = 1 \Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \underline{\theta = \frac{\pi}{3}}$ Ans

Ans 9. $|\vec{a}| = \sqrt{2^2 + (-1)^2 + (2)^2} = 3$

$\therefore$ Req Ans $= \frac{5}{3}(2\hat{i} - \hat{j} + 2\hat{k})$

Ans 10. here, $(-3)(3k) + 2k(1) + 2(-5) = 0 \Rightarrow k = -\frac{10}{7}$ Ans.

Section - B

Ans 11. Use $R_1 \to R_1 + R_2 + R_3$. Consult NCERT

Ans 12. for 1-1 let $x_1, x_2 \in R_+$ (domain)

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 + 4 = x_2^2 + 4 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2 \Rightarrow x_1 = x_2$

as $x_1, x_2 \in R_+$

$\Rightarrow f$ is 1-1

for onto. $x^2 \geq 0 \Rightarrow x^2 + 4 \geq 4 \Rightarrow f(x) \geq 4 \Rightarrow f(x) \in [4, \infty)$

i.e. $R_f =$ Co-domain. $\Rightarrow f$ is onto.

Thus f is 1-1 and onto & hence invertible.

for f^{-1} let $y = f(x) \Rightarrow y = x^2 + 4$

$\Rightarrow x = \pm\sqrt{y-4}$

$\Rightarrow x = \sqrt{y-4}$ as $x \in R_+$ —— ①

Since, $y = f(x) \Rightarrow x = f^{-1}(y)$ —— ②

from ① & ②,

$x = f^{-1}(y) = \sqrt{y-4}$

$\therefore f^{-1}(x) = \sqrt{x-4}$. Hence shown.

Ans 13. $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$

$\Rightarrow \tan^{-1}\left(\frac{x+1+x-1}{1-(x+1)(x-1)}\right) = \tan^{-1}\frac{8}{31}$; $(x+1)(x-1) < 1$

$\Rightarrow \tan^{-1}\frac{2x}{2-x^2} = \tan^{-1}\frac{8}{31}$; $x^2 < 2$

$\Rightarrow \frac{2x}{2-x^2} = \frac{8}{31}$; $x^2 < 2$

$\Rightarrow 4x^2+31x-8=0 \Rightarrow (x+8)(4x-1)=0 \Rightarrow x=-8$ or $\frac{1}{4}$

Since, $(-8)^2 \nless 2$ $\therefore$ $x=-8$ rejected

$\therefore x = \frac{1}{4}$ Ans.

Ans 14. clearly f is cts. at $x=2$ and 4.

Since f is cts. at $x=2$

$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\Rightarrow \lim_{h\to 0} f(2-h) = \lim_{h\to 0} f(2+h) = f(2)$

$\Rightarrow \lim_{h\to 0} [(2-h)^2 + a(2-h)+b] = \lim_{h\to 0} [3(2+h)+2] = 3(2)+2$

$\Rightarrow 2a+b+4=8 \Rightarrow 2a+b=4$ —— ①

Again, f is cts. at $x=4$

$\Rightarrow \lim_{h\to 0} f(4-h) = \lim_{h\to 0} f(4+h) = f(4)$

$\Rightarrow \lim_{h\to 0} [3(4-h)+2] = \lim_{h\to 0} [2a(4+h)+5b] = 3(4)+2$

$\Rightarrow 8a+5b=14$ —— ②

solving ① and ②, $a=3$, $b=-2$ Ans.

