ASSIGNMENT CLASS XII VECTOR ALGEBRA

1. In a regular hexagon ABCDEF, if $\overrightarrow{AB}=\vec{a}$ and $\overrightarrow{BC}=\vec{b}$, then express $\overrightarrow{CD},\overrightarrow{DE},\overrightarrow{EF},\overrightarrow{FA},\overrightarrow{AC},\overrightarrow{AD},\overrightarrow{AE}$ and $\overrightarrow{CF}$ in terms of $\vec{a}$ *and* $\vec{b}$.

2. If $\vec{a}=\hat{i}+\hat{j}$, $\vec{b}=\hat{j}+k$, $\vec{c}=k+\hat{i}$, find a unit vector in the direction of $\vec{a}+\vec{b}+\vec{c}$.

3. The position vectors of the points P, Q and R are $\hat{i}+2\hat{j}+3k, -2\hat{i}+3\hat{j}+5k, 7\hat{i}-k$ respectively. Prove that P, Q and R are collinear.

4. If $\vec{a}=\hat{i}+2\hat{j}+3k$, $\vec{b}=2\hat{i}+4\hat{j}-5k$ represents two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

5. Prove that the points $\hat{i}-\hat{j}$, $4\hat{i}+3\hat{j}+k$, $2\hat{i}-4\hat{j}+5k$ are the vertices of a right angled triangle.

6. If the position vectors of the vertices of a triangle ABC are $\hat{i}+2\hat{j}+3k, 2\hat{i}+3\hat{j}+k, 3\hat{i}+\hat{j}+2k$, prove that ΔABC is an equilateral triangle.

7. Write the position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ externally in the ratio 1: 4, where $\vec{a}=2\hat{i}+3\hat{j}+4k$ and $\vec{b}=-\hat{i}+\hat{j}+k$.

8. Find the projection of $\vec{b}+\vec{c}$ on $\vec{a}$, where $\vec{a}=2\hat{i}-2\hat{j}+k$, $\vec{b}=\hat{i}+2\hat{j}-2k$ and $\vec{c}=2\hat{i}-\hat{j}+4k$.

9. If $\vec{a}=\hat{i}+\hat{j}+2k$ and $\vec{b}=3\hat{i}+2\hat{j}-k$, find the value of $\left(\vec{a}+3\vec{b}\right).\left(2\vec{a}-\vec{b}\right)$.

10. Find a vector whose magnitude is 3 units and which is perpendicular to each of the vectors $\vec{a}=3\hat{i}+\hat{j}-4k$ and $\vec{b}=6\hat{i}+5\hat{j}-2k$.

11. If $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=3$, $|\vec{b}|=5$, $|\vec{c}|=7$, find angle between $\vec{a}$ and $\vec{b}$.

12. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}|=2$, $|\vec{b}|=3$ and $\vec{a}.\vec{b}=4$, find $|\vec{a}-\vec{b}|$ and $|\vec{a}+\vec{b}|$.

13. If a and $\hat{b}$ are unit vectors and θ is the angle between them, prove that $\sin\frac{\theta}{2}=\frac{1}{2}|a-\hat{b}|$ and $\cos\frac{\theta}{2}=\frac{1}{2}|a+\hat{b}|$.

14. Show that the points A, B and C with position vectors $2\hat{i}-\hat{j}+k$, $\hat{i}-3\hat{j}-5k$, $3\hat{i}-4\hat{j}-4k$ respectively, are the vertices of the right triangle. Also, find the remaining angles of the triangle.

15. If $\vec{a}=\hat{i}+2\hat{j}-3k$ and $\vec{b}=3\hat{i}-\hat{j}+2k$, then show that $\vec{a}+\vec{b}$ is perpendicular to $\vec{a}-\vec{b}$.

16. Find the angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, if $\vec{a}=2\hat{i}-\hat{j}+3k$ and $\vec{b}=3\hat{i}+\hat{j}-2k$.

17. Express the vectors $\vec{a}=5\hat{i}-2\hat{j}+5k$ as sum of two vectors such that one is parallel to the vector $\vec{b}=3\hat{i}+k$ and the other is perpendicular to $\vec{b}$.

