CRPF PUBLIC SCHOOL, ROHINI

THIRD Intra School MatheMaticS olyMpiad 2012

CLASS XII

Max. Marks: 50 **Max. Time: 1 hour 30 minutes**

General Instructions:

1. Q1-15 (Section A) each MCQ carries 2 mark. Each question has five choices (A, B, C, D or E). Select the correct answer to each question and darken the corresponding circle in the Answer Sheet provided to you. **THERE IS NO NEGATIVE MARKING.** Marking of more than one circle for an answer shall be awarded zero mark.

2. Q16-20 (Section B) each question carries 4 mark. You are to give the complete solution. Marking will be done stepwise.

SECTION – A

Q1. How may integer values of x satisfy $\frac{x-2}{5} < \frac{8}{3} < \frac{x+6}{7}$.

(A) 2 (B) 3 (C) 4 (D) 5 (E) None of these

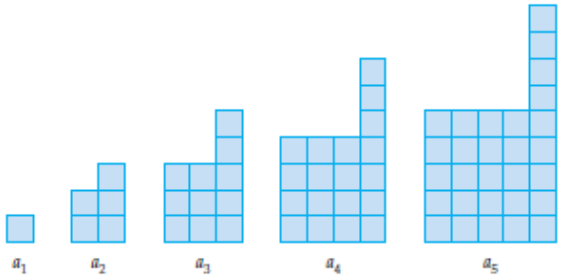
Q2. Given that $m < 0, n > 0$ and are the roots of the equation $18x^2 - 7x - 1 = 0$. What is the numerical value of $\left(m+\frac{1}{3}\right)\left(n+\frac{1}{3}\right)$?

(A) $\frac{5}{27}$ (B) (C) (D) (E) None of these

Q3. At present, the sum of the ages of a father and his son is 33 years. Determine the smallest number of years until the father's age is 4 times the son's age.

(A) 1 (B) 1.5 (C) 2 (D) 4 (E) None of these

Q4. Assume the pattern shown by the square tiles in the following figure continues. How many tiles are in the tenth figure of the sequence?



(A) 99 (B) 100 (C) 109 (D) 119 (E) None of these

Q5. The value of $\frac{1}{2^2-1}+\frac{1}{4^2-1}+\frac{1}{6^2-1}+...+\frac{1}{20^2-1}$ is:

(A) $\frac{10}{21}$ (B) $\frac{13}{27}$ $\frac{15}{22}$ (C) (D) (E) None of these

Q6. Consider the following pattern:

$$\sqrt{1+1\times2\times3\times4}=5$$
$$\sqrt{1+2\times3\times4\times5}=11$$
$$\sqrt{1+3\times4\times5\times6}=19$$
$$\sqrt{1+4\times5\times6\times7}=29$$
.....

The value of $\sqrt{1+19\times20\times21\times22}$ is:

(A) 419 (B) 381 (C) 409 (D) 425 (E) None of these

Q7. If $f(x)=2x+1$ and $g(f(x))=4x^2+1$, the expression for $g(x)$ is:

(A) x^2-2x+2 (B) (C) (D) (E) None of these

Q8. In a rectangle $ABCD$, F is on the diagonal BD so that $AF\perp BD$. Also, $BC=30, CD=40$ and $AF=x$. The value of x is:

(A) 24 (B) 25 (C) 35 (D) 36 (E) None of these

Q9. An integer n with $100\le x\le 999$, is chosen at random. The probability that the sum of the digits of n is 24 is:

(A) $\frac{1}{90}$ (B) (C) (D) (E) None of these

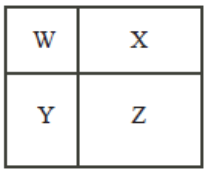
Q10. Which of the following must be an even integer?

(A) The average of two even integers (B) The average of two prime numbers

(C) The average of two perfect squares (D) The average of two multiples of 4

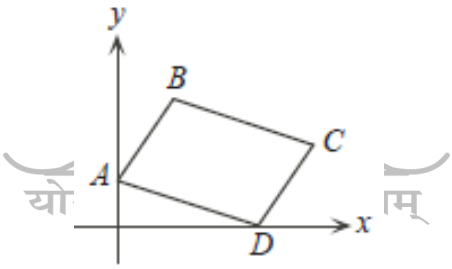
(E) The average of three consecutive integers

Q11. A rectangle is divided into four smaller rectangles, labelled W, X, Y, and Z, as shown. The perimeters of rectangles W, X and Y are 2, 3 and 5, respectively. The perimeter of rectangle Z is:



(A) 6 (B) 8 (C) 10 (D) 12 (E) None of these

Q12. In the diagram, the four points have coordinates A(0, 1), B(1, 3), C(5, 2), and D(4, 0). The area of quadrilateral ABCD (in sq. units) is:



(A) 8 (B) 9 (C) 10 (D) 12 (E) None of these

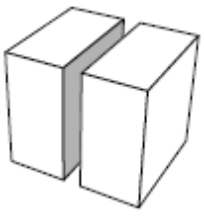
Q13. An unusual die has the numbers 2, 2, 3, 3, 5, and 8 on its six faces. Two of these dice are rolled. The two numbers on the top faces are added. The total numbers of different sums possible are:

(A) 9 (B) 10 (C) 8 (D) 7 (E) 12

Q14. The smallest number n such that $n!$ is divisible by 990 is:

(A) 11 (B) 10 (C) 12 (D) 9 (E) None of these

Q15. A cube is sliced up by planes parallel with its faces. How many planes do we use to slice if the total of the surface areas of the resulting pieces is twice the surface area of the original cube?



(A) 3 (B) 2 (C) 4 (D) 5 (E) 6

SECTION – B

Q16. In the expression $S = \sqrt{x_1 + x_2 - x_3 - x_4}$, the variables x_1, x_2, x_3, x_4 are replaced by 1, 2, 3, and 4 with no repetitions allowed. There are 24 possible replacements. Determine the number of times *S* will be a real number.

Q17. The functions f and g satisfy $f(x)+g(x) = 3x+5$ and $f(x)-g(x) = 5x+7$ for all values of x. Determine the value of $2f(2)g(2)$.

Q18. Find the least and the greatest value of $\frac{x^2+14x+9}{x^2+2x+3}$ for all real values of x.

Q19. Two cars start together in the same direction from the same place. The first goes with uniform speed of 10 km/h. The second goes at a speed of 8 km/h in the first hour and increases the speed by $\frac{1}{2}$ km in each succeeding hour. After how many hours will the second car overtake the first car if both cars go nonstop?

Q20. The edges of a rectangular box are 2, 3, and 4. What is the length of the shortest path from one vertex to the opposite vertex on the surface of the box?

**********END OF PAPER**********

NOTE: The **Solution Key** of this paper will be available on School's blog **www.crpfpsrohini.blogspot.in** today after 6 pm. The **Result** will be declared on 30 November 2012 and will be available on School's blog.