

SECOND TERMINAL EXAMINATION 2012-13

MATHEMATICS

Class XII

Time : 3 Hours

Max. Marks : 100

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

SECTION – A

1. Let * be a binary operation on the set Q of non-zero rational numbers defined as $a*b = \frac{ab}{3}$. Write the identity element for *, if any.
2. Using the principal values, write the value of $\cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$.
3. If A is a matrix of order 3×4 and B is a matrix of order 4×3 , find the order of the matrix (AB) .
4. If the matrix $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew-symmetric, write the values of $a+b+c$.
5. A square matrix A , of order 2, has $|A| = 3$, write the value of $A.(adj A)$.
6. A and B appear for an interview for two posts. The probabilities of their selection are respectively $\frac{1}{3}$ and $\frac{2}{5}$. What is the probability that only one of them will be selected?
7. Write the position vector of a point dividing the line segment joining points A and B with position vectors \vec{a} and \vec{b} externally in the ratio 2:1, where $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$.

8. Write the value of $|\vec{a} - \vec{b}|$, if two vectors \vec{a} and \vec{b} are such that $|\vec{a}|=2$, $|\vec{b}|=3$ and $\vec{a} \cdot \vec{b}=4$.

9. Find the value of λ such that the line $\frac{x-2}{9} = \frac{y-1}{\lambda} = \frac{z-3}{-6}$ is perpendicular to the plane $3x - y - 2z = 7$.

10. Write the value of $\int \frac{e^{5 \log x} - e^{3 \log x}}{x^2 - 1} dx$.

SECTION B

11. Using properties of determinants, show that:

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy + yz + zx).$$

12. Let $f : R - \{3\} \rightarrow R - \{1\}$ be a function given by $f(x) = \frac{x-2}{x-3}$. Check that f is invertible or not?

If yes, find its inverse.

OR

Define a binary operation $*$ on the set $\{0, 1, 2, 3, 4, 5\}$ as $a * b = \begin{cases} a+b & \text{if } a+b < 6 \\ a+b-6 & \text{if } a+b \geq 6 \end{cases}$

Show that zero is the identity for this operation and each element a of the set is invertible with $6 - a$ being the inverse of a .

13. Solve for x :

$$2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x).$$

14. If $\log(\sqrt{x^2 + y^2}) = \tan^{-1} \frac{y}{x}$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

OR

If $y = e^x (\sin x + \cos x)$, prove that $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$.

15. Find the values of x for which $f(x) = (x(x-2))^2$ is an:

(a) increasing function

(b) decreasing function

16. Using differentials, find the approximate value of $(81.5)^{1/4}$

OR

Find the equation of the tangent to the curve $y = \frac{x-7}{x^2-5x+6}$ at the point where it cuts the x-axis.

17. Solve the following differential equation:

$$x^2 dy + (xy + y^2) dx = 0, \text{ given } x=1 \text{ when } y=1$$

OR

$$(1+x^2) dy + 2xy dx = \cot x dx; x \neq 0.$$

18. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ .

19. A coin is biased such that a head is three times as likely to occur than a tail. When it is tossed twice, find the probability distribution of number of heads.

In your daily life, which is the most popular application of tossing a coin?

20. Evaluate the following: $\int \frac{x+3}{\sqrt{5-4x-x^2}} dx$

21. Evaluate the following: $\int \frac{\sin x + \cos x}{\sqrt{\sin x \cdot \cos x}} dx$

OR

Evaluate the following: $\int \frac{2 + \sin 2x}{1 + \cos 2x} e^x dx$

22. Find the shortest distance between the following pair of lines:

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}) \text{ and } \vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$$

SECTION C

23. If $A = \begin{pmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{pmatrix}$, then find A^{-1} . Hence solve the following system of equations:

$$3x + 2y + z = 6; 4x - y + 2z = 5; 7x + 3y - 3z = 7$$

24. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Suggest two measures one must follow to avoid accidents.

25. Using the properties of definite integral, evaluate the following: $\int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x}) dx$.

26. Find the equation of the plane which contains line of intersection of planes

$$\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0, \quad \vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$$
 and which is perpendicular to the plane

$$\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0.$$

OR

Find the image of the point $(1, 2, 3)$ in the plane $x + 2y + 4z = 38$. Also find the distance of the given point from the plane.

27. Find the area of the region enclosed between the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

OR

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the line $\frac{x}{4} + \frac{y}{3} = 1$.

28. Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12 cm is 16 cm.

29. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 5760 to invest and has space for at most 20 items. A fan costs him Rs 360 and a sewing machine costs him Rs 240. He expects to sell a fan a profit of Rs 22 and a sewing machine at a profit of Rs 18. Assuming that he can sell all the items that he buys, how should he invest his money to maximize the profit? What is the maximum profit?

In your opinion what are the main qualities a person must possess to become a successful businessman/businesswoman?