ASSIGNMENT CLASS XII

CONTINUITY AND DIFFERENTIABILITY

Important Formulas

1. A function $f(x)$ is continuous at $x=a$ iff $\lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)=f(a)$.

2. A function $f(x)$ is differentiable at $x=a$ iff $\lim_{x\to a}\dfrac{f(x)-f(a)}{x-a}$ exists finitely ie.

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}.$$

1. Show that the function $f(x)=\begin{cases} x\sin\dfrac{1}{x}, & x\neq 0 \\ 0 & x=0\end{cases}$ is continuous at $x=0$.

2. Show that the function $f(x)=\begin{cases} \dfrac{\sin x}{x}+\cos x, & x\neq 0 \\ 2 & x=0\end{cases}$ is continuous at $x=0$.

3. Show that the function $f(x)=\begin{cases} 5x-4 & \text{when } 0<x\leq 1 \\ 4x^3-3x & \text{when } 1<x<2\end{cases}$ is continuous at $x=1$.

4. Show that the function $f(x)=2x-|x|$ is continuous at $x=0$.

5. Show that the function $f(x)=\begin{cases} \dfrac{x-|x|}{x}, & x\neq 0 \\ 2 & x=0\end{cases}$ is discontinuous at $x=0$.

6. If f is defined as $f(x)=\begin{cases} \dfrac{|x-4|}{x-4}, & x\neq 4 \\ 0 & x=4\end{cases}$. Show that f is everywhere continous except at $x=4$.

7. Show that the function $f(x)=\begin{cases} \dfrac{|\sin x|}{x}, & x\neq 0 \\ 1 & x=0\end{cases}$ is discontinuous at $x=0$.

8. Find the value of k so that the function f is continous at the indicated point:

(a) $f(x)=\begin{cases} \dfrac{1-\cos 2x}{2x^2}, & x\neq 0 \\ k & x=0\end{cases}$ at $x=0$. (b) $f(x)=\begin{cases} \dfrac{x^2-25}{x-5}, & x\neq 5 \\ k & x=5\end{cases}$ at $x=5$.

9. Show that the function $f(x)=\begin{cases} x-1 & \text{if } x<2 \\ 2x-3 & \text{if } x\geq 2\end{cases}$ is not differentiable at $x=2$.

10. Discuss the continuity and differentiability of $f(x)=|x-1|+|x-2|$.

ANSWERS

8.(a) 1 (b) 10 **10**. continous but not differentiable at $x=1,2$