18. The dot products of a vector with the vectors $\hat{i}+\hat{j}-3k$, $\hat{i}+3\hat{j}-2k$ and $2\hat{i}+\hat{j}+4k$ are 0, 5, 8 respectively. Find the vector.

19. Find a unit vector perpendicular to each of the vectors $\vec{a}=4\hat{i}-\hat{j}+3k$ and $\vec{b}=2\hat{i}+2\hat{j}-k$.

20. If $|\vec{a}|=\sqrt{26}$, $|\vec{b}|=7$ and $|\vec{a}\times\vec{b}|=35$, find $\vec{a}.\vec{b}$.

21. Find the area of the triangle whose adjacent sides are determined by the vectors $\vec{a}=-2\hat{i}-5k$ and $\vec{b}=\hat{i}-2\hat{j}-k$.

22. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=3\hat{i}+\hat{j}-2k$ and $\vec{b}=\hat{i}-3\hat{j}+4k$.

23. Find the area of the parallelogram whose diagonals are determined by the vectors $\vec{a}=2\hat{i}+3\hat{j}-6k$ and $\vec{b}=3\hat{i}-4\hat{j}-\hat{k}$.

24. Show that points whose position vectors are $\vec{a}=5\hat{i}+6\hat{j}+7k$, $\vec{b}=7\hat{i}-8\hat{j}-9k$, $\vec{c}=3\hat{i}+20\hat{j}+5k$ are collinear.

25. Let $\vec{a}=\hat{i}-\hat{j}$, $\vec{b}=3\hat{j}-k$, $\vec{c}=7\hat{i}-k$. Find a vector $\vec{d}$ such that it is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d}=1$

26. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the position vectors of the vertices A, B and C of a ΔABC respectively, find an expression for the area of ΔABC and hence deduce the condition for the points A, B and C to be collinear.

27. If $\vec{a}=\hat{i}+\hat{j}+k$, $\vec{c}=\hat{j}-k$ are given vectors, then find a vector $\vec{b}$ satisfying equations $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}.\vec{b}=3$.

28. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, then prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

29. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}, \vec{b}\neq\vec{c}$.

30. If $\vec{a}.\vec{b}=\vec{a}.\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ and $\vec{a}\neq\vec{0}$, then show that $\vec{b}=\vec{c}$.

ANSWERS

1. $\overrightarrow{CD}=\vec{b}-\vec{a}$, $\overrightarrow{DE}=-\vec{a}$, $\overrightarrow{EF}=-\vec{b}$, $\overrightarrow{FA}=\vec{a}-\vec{b}$, $\overrightarrow{AC}=\vec{a}+\vec{b}$, $\overrightarrow{AD}=2\vec{b}$, $\overrightarrow{AE}=2\vec{b}-\vec{a}$, $\overrightarrow{CF}=-2\vec{a}$

2. $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+k\right)$ 4. $\frac{1}{7}\left(3\hat{i}+6\hat{j}-2k\right)$, $\frac{1}{\sqrt{69}}\left(\hat{i}+2\hat{j}-8k\right)$ 7. $3\hat{i}+\frac{11}{3}\hat{j}+5k$ 8. 2 9. –15

10. $2\hat{i}-2\hat{j}+k$ 11. 60^0 12. $\sqrt{5}$, $\sqrt{21}$ 14. $\cos^{-1}\left(\sqrt{\frac{35}{41}}\right), \cos^{-1}\left(\sqrt{\frac{6}{41}}\right)$ 16. $\frac{\pi}{2}$

17. $6\hat{i}+2k, -\hat{i}-2\hat{j}+k$ 18. $\hat{i}+2\hat{j}+k$ 19. $\frac{1}{3}\left(-\hat{i}+2\hat{j}+2k\right)$ 20. 7 21. $\frac{1}{2}\sqrt{165}$ sq.units

22. $10\sqrt{3}$ sq.units 23. $\frac{1}{2}\sqrt{1274}$ sq.units 25. $\frac{1}{4}\left(\hat{i}+\hat{j}+3k\right)$ 26. $ar(\Delta ABC)=\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$;

$\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$ 27. $\vec{b}=\frac{1}{3}\left(5\hat{i}+2\hat{j}+2k\right)$