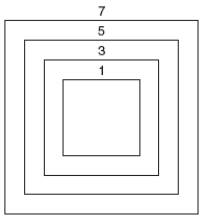
Try these!

1. You have a lot of Re 1, Rs 2 and Rs 5 coins in your pocket. You reach in and pull out three of the coins without looking. What are the different amounts of money you could have taken from your pocket?

2. If 3 chickens can lay 4 eggs in 5 days, how many days will it take 12 chickens to lay 48 eggs?

3. There are 6 points, no three of which lie on the same line. How many straight line segments are needed to connect every possible pair of dots?

4. Sanjeev works for an art gallery. He is designing a large wall covering for a client. The entire design is made up of 50 concentric squares (squares with the same center and sides parallel). Figure shows the first four squares of his design and gives the length of one side of each square. Sanjeev is going to outline the perimeter of each square with wool. How many feet of wool does he need to outline all 50 squares?



5. Sahrukh Khan has 35 feet of flat board. He wants to make bookshelves that will be exactly 5 feet long. It takes him 2 minutes to make one cut through the board. How long will it take him to make the shelves?

6. There are four finalists, Seema, Bhavya, Anisha and Shallu, who are competing for two prizes in a photo contest. In how many different ways can the first and second prizes be awarded?

7. A zookeeper has ostriches and elephants in one part of the zoo. Altogether the animals account for 60 heads and 180 legs. How many of each animal does he have?

8. How many different four-digit numbers can be made using the digits 1, 2, 3 and 4? What is the sum of all the number so formed?

S. D. School Workshop Try these! SOLUTIONS

Ans 1.

Re 1	Rs 2	Rs 5	Sum
3	0	0	Rs 3
2	1	0	Rs 4
2	0	1	Rs 7
1	2	0	Rs 5
1	1	1	Rs 8
1	0	2	Rs 11
0	3	0	Rs 6
0	2	1	Rs 9
0	1	2	Rs 12
0	0	3	Rs 15

Check to make sure all the amounts are different. In this problem, the list is the actual answer.

Answer: There are 10 different sums you could make with three coins.

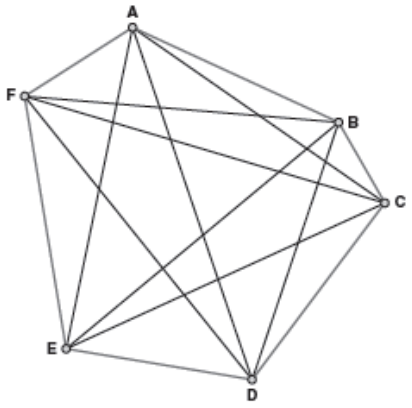
Ans 2.

Solution: Rather than consider this problem in its entirety, we solve it as a pair of simpler problems. If 3 chickens lay 4 eggs in 5 days (that is, 3 chickens = 4 eggs in 5 days), then multiplying the number of eggs and the number of chickens by 4 to gives us: 12 chickens lay 16 eggs in the same 5 days (that is, 12 chickens = 16 eggs in 5 days). To get 48 eggs, we need to multiply 16 eggs by 3. To do this we multiply the number of days and the number of eggs by 3 to get 12 chickens lay 48 eggs in 15 days.

Answer: It will take the chickens 15 days to lay 48 eggs.

Ans 3.

Solution: By organizing data, we take the six points and begin by connecting point *A* to each of the other points. Then we have 5 line segments, *AB*, *AC*, *AD*, *AE*, and *AF* (see Figure 2.1).



Now consider point *B*. In similar fashion, we have *BA*, *BC*, *BD*, *BE*, and *BF*. But *BA* is the same line segment as *AB*, so it need not be drawn again. Thus, from point *B*, we have only 4 segments. If we continue this pattern, we have 3 segments from point *C* (*CD*, *CE*, and *CF*, while *AC* and *BC* have been drawn). Thus we have $5 + 4 + 3 + 2 + 1 = 15$ lines drawn.

Answer: There will be 15 line segments to connect the points as required.

Ans4.

Solution: We examine the data and see if we can find a pattern to help us. Because a square has four equal sides, we can find the sum of one side of each square and then multiply by four. This will let us use smaller numbers. Starting from the smallest square and moving outward, the lengths of the sides form a sequence of the first 50 odd numbers: 1, 3, 5, 7, . . . , 99. We need to compute the sum $1 + 3 + 5 + 7 + \ldots + 99$.

$1 = \mathbf{1}$

$1 + 3 = \mathbf{4}$

$1 + 3 + 5 = \mathbf{9}$

$1 + 3 + 5 + 7 = \mathbf{16}$

$1 + 3 + 5 + 7 + 9 = \mathbf{25}$

Notice that the sums are perfect squares. In fact, each sum is the square of the number of terms being added.