Ans 15. $x = \sqrt{a^{\sin^{-1}t}}$ —— ①

$y = \sqrt{a^{\cos^{-1}t}}$ —— ②

multiplying ① & ②,

$xy = \sqrt{a^{\sin^{-1}t} \times a^{\cos^{-1}t}} = \sqrt{a^{\sin^{-1}t+\cos^{-1}t}} = \sqrt{a^{\pi/2}}$

diff., $x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$ H. proved

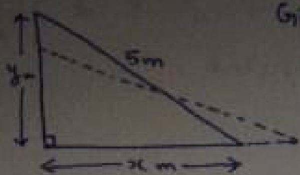
OR

$y = \sin^{-1}x$ diff., $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2}\cdot\frac{dy}{dx} = 1$

diff. again, $\sqrt{1-x^2}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{1}{2\sqrt{1-x^2}} \times (-2x) = 0$

Simplifying, $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$ H. Proved

Ans 16



Given: $\frac{dx}{dt} = 2\ m/s$ To find: $\left.\frac{dy}{dt}\right]_{x=4m}$

here, $x^2 + y^2 = 5^2$

diff., $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\Rightarrow x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

Now, when $x = 4$, then $y = 3\ m$

$\therefore 4(2) + 3.\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{8}{3}$

$\therefore$ height is decreasing at rate of $\frac{8}{3}$ m/s Ans.

OR

$y = \log(x+1) - \frac{2x}{2+x}$; $x > -1$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x+1} - \frac{(2+x)(2) - 2x(1)}{(2+x)^2}$

$= \frac{1}{x+1} - \frac{4}{(2+x)^2} = \frac{(2+x)^2 - 4(x+1)}{(x+1)(2+x)^2} = \frac{x^2}{(x+1)(2+x)^2}$

$= \left(\frac{x}{2+x}\right)^2 \times \frac{1}{x+1}$

Now, $\left(\frac{x}{2+x}\right)^2 > 0$. also, $x > -1$ (given)

$\Rightarrow x+1 > 0 \Rightarrow \frac{1}{x+1} > 0$

Thus, sign of $\frac{dy}{dx}$ = (+ve) × (+ve) = +ve

ie. $\frac{dy}{dx} > 0$ $\therefore$ y is inc. fn. H. proved.

Ans 17. here, $P(\text{six}) = \frac{1}{6}$, $P(\text{not six}) = \frac{5}{6}$

clearly X can take values 0, 1, 2

Now, $P(X=0) = P(\text{no sixes}) = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

$P(X=1) = P(\text{one six}) = 2 \times \frac{5}{6} \times \frac{1}{6} = \frac{10}{36}$

$P(X=2) = P(\text{both sixes}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$\therefore$ P.D. table is

X	0	1	2
P(X)	25/36	10/36	1/36

II part: No. multiple responses.

Ans 18. here, $\vec{OA} = 3\hat{i} + 2\hat{j} + \hat{k}$, $\vec{OB} = 4\hat{i} + x\hat{j} + 5\hat{k}$

$\vec{OC} = 4\hat{i} + 2\hat{j} - 2\hat{k}$, $\vec{OD} = 6\hat{i} + 5\hat{j} - \hat{k}$

$\therefore \vec{AB} = \vec{OB} - \vec{OA} = \hat{i} + (x-2)\hat{j} + 4\hat{k}$

$\vec{AC} = \hat{i} - 3\hat{k}$ and $\vec{AD} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

If A, B, C and D are coplanar, then

$\vec{AB} \cdot (\vec{AC} \times \vec{AD}) = 0$

$$\Rightarrow \begin{vmatrix} 1 & x-2 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

expanding, $\underline{x = 5}$ Ans.

OR

Since $\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow \vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} + \vec{c}\cdot\vec{b} + \vec{c}\cdot\vec{c} = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow (1)^2 + (4)^2 + (2)^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{21}{2}$ Ans.

Ans 19. $I = \int x\sqrt{1+x-x^2}\,dx$

Put $x = A(1-2x) + B$

$\Rightarrow x = A + B - 2Ax$

Comparing, $A + B = 0$ and $-2A = 1$

$\Rightarrow B = -A$ $\Rightarrow A = -\frac{1}{2}$

$\Rightarrow B = \frac{1}{2}$

$\therefore I = \int\left[-\frac{1}{2}(1-2x) + \frac{1}{2}\right] \cdot \sqrt{1+x-x^2}\,dx$

$= -\frac{1}{2}\int (1-2x)\sqrt{1+x-x^2}\,dx + \frac{1}{2}\int \sqrt{1+x-x^2}\,dx$

