ASSIGNMENT CLASS XII RELATIONS AND FUNCTIONS

Important Formulas

If A and B are finite sets containing m and n elements, then

1. Total number of relations from the set A to set B is 2^{mn}.
2. Total number of relations on the set A is 2^{m^2}.
3. Total number of functions from the set A to set B is n^m.
4. Total number of bijective functions from the set A to set B is $m!, \text{if } m = n$, otherwise 0.
5. Total number of one-one functions from the set A to set B is ${}^nP_m$ if $n \geq m$, otherwise 0.
6. Total number of onto functions from set A to set B is $\sum_{r=1}^{n} (-1)^{n-r}\, {}^nC_r\, r^m$ if $m \geq n$, otherwise 0.

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7. Total number of binary operations on A is m^{m^2}

Q1. Prove that the relation R on the set $N \times N$ defined by $(a,b)R(c,d) \Leftrightarrow a+d=b+c$ for all $(a,b),(c,d) \in N \times N$ is an equivalence relation.

Q2. Let R be a relation defined on the set of natural numbers N as $R = \{(x,y): x,y \in N, 2x+y=41\}$. Show that R is neither reflexive nor symmetric but it is transitive.

Q3. Let S be a relation on the set R of all the real numbers defined by $S = \{(a,b) \in R \times R : a^2+b^2=1\}$. Prove that S is not an equivalence relation on R.

Q4. Le N be the set of all natural numbers and let R be a relation in N, defined by $R = \{(a,b): a \text{ is a factor of } b\}$. Show that R is reflexive and transitive but not symmetric.

Q5. Test whether the following relations R_1, R_2 are (i) reflexive (ii) symmetric (iii) transitive:

(i) R_1 on Q_0 defined by $(a,b) \in R_1 \Leftrightarrow a = 1/b$ (ii) R_2 on Z defined by $(a,b) \in R_2 \Leftrightarrow |a-b| \leq 5$

Q6. Classify the following functions as injection, surjection or bijection:

(i) $f: R \to R, \quad f(x) = |x|$

(ii) $f: Z \to Z, \quad f(x) = x^2 + x$

(iii) $f: Z \to Z, \quad f(x) = x - 5$

(iv) $f: R \to R, \quad f(x) = \sin x$

(v) $f: R \to R, \quad f(x) = x^3 + 1$

(vi) $f: R \to R, \quad f(x) = x^3 - x$

(vii) $f: R \to R, \quad f(x) = \sin^2 x + \cos^2 x$

$(viii)$ $f: Q - \{3\} \to Q, \quad f(x) = \frac{2x+3}{x-3}$

(ix) $f: Q \to Q, \quad f(x) = x^3 + 1$

(x) $f: R \to R, \quad f(x) = 5x^3 + 4$

Q7. Let $f: R - \{2\} \to R - \{1\}$ is a mapping defined by $f(x) = \frac{x-1}{x-2}$, show that f is bijective.

Q8. Find gof and fog when and $f: R \to R$ and $g: R \to R$ are defined by:

(i) $f(x) = 2x + 3$ *and* $g(x) = x^2 + 5$ (ii) $f(x) = 2x + x^2$ *and* $g(x) = x^3$

Q9. Let A be a set of all real numbers i.e. $A = R - \{-1\}$. Let $*$ be defined on A as $a * b = a + b + ab$.
Prove that (i) $*$ is a binary operation on A. (ii) $*$ is commutative and associative.
(iii) 0 is the identity element. (iv) $\frac{-a}{1+a}$ is the inverse of a.

Q10. Let R^+ be the set of all positive reals. Define an operation * on R^+ by $a*b = \frac{ab}{5}$ $\forall\, a, b \in R^+$. Show that the operation * is commutative as well as associative. Also, find the identity element and inverse of a.

ANSWERS

5. (i) R_1 is symmetric but neither reflexive nor transitive. (ii) R_2 is reflexive and symmetric but not transitive.

6. (i) Neither an injection nor a surjection (ii) Neither an injection nor a surjection (iii) bijective (iv) Neither an injection nor a surjection (v) bijective (vi) surjective but not injective (vii) Neither an injection nor a surjection $(viii)$ injective but not surjective (ix) injective (x) bijective 8.(i) $4x^2 + 12x + 14$, $2x^2 + 13$ (ii) $(x^2+2x)^3$, $2x^3 + x^6$ 10. $e = 5$, $b = \frac{25}{a}$