ASSIGNMENT CLASS XII DETERMINANTS

1. Find the values of x , if (i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ (ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$ (iii) $\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$

2. Using properties of determinants, show that:

(i) $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$ (ii) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

(iii) $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2(a+b+c)(ab+bc+ca-a^2-b^2-c^2)$ (iv) $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

(v) $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$ (vi) $\begin{vmatrix} 1 & a & a^2+bc \\ 1 & b & b^2+ca \\ 1 & c & c^2+ab \end{vmatrix} = 2(a-b)(b-c)(c-a)$

(vii) $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$ (viii) $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

(ix) $\begin{vmatrix} 1 & x+y & x^2+y^2 \\ 1 & y+z & y^2+z^2 \\ 1 & z+x & z^2+x^2 \end{vmatrix} = (x-y)(y-z)(z-x)$ (x) $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a+b)$

(xi) $\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = (a-b)(b-c)(a-c)(a^2+b^2+c^2)$ (xii) $\begin{vmatrix} -\alpha^2 & \alpha\beta & \gamma\alpha \\ \alpha\beta & -\beta^2 & \beta\gamma \\ \gamma\alpha & \beta\gamma & -\gamma^2 \end{vmatrix} = 4\alpha^2\beta^2\gamma^2$

(xiii) $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$ (xiv) $\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix} = (a+b+c)(a-c)^2$

(xv) $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$ (xvi) $\begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix} = 2a^3b^3c^3$

3. Without expanding the determinants, show that:

(i) $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ (ii) $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$ (iii) $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

4. Using properties of determinants, solve for x:

(i) $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$ (ii) $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$ (iii) $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$

Ans: $0, 3a$ Ans: $2/3, 11/3$ Ans: $1, 2, -3$

5. Using determinants, find the area of the triangle whose vertices are $(-2,4),(2,-6)$ and $(5,4)$. Are the given points collinear?

6. Find the equation of the line joining $A(1,3)$ and $B(0,0)$ using determinants and find k if $D(k,0)$ is a point such that $ar(\Delta ABD)$ is 3 sq. units.

7. Find the value of x if the area of the triangle with vertices $(x,4),(2,-6)$ and $(5,4)$ be 70 sq. cm.

8. Find the value of λ so that the points $(\lambda,2-2\lambda),(-\lambda+1,2\lambda)$ and $(-4-\lambda,6-2\lambda)$ are collinear?

9. Let $A=\begin{bmatrix}1 & -2 & 1\\ -2 & 3 & 1\\ 1 & 1 & 5\end{bmatrix}$, verify that: $(i)\ A(adjA)=(adjA)A=|A|I$ $\quad (ii)\ (adjA)^{-1}=adj(A^{-1})$ $\quad (iii)\ (A^{-1})^{-1}=A$

$(iv)\ (A^T)^{-1}=(A^{-1})^T$ $\quad (v)\ adjA^T=(adjA)^T$

10. If $A=\begin{bmatrix}2 & 2 & 1\\ -2 & 1 & 2\\ 1 & -2 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3 & 2\\ 1 & 1 & 1\\ 2 & -3 & -1\end{bmatrix}$, verify that: $(i)\ (AB)^{-1}=B^{-1}A^{-1}$ $\quad (ii)\ adj(AB)=(adjB)(adjA)$

11. Given $A=\begin{bmatrix}5 & 0 & 4\\ 2 & 3 & 2\\ 1 & 2 & 1\end{bmatrix}$ and $B^{-1}=\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$. Compute $(AB)^{-1}$. Ans: $(AB)^{-1}=\begin{bmatrix}-2 & 19 & -27\\ -2 & 18 & -25\\ -3 & 29 & -42\end{bmatrix}$

12. If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, prove that $A^2-4A-5I=0$. Hence find A^{-1}. Ans: $A^{-1}=\frac{1}{5}\begin{bmatrix}-3 & 2 & 2\\ 2 & -3 & 2\\ 2 & 2 & -3\end{bmatrix}$

13. Find A, so that $(i)\ A\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}=\begin{bmatrix}14 & 7\\ 7 & 7\end{bmatrix}$ $\quad (ii)\ \begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $\quad (iii)\ \begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}A=\begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$

Ans: $A=\begin{bmatrix}3 & 1\\ 1 & -2\end{bmatrix}$ $\quad A=\begin{bmatrix}25 & 15\\ -37 & -22\end{bmatrix}$ $\quad A=\begin{bmatrix}1 & -2 & -5\\ 3 & 4 & 0\end{bmatrix}$

14. Find whether the following system of equations is consistent or not, find the solution of the system also:

(i) $3x-y+2z=3$, $x-2y-z=1$, $2x+y+3z=5$

(ii) $5x-7y+z=11$, $6x-8y-z=15$, $3x+2y-6z=7$

(iii) $x+y+z=6$, $x+2y+3z=14$, $x+4y+7z=30$

Ans: inconsistent; consistent $x=1, y=-1, z=-1$; consistent, $x=k-2, y=8-2k, z=k$

15. Using matrix method, solve the following system of linear equations:

(i) $4x+2y+3z=2$, $x+y+z=1$, $3x+y-2z=5$

(ii) $x+2y+z=7$, $x+3z=11$, $2x-3y=1$

(iii) $x-y+z=2$, $2x-y=0$, $2y-z=1$

(iv) $x+y-z=1$, $3x+y-2z=3$, $x-y-z=-1$

(v) $x-y=3$, $2x+3y+4z=17$, $y+2z=7$

Ans: $x=\frac{1}{2}, y=\frac{3}{2}, z=-1$ $\quad x=2, y=1, z=3$ $\quad x=1, y=2, z=3$ $\quad x=2, y=1, z=2$ $\quad x=2, y=-1, z=4$

16. Find the product of matrices $A=\begin{bmatrix}-5 & 1 & 3\\ 7 & 1 & -5\\ 1 & -1 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 1 & 2\\ 3 & 2 & 1\\ 2 & 1 & 3\end{bmatrix}$ and use it for solving the equations $x+y+2z=1, 3x+2y+z=7, 2x+y+3z=2$. Ans: $AB=4I$, $x=2, y=1, z=-1$

17. If $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$, find A^{-1}. Hence solve the following: $x+2y+z=4$, $-x+y+z=0$, $x-3y+z=2$.

Ans: $x=9/5, y=2/5, z=7/5$