$= -\frac{1}{2} I_1 + \frac{1}{2} I_2$ —— (1)

Now, $I_1 = \int (1-2x).\sqrt{1+x-x^2}\,dx$

Put $1+x-x^2 = t \Rightarrow (1-2x)dx = dt$

$\therefore I_1 = \int \sqrt{t}\,dt = \frac{2}{3}t^{3/2} + c_1$

$= \frac{2}{3}(1+x-x^2)^{3/2} + c_1$ —— ②

Now, $I_2 = \int \sqrt{1+x-x^2}\,dx$

Consider, $1+x-x^2 = -(x^2-x-1) = -\left(x^2-x+\frac{1}{4}-\frac{1}{4}-1\right)$

$= -\left[\left(x-\frac{1}{2}\right)^2-\frac{5}{4}\right] = \left(\frac{\sqrt{5}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2$

$\therefore I_2 = \int \sqrt{\left(\frac{\sqrt{5}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}\,dx$

$= \frac{1}{2}\left(x-\frac{1}{2}\right).\sqrt{1+x-x^2} + \frac{5}{4}\times\frac{1}{2}\sin^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{5}}{2}}\right) + c_2$

$= \frac{2x-1}{4}.\sqrt{1+x-x^2} + \frac{5}{8}\sin^{-1}\left(\frac{2x-1}{\sqrt{5}}\right) + c_2$ —— ③

from ①, ② and ③,

$I = +\frac{1}{8}(2x-1)\sqrt{1+x-x^2} + \frac{5}{16}\sin^{-1}\left(\frac{2x-1}{\sqrt{5}}\right) - \frac{1}{3}(1+x-x^2)^{3/2} + c$ Ans.

OR

$I = \int \frac{3x+5}{(x+1)(x-1)^2}\,dx$

Let $\frac{3x+5}{(x+1)(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$

$\Rightarrow 3x+5 = A(x^2-1) + B(x+1) + C(x^2+1-2x)$

equating coeff, $A+C=0$; $B-2C=3$ and $B=4$

solving, $A=-\frac{1}{2}$, $B=4$, $C=\frac{1}{2}$

$\therefore I = -\frac{1}{2}\int\frac{1}{x-1}\,dx + 4\int\frac{dx}{(x-1)^2} + \frac{1}{2}\int\frac{1}{x+1}\,dx$

$= -\frac{1}{2}\log|x-1| + 4\left(\frac{-1}{x-1}\right) + \frac{1}{2}\log|x+1| + c$

$= \frac{1}{2}\log\left|\frac{x+1}{x-1}\right| - \frac{4}{x-1} + c$ Ans.

Ans 20. Let $I = \int_0^{\pi/4} \log(1+\tan x)$

$$= \int_0^{\pi/4} \log\left(1+\tan\left(\pi/4 - x\right)\right) dx$$

$$= \int_0^{\pi/4} \log\left(1+\frac{1-\tan x}{1+\tan x}\right) dx$$

$$= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right) dx$$

$$= \int_0^{\pi/4} \log 2 \, dx - \int_0^{\pi/4} \log(1+\tan x)\, dx$$

$$= \log 2 . [x]_0^{\pi/4} - I$$

$$\Rightarrow 2I = \frac{\pi}{4}\log 2 \quad \Rightarrow I = \frac{\pi}{8}\log 2 \quad \text{Ans.}$$

Ans 21. $\int_0^2 (x^2+3x)\,dx$

here, $a=0,\ b=2 \Rightarrow nh = b-a = 2-0 \Rightarrow nh = 2$

Let $f(x) = x^2+3x$

Now, $\int_0^2 (x^2+3x)\,dx = \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+\dots+f(0+(n-1)h)\right]$