$1 = 1 = \mathbf{1^2}$

$1 + 3 = 4 = \mathbf{2^2}$

$1 + 3 + 5 = 9 = \mathbf{3^2}$

$1 + 3 + 5 + 7 = 16 = \mathbf{4^2}$

$1 + 3 + 5 + 7 + 9 = 25 = \mathbf{5^2}$

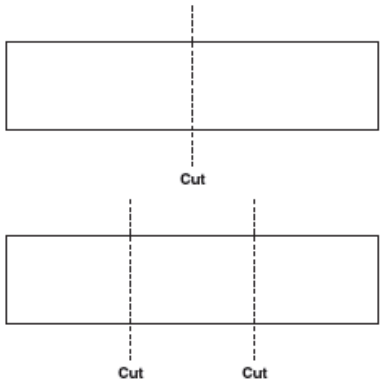
Using this pattern, the sum of all 50 terms will be 50^2, or 2,500. Now we multiply by 4 to find the perimeter and arrive at 10,000 feet of wool.

You can also look at this problem from a different point of view, using a different pattern (or, if you wish, organizing data). We need to compute the sum $1 + 3 + 5 + 7 + \ldots + 99$. Instead of adding the numbers in the order in which they are written here, consider the partial sums: $1 + 99 = 100$, $3 + 97 = 100$, $5 + 95 = 100$, . . . , $49 + 51 = 100$. The total of these sums is $25 \times 100 = 2{,}500$. As before, we then multiply by 4 to get 10,000.

Ans: Sanjeev will need 10,000 feet of wool for the wall covering.

Ans 5.

Sahrukh has 35 feet of board. If each shelf is to be 5 feet long, he will make 7 shelves. We have to find out how many cuts he must make. To do this, let's look at simpler versions of the problem. Suppose Sahrukh had a board and needed to cut it into two parts. He would need 1 cut. How about cutting a board into three parts? He would need 2 cuts.



It looks like he needs one cut fewer than the number of pieces. There is a pattern to be seen. Now we can solve our problem. To make 7 shelves, Art must make 6 cuts. Because each cut takes him 2 minutes, it will take him 12 minutes to cut the shelves.

Answer: Sahrukh will take 12 minutes to cut the board into seven 5-foot shelves.

Ans 6.

Solution: One way to solve this problem is to draw a tree diagram as shown in Figure 10.2.

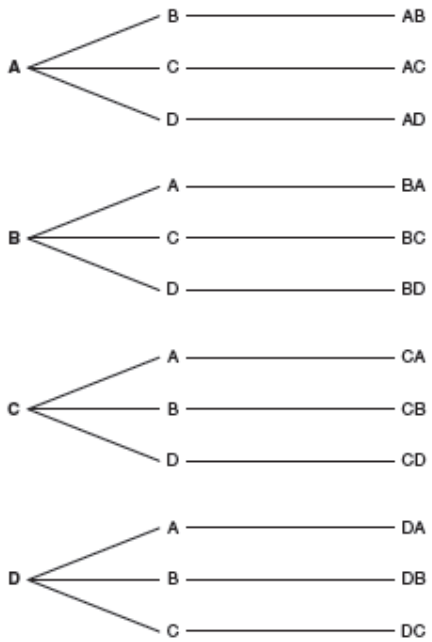


Figure 10.2

This gives the answer, 12.

Let's approach the problem from a different point of view. The first prize can be awarded to any one of the 4 people. Once it has been awarded, the next prize can be awarded to each of 3 people. Thus there are 4×3 or 12 ways to award the two prizes.

Ans 7.

Solution: We can reduce the complexity of the problem and work with a simpler but equivalent set of numbers. Divide by 10. Let's try to solve the problem for 6 heads and 18 legs. We will work with these smaller numbers, and then we will go back to the original numbers.

Ostriches have 2 legs and elephants have 4 legs. Make a drawing and represent the heads with a **0**.

0	0	0	0	0	0

Now, whether it's an elephant or an ostrich, it has *at least* 2 legs. Let's put these "//" on each of the heads (**0**).

0	0	0	0	0	0
//	//	//	//	//	//

That accounts for 12 legs. The rest go on in pairs on three of the heads.

//	//	//			
0	0	0	0	0	0
//	//	//	//	//	//

There are 3 elephants and 3 ostriches. Now multiply by 10 to find the actual answers. (Remember, we divided by 10 to get smaller numbers to work with.)

Notice that this problem also makes use of the Make a Drawing strategy. (As we have stated earlier, more than one strategy will often be used to solve a problem.)

Answer: He has 30 ostriches and 30 elephants.

Ans 8.

Here, unit digit can take any of the four digits. Then ten's place can take any value from remaining three digits. Similarly hundred and thousand place can take remaining digits with values two and one respectively. So, total number formed is 4 x 3 x 2 x 1 = 24.

Sum of all numbers =

6(1 + 2 + 3 + 4) x (1 + 10 + 100 + 1000) = 60 x 1111 = 66660 (think how!)