FIRST TERMINAL EXAMINATION 2012-13

MATHEMATICS

Class XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

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SECTION – A

Q1. The binary operation $*: R \times R \to R$ is defined as $a*b = 2a + b$. Find $(2*3)*4$.

Q2. Evaluate: $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$.

Q3. Express $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$; $|x|>1$ in the simplest form.

Q4. If $adj\, A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $adj\, B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$, write the value of $adj(AB)$.

Q5. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$.

Q6. Find the derivative of $y = \log_7 x$ w.r.t. x.

Q7. Find $\left.\frac{dy}{dx}\right|_{x=\pi/2}$ where $y = e^{\sin x}$.

Q8. What is the slope of the normal to the curve $y = x^3 - 3x$ at $x = 2$?

Q9. What is the value of the integral $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$.

Q10. Evaluate: $\int \frac{x^2}{1+x^3} dx$

SECTION – B

Q11. Express the matrix $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$ as the sum of symmetrc and skew-symmetric matrix.

Q12. Examine which of the following is a binary operation:

$(a)\ a*b=\dfrac{a+b}{2}\ ;\ a,b\in N$

$(b)\ a*b=\dfrac{a+b}{2}\ ;\ a,b\in Q$

For binary operation check the commutative and associative property.

OR

Show that the relation R on the set Z of integers given by $R=\{(a,b):4 \text{ divides } a-b\}$, is an equivalence relation.

Q13. Solve the following equation :

$$\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}(3x)$$

OR

Prove that: $\sin^{-1}\left(\dfrac{8}{17}\right)+\sin^{-1}\left(\dfrac{3}{5}\right)=\cos^{-1}\left(\dfrac{36}{85}\right)$

Q14. Using properties of determinants, prove that:

$$\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^3 & b^3 & c^3\end{vmatrix}=(a-b)(b-c)(c-a)(a+b+c)$$

Q15. If the function $f(x)=\begin{cases}3ax+b & \text{for } x>1\\ 11 & \text{for } x=1\\ 5ax-2b & \text{for } x<1\end{cases}$, is continous at $x=1$, find the values of a and b.

Q16. If $\cos y=x\cos(a+y)$ with $\cos a\neq\pm1$, prove that $\dfrac{dy}{dx}=\dfrac{\cos^2(a+y)}{\sin a}$.

Q17. If $y=3\cos(\log x)+4\sin(\log x)$, then show that $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y=0$.

OR

If $(\cos x)^y=(\sin y)^x$, find $\dfrac{dy}{dx}$.

Q18. Prove that $y=\dfrac{4\sin\theta}{2+\cos\theta}-\theta$ is an increasing function in $\left[0,\dfrac{\pi}{2}\right]$.

Q19. Find the point on the curve $y=x^3-11x+5$ at which the equation of tangent is $y=x-11$.

Q20. Evaluate the following: $\displaystyle\int\frac{x+2}{\sqrt{(x-2)(x-3)}}dx$

Q21. Evaluate the following: $\int\sin x\sin 2x\sin 3x\,dx$

Q22. Evaluate the following: $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

OR

Evaluate the following: $\int\left(\frac{\sin 4x-4}{1-\cos 4x}\right)e^x\,dx$

SECTION – C

Q23. Using Matrix Method , solve the given system of equations:

$$x+2y-3z=-4$$
$$2x+3y+2z=2$$
$$3x-3y-4z=11$$

Q24. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

OR

Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

Q25. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

Q26. Let $f:N\to R$ be a function defined as $f(x)=4x^2+12x+15$. Show that $f:N\to S$, where,

S is the range of *f*, is invertible. Find the inverse of *f*.

Q27. (a) Prove that : $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}\ ;\ x\in\left(0,\frac{\pi}{4}\right)$

(b) Find the value of the following: $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]\ ;\ |x|<1,\ y>0\ ,\ xy<1$

Q28. (a) Find $\frac{dy}{dx}$ if $x=a(\cos\theta+\theta\sin\theta)\ ;\ y=a(\sin\theta-\theta\cos\theta)$.

(b) Verify Rolle's Theorem for the function $f(x)=x^2+2x-8\ ,\ x\in[-4\ ,\ 2]$.

Q29. Evaluate the following: $\int\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$

OR

Evaluate the following: $\int\frac{x^2+1}{(x-1)^2(x+3)}\,dx$