$f(0) = 0$

$f(h) = h^2+3h$

$f(2h) = 4h^2+6h$

$\dots\dots$

$f(0+(n-1)h) = (n-1)^2h^2+3(n-1)h$

$$\therefore \int_0^2 (x^2+3x)\,dx = \lim_{h\to 0} h.\left[0+(h^2+3h)+(4h^2+6h)+\dots((n-1)^2h^2+3(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[h^2(1^2+2^2+\dots+(n-1)^2) + 3h(1+2+\dots+(n-1))\right]$$

$$= \lim_{h\to 0} h.\left[h^2.\frac{n(n-1)(2n-1)}{6} + 3h.\frac{n(n-1)}{2}\right]$$

$$= \lim_{h\to 0} \frac{(nh)(nh-h)(2nh-h)}{6} + \frac{3}{2}(nh)(nh-h)$$

$$= \lim_{h \to 0} \frac{2(2-h)(4-h)}{6} + \frac{3}{2}(2)(2-h)$$

$$= \frac{2 \times 2 \times 4}{6} + \frac{3}{\cancel{2}} \times 2 \times \cancel{2} = \frac{8}{3} + 6 = \frac{26}{3} \text{ Ans.}$$

Ans 22. here, $\vec{a}_1 = 2\hat{i} - 3\hat{j} + 5\hat{k}$, $\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$

$\vec{a}_2 = -\hat{i} - \hat{j} + 5\hat{k}$, $\vec{b}_2 = 2\hat{i} + 4\hat{j} - 3\hat{k}$

Now, $\vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{j}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 4 & -3 \end{vmatrix} = -\hat{i} + 5\hat{j} + 6\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + (5)^2 + (6)^2} = \sqrt{62}$$

Now, $$S.D. = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$= \left| \frac{(-\hat{i} + 5\hat{j} + 6\hat{k}) \cdot (-3\hat{i} + 2\hat{j})}{\sqrt{62}} \right| = \frac{13}{\sqrt{62}} = \frac{13\sqrt{62}}{62} \text{ units Ans}$$

Section - C

Ans 23. Let $A = IA$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

Applying, $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 + 2R_1$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} A$$

$R_2 \to R_2 - R_3$, $R_1 \to R_1 - 3R_3$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -5 & 0 & -3 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix} \cdot A$$

$R_1 \to R_1 - 2R_2$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix} . A$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix} \quad \text{Ans.}$$

<u>OR</u>

here, $AB = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix} \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow AB = I \Rightarrow A^{-1} = B$$

Given system can be written as $AX = C$

i.e. $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

Now, $AX = C \Rightarrow X = A^{-1}C$

$$= \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix}$$

$\therefore$ $\underline{x = 0}$, $\underline{y = 5}$ and $\underline{z = 3}$ Ans.

<u>Ans 24</u>.

Given: Volume

Min : CSA when $h = \sqrt{2}\, r$

Now, $V = \frac{1}{3}\pi r^2 h \Rightarrow h = \frac{3V}{\pi r^2}$

$$A = \pi r l = \pi r \sqrt{r^2 + h^2} = \pi r \sqrt{r^2 + \frac{9V^2}{\pi^2 r^4}}$$

$$\Rightarrow A^2 = A' = \pi^2 r^2 \left(r^2 + \frac{9V^2}{\pi^2 r^4} \right) = \pi^2 r^2 \left(\frac{\pi^2 r^6 + 9V^2}{\pi^2 r^4} \right)$$

$$\Rightarrow A' = \pi^2 r^4 + \frac{9V^2}{r^2}$$

A will be max./min. as A' is max./min.

(6)

Now, $\frac{dA'}{dr} = 4\pi^2 r^3 + 9V^2\left(-\frac{2}{r^3}\right)$

for max./min, $\frac{dA'}{dr} = 0 \Rightarrow 4\pi^2 r^3 = \frac{18V^2}{r^3}$

$\Rightarrow 2\pi^2 r^6 = 9 \cdot \frac{1}{9}\pi^2 r^4 h^2$

$\Rightarrow h^2 = 2r^2$

$\Rightarrow h = \sqrt{2}\, r$

again, $\frac{d^2A'}{dr^2} = 12\pi^2 r^2 - 18V^2\left(\frac{-3}{r^4}\right)$

$= 12\pi^2 r^2 + \frac{54V^2}{r^4} > 0$

$\therefore$ A' and hence A is min. when $h = \sqrt{2}\, r$. H. proved.

