THIRD InTRa ScHool MaTHeMaTIcS olyMpIaD 2012

CLASS IX

SolutionS

SECTION A

Q1.A
Q2.E
Q3.

Solution 1
The first stack is made up of $1+2+3+4+5=15$ blocks.
The second stack is made up of $1+2+3+4+5+6=21$ blocks.
There are 36 blocks in total.
We start building the new stack from the top.
Since there are more than 21 blocks, we need at least 6 rows.
For 7 rows, $1+2+3+4+5+6+7=28$ blocks are needed.
For 8 rows, $1+2+3+4+5+6+7+8=36$ blocks are needed.
Therefore, Clara can build a stack with 0 blocks leftover.

Solution 2
Since the new stack will be larger than the second stack shown, let us think about adding new rows to this second stack using the blocks from the first stack.
The first stack contains $1+2+3+4+5=15$ blocks in total.
The first two rows that we would add to the bottom of the second stack would have 7 and 8 blocks in them, for a total of 15 blocks.
This uses all of the blocks from the first stack, with none left over, and creates a similar stack.
Therefore, there are 0 blocks left over.

ANSWER: (A)

Solution 1

Q4.

Since each of five friends paid an extra \$3 to cover Luxmi's portion of the bill, then Luxmi's share was $5 \times \$3 = \$15$.
Since each of the six friends had an equal share, then the total bill is $6 \times \$15 = \$90$.

ANSWER: (A)

Q5.

To maximize the value of the given expression, we must make the individual fractions as large as possible. We do this by selecting the largest value possible for the numerator of a fraction and then the smallest possible value from the numbers remaining, as its denominator. Using this principle, the first 'fraction' would be $\frac{6}{1}$, the second $\frac{5}{2}$ and the third $\frac{4}{3}$. This gives, $6+\frac{5}{2}+\frac{4}{3}=\frac{36+15+8}{6}=\frac{59}{6}=9\frac{5}{6}$.

ANSWER: (B)

Q6.

The first ten balloons are popped in the following order: *C*, *F*, *I*, *L*, *D*, *H*, *A*, *G*, *B*, and *K*. The remaining two balloons are *E* and *J*. ANSWER: (D)

Q7.

We need first to realize that each zero at the end of any product results from the product containing a factor of 2 and a factor of 5.
Let's look at the number 90:
90 can be written as $15\times 6=90$, which ends in a zero.
Because $15=3\times 5$ contains the factor 5 and
$6=3\times 2$ contains the factor 2,
we could look at it like this:

$$15\times 6=3\times 5\times 3\times 2=(3\times 3)\times(5\times 2)=9\times 10=90.$$

Another example is 160, which can be written as

$$8\times 20=(2\times 2\times 2)\times(2\times 2\times 5)=(8\times 2)\times(2\times 5)=16\times 10=160$$

Knowing this, we need to work out the number of factors of 5 in the product: $1\times 2\times 3\times.....\times 199\times 200$.
Clearly there are 40 multiples of 5, but there are also some of these multiples of 5 with additional factors of 5.
They are: 25, 50, 75, 100, 150, 175, 200, seven of them which each have 1 extra factor of 5, and 125 which has 2 extra factors of 5.
$\therefore$ The number of factors of 5 in the product is

$$40+7+2=49$$

There are over 100 factors of 2 in the product (i.e. all the even numbers!), so we know that for each factor of 5, there is at least one factor of 2. The product must therefore end with 49 zeros.

ANSWER (D)

Q8.

ANSWER: A
Let length = l and breadth = 1 when closed. Now when opened length = 2 and breadth = l. We have $\frac{l}{1}=\frac{2}{l}$ $\therefore l^2=2$ $\therefore l=\sqrt{2}$ if $b=1$ and therefore the ratio of length to breadth is $\sqrt{2}:1$

Q9.(B)
Q10.In container 1 milk=12x/5, water=3x/5
In container 2 milk=3x, water=x
In container 3 milk=25x/7, water=10x/7
In container 4 total milk=314x/35
Total water=106x/35
Ratio=157:53
ANSWER (C)

Q11.(D)

The first figure has 8 unshaded squares. The second figure has 12 unshaded squares. The third figure has 16 unshaded squares. So the number of unshaded squares increases by 4 with each new figure. So the number of unshaded squares in the tenth figure should be $8+4(9)=44$ (we add 4 nine times to get from the first figure to the second, from the second to the third, and so on).

Q12. $\frac{40x}{100}=80$

X=200 ANSWER(C)

Q13. Mean=48/11=4.36

Median=6th term=5

Mode=5 ANSWER (B)

Q14.Every time+7 and -1 ANSWER (D)

Q15.(E)

SECTION B

Q16. $\sqrt{(\sqrt{3})^2+(\sqrt{2})^2+2\sqrt{3}\sqrt{2}}$

$=\sqrt{(\sqrt{3}+\sqrt{2})^2}$

$=\sqrt{3}+\sqrt{2}$

=3.14

Q17.p (-1)=19 p(1)=5

a+b=13 b-a=3

2b=16 b=8 a=5 p (2)=10

Q18.Area of a rhombus=2area of a triangle=2×432

Each cow will get=2×432/18=48m²

Q19.

Let v_1 and v_2 be the speeds of bus 1 and bus 2 respectively, let t_1 and t_2 be the times at which they pass each other and let x be the distance between the towns. By considering the first time they pass we see that $\frac{7}{t_1}=v_1$, $\frac{x-7}{t_1}=v_2$, which implies that $\frac{v_2}{v_1}=\frac{x-7}{7}$. By considering the second time they pass we get $\frac{x+4}{t_2}=v_1$, $\frac{2x-4}{t_2}=v_2$, which implies that $\frac{v_2}{v_1}=\frac{2x-4}{x+4}$. Thus we have $\frac{x-7}{7}=\frac{2x-4}{x+4}$, which gives $x^2=17x$ and so $x=17$.

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Q20.

The side length of the square is 2 and thus the diameter of the circle has length $2\sqrt{2}$ which is also the length of the diagonal AC (or BD). The area of the circle is thus $\pi\left(\sqrt{2}\right)^2=2\pi$. Since the side length of the square is 2, it will have an area of 4. From this, we calculate the area of the circle outside the square to be $2\pi-4$. To calculate the shaded area, we first calculate the area of each semi-circle. Each of the semi circles has a radius of 1 meaning that each semi-circle will have an area of $\frac{1}{2}\left[\pi(1)^2\right]=\frac{1}{2}\pi$. In total, the four semi-circles have an area of 2π. Thus the shaded area has an area of $2\pi-(2\pi-4)=4$. ANSWER: (B)