Ans 25. $\frac{dy}{dx} + y \cdot \cot x = 2x + x^2 \cot x$

clearly given eqn is linear diff. eqn.

here, $P = \cot x$, $Q = 2x + x^2 \cot x$

$I.F. = e^{\int \cot x\, dx} = e^{\log(\sin x)} = \sin x$

sol. is given by ;

$y \cdot \sin x = \int (2x + x^2 \cot x) \cdot \sin x\, dx + c$

$= 2\int x \sin x\, dx + \int x^2 \cos x\, dx + c$

$= \cancel{2\int x \sin x\, dx} + x^2 \cdot \sin x - \cancel{\int 2x \cdot \sin x\, dx} + c$

$= x^2 \sin x + c$

$\Rightarrow y = x^2 + c \operatorname{cosec} x$

Now, $y = 0$ when $x = \pi/2$

$\therefore 0 = \left(\frac{\pi}{2}\right)^2 + c \cdot 1 \Rightarrow c = -\frac{\pi^2}{4}$

$\therefore y = x^2 - \frac{\pi^2}{4} \operatorname{cosec} x$ Ans.

OR

Given eqn is $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$

$$\Rightarrow \frac{dy}{dx} = \frac{y\cos\left(\frac{y}{x}\right) + x}{x\cos\left(\frac{y}{x}\right)} = \frac{y}{x} + \frac{1}{\cos\left(\frac{y}{x}\right)} \quad \text{—— (1)}$$

Consider, $f(x,y) = \frac{y}{x} + \frac{1}{\cos\left(\frac{y}{x}\right)}$

here, $f(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} + \frac{1}{\cos\left(\frac{\lambda y}{\lambda x}\right)} = \frac{y}{x} + \frac{1}{\cos\left(\frac{y}{x}\right)} = f(x,y)$

∴ Given eqn is homogeneous diff. eqn.

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Using in eq- (1),

$\cancel{v} + x\frac{dv}{dx} = \cancel{v} + \frac{1}{\cos v} \Rightarrow \cos v \, dv = \frac{dx}{x}$

$\Rightarrow \int \cos v \, dv = \int \frac{dx}{x} \Rightarrow \sin v = \log|x| + \log c$

$\Rightarrow \sin\frac{y}{x} = \log|cx|$ Ans.

Ans 26. Let req. eqn of plane be

$a(x-1) + b(y-1) + c(z+1) = 0$ —— (1)

where, a, b, c are d.r. of normal to the plane.

Since plane (1) is ⊥ to $x + 2y + 3z - 7 = 0$ and $2x - 3y + 4z = 0$

∴ $a + 2b + 3c = 0$ —— (2)

& $2a - 3b + 4c = 0$ —— (3)

solving (2) + (3),

$\frac{a}{8+9} = \frac{-b}{4-6} = \frac{c}{-3-4} = \lambda \Rightarrow \frac{a}{17} = \frac{b}{2} = \frac{c}{-7} = \lambda$

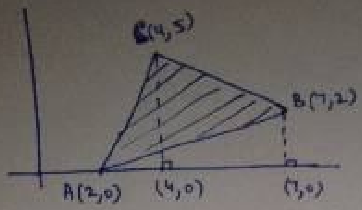
$\Rightarrow a = 17\lambda, \; b = 2\lambda, \; c = -7\lambda$

using in eq. (1),

$17\lambda(x-1) + 2\lambda(y-1) - 7\lambda(z+1) = 0$

$\Rightarrow 17x + 2y - 7z = 26$ Ans.

Ans 27 Point of intersection of given 3 lines are (4,5), (7,2) and (2,0).



Req. Area

$$= \int_2^4 (y \text{ of } AC)\,dx + \int_4^7 (y \text{ of } BC)\,dx - \int_2^7 (y \text{ of } AB)\,dx$$

$$= \int_2^4 \frac{5}{2}(x-2)\,dx + \int_4^7 (9-x)\,dx - \int_2^7 \frac{2}{5}(x-2)\,dx$$

$$= \frac{5}{2}\left(\frac{x^2}{2}-2x\right)_2^4 + \left|9x-\frac{x^2}{2}\right|_4^7 - \frac{2}{5}\left(\frac{x^2}{2}-2x\right)_2^7$$

$$= \frac{5}{2}(6-2) + \left(27-\frac{33}{2}\right) - \frac{2}{5}\left(\frac{45}{2}-10\right) = 10+\frac{21}{2}-5 = \frac{31}{2} \text{ sq. units Ans}$$

Ans 28. Let E_1: hostelier ; E_2: day-scholar ; A: A grade.

here, $P(E_1) = \frac{60}{100}$; $P(E_2) = \frac{40}{100}$

$P(A|E_1) = \frac{30}{100}$; $P(A|E_2) = \frac{20}{100}$

Now, $$P(E_1|A) = \frac{P(E_1).P(A|E_1)}{P(E_1).P(A|E_1) + P(E_2).P(A|E_2)}$$

$$= \frac{\frac{60}{100}\times\frac{30}{100}}{\frac{60}{100}\times\frac{30}{100} + \frac{40}{100}\times\frac{20}{100}}$$

$$= \frac{9}{9+4} = \frac{9}{13} \text{ Ans.}$$

II part: Multiple responses.

Grading is better as it reduces mal classification of students on basis of unreliable marks.

Ans 29. Let no. of trunks of I type = x & of second type = y

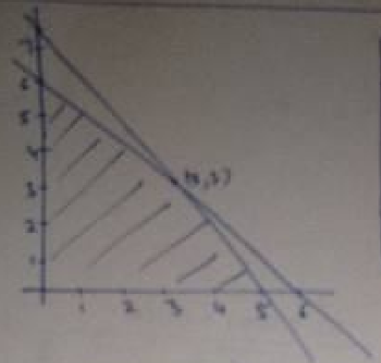
Max. $30x + 25y$

subject to

$3x+3y \le 18 \Rightarrow x+y \le 6$

$3x+2y \le 15$

$x \geq 0,\ y \geq 0$



Point	$Z = 30x + 25y$
(0,0)	0
(0,6)	150
(3,3)	165 → max
(5,0)	150

∴ max. profit is Rs 165 which is achieved when 3 units of each type of trunk is produced.

II. part: Qualities:

(a) takes less energy (electricity)

(b) does more work in less time

(c) easy to operate

(d) do not produce any radiation or harmful gases etc.

SET-B

Only those questions which are not in set-A are solved.

Section-A

Ans 1 $f(f(x)) = f(3x+4) = 3(3x+4) + 4 = \underline{9x+16}$ Ans

Ans 2 clearly, $x = \frac{\pi}{2}$

Ans 3

Ans 4 $|adj\, A| = |A|^{n-1} \Rightarrow 49 = |A|^{3-1} \Rightarrow |A|^2 = 49$

$\Rightarrow |A| = \underline{\pm 7}$ Ans.

Ans 5 here, $|A| = 0 \Rightarrow 4(3-2k) - 2(k+1) = 0$

$\Rightarrow 12 - 8k - 2k - 2 = 0$

$\Rightarrow 10 = 10k \Rightarrow \underline{k = 1}$ Ans

Ans 6 $\tan^{-1}(\cot x) = \tan^{-1}\left(\tan\left(\frac{\pi}{2}-x\right)\right) = \frac{\pi}{2} - x$

$\therefore$ Req. Ans = $\int\left(\frac{\pi}{2} - x\right)dx = \frac{\pi}{2}x - \frac{x^2}{2} + c$ Ans.

Ans 7 $\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$ integrating,

$\tan^{-1}y = \tan^{-1}x + c$ Ans.

Ans 8 Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{(1)(1)+(1)(0)+(0)(1)}{\sqrt{1^2+0^2+1^2}} = \frac{1}{\sqrt{2}}$ Ans.

Ans 9. here, $|\vec{a}| = 1$

Now, $(\vec{x}-\vec{a}).(\vec{x}+\vec{a}) = 15$

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 15 \Rightarrow |\vec{x}|^2 - 1 = 15 \Rightarrow |\vec{x}|^2 = 16 \Rightarrow |\vec{x}| = \underline{4}$

Ans 10. Req. eqn is $\frac{x-5}{3} = \frac{y-2}{2} = \frac{z+4}{-4} = \lambda$ Ans.

Section - B

Ans 15. $y = (\tan^{-1}x)^2$

differentiating,

$\frac{dy}{dx} = 2.\tan^{-1}x.\frac{1}{1+x^2}$

$\Rightarrow (1+x^2).\frac{dy}{dx} = 2\tan^{-1}x$

diff. again both sides,

$(1+x^2).\frac{d^2y}{dx^2} + \frac{dy}{dx}.(2x) = 2.\frac{1}{1+x^2}$

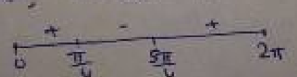
$\Rightarrow (1+x^2).\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} = 2$ H. proved.

Ans 16 $f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x$

for inc/dec $f'(x) = 0 \Rightarrow \cos x = \sin x \Rightarrow \tan x = 1$

Since, $x \in [0, 2\pi]$ $\therefore x = \frac{\pi}{4}, \frac{5\pi}{4}$



f is inc. in $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$

and f is dec. in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ Ans.

Ans 18. Let req. vector be $\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$

Now, $\vec{a} \perp \vec{d} \Rightarrow \vec{a}.\vec{d} = 0 \Rightarrow x + 4y + 2z = 0$ —— ①

$\vec{b} \perp \vec{d} \Rightarrow \vec{b}.\vec{d} = 0 \Rightarrow 3x - 2y + 7z = 0$ —— ②

also, $\vec{c}.\vec{d} = 18 \Rightarrow 2x - y + 4z = 18$ —— ③

solving ①, ② & ③,

$$\begin{array}{l} \cancel{2x} + 8y + \cancel{4z} = 0 \\ \underline{\cancel{2x} - y + \cancel{4z} = 18} \\ \quad 9y = -18 \end{array} \Rightarrow y = -2$$

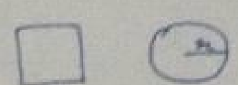
using $y = -2$ in eq. ① + ②,

$x + 2z = 8$
$3x + 7z = -4$ $\Big\}$ $\therefore x = 64$, $z = -28$

$\therefore \vec{d} = 64\hat{i} - 2\hat{j} - 28\hat{k}$ Ans.

Section-C

Ans 24



Given: $P = 4x + 2\pi r = k$

$\Rightarrow x = \frac{k - 2\pi r}{4}$

(To min. when $x = 2r$)

Now, $A = x^2 + \pi r^2$

$= \frac{1}{16}(k - 2\pi r)^2 + \pi r^2$

$\frac{dA}{dr} = \frac{1}{16} \times 2(k - 2\pi r)(-2\pi) + 2\pi r$

for max/min, $\frac{dA}{dr} = 0 \Rightarrow \frac{1}{4}\cancel{\pi}(k - 2\pi r) = 2\cancel{\pi} r$

$\Rightarrow k - 2\pi r = 8r \Rightarrow k = (2\pi + 8)r \Rightarrow r = \frac{k}{2(\pi+4)}$

$\frac{d^2A}{dr^2} = 4\pi^2 \times \frac{1}{8} + 2\pi = \frac{\pi^2}{2} + 2\pi > 0 \therefore A$ is min.

Now, $x = \frac{1}{4}\left(k - \cancel{2}\pi \cdot \frac{k}{\cancel{2}(\pi+4)}\right) = \frac{1}{4}\left(\frac{\cancel{\pi k} + 4k - \cancel{\pi k}}{\pi+4}\right) = \frac{k}{\pi+4}$

clearly, $x = 2r$ Hence proved.