ASSIGNMENT CLASS XII INDEFINITE INTEGRALS

Evaluate the following Integrals:

1. $e^{x\log a}+e^{a\log x}+e^{a\log a}$
2. $\dfrac{1}{1+\cos x}$
3. $\dfrac{\sin x}{1+\sin x}$
4. $\tan^{-1}(\sec x+\tan x)$
5. $\dfrac{\sin^6 x+\cos^6 x}{\sin^2 x\cos^2 x}$
6. $\tan^{-1}\sqrt{\dfrac{1-\sin x}{1+\sin x}}$
7. $\dfrac{1}{\sqrt{3x+4}-\sqrt{3x+1}}$
8. $\dfrac{x^3}{x+2}$
9. $\sin^3 x\cos^3 x$
10. $\cos^4 x$
11. $\cos 2x\cos 4x$
12. $\dfrac{x^4+3}{x^2+1}$
13. $\sin 4x\cos 7x$
14. $\dfrac{1+\cos x}{1-\cos x}$
15. $\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$
16. $\dfrac{\sin 2x}{a^2\sin^2 x+b^2\cos^2 x}$
17. $\dfrac{\sin x}{\sin(x-a)}$
18. $\dfrac{1}{\sin(x-a)\sin(x-b)}$
19. $\dfrac{1-\cot x}{1+\cot x}$
20. $\dfrac{1}{e^x+1}$
21. $\dfrac{1}{\sqrt{x}\left(\sqrt{x}+1\right)}$
22. $\dfrac{\sin 2x}{(a+b\cos x)^2}$
23. $\dfrac{\sec^2\left(2\tan^{-1}x\right)}{1+x^2}$
24. $\sin^2 x\cos^5 x$
25. $\sin^5 x$
26. $\tan^3 x$
27. $\left(\dfrac{x+1}{x}\right)(x+\log x)^2$
28. $\dfrac{\cos^5 x}{\sin x}$
29. $\dfrac{1}{\sqrt{9-25x^2}}$
30. $\dfrac{x^4+1}{x^2+1}$
31. $\dfrac{1}{3+2x-x^2}$
32. $\dfrac{1}{x^2+8x+20}$
33. $\dfrac{e^x}{e^{2x}+6e^x+5}$
34. $\dfrac{1}{x\left(x^n+1\right)}$
35. $\dfrac{x}{x^4-x^2+1}$
36. $\dfrac{1}{\sqrt{9+8x-x^2}}$
37. $\dfrac{1}{\sqrt{16-6x-x^2}}$
38. $\dfrac{2x}{\sqrt{1-x^2-x^4}}$
39. $\dfrac{2x-3}{x^2+3x-18}$
40. $\dfrac{2\sin 2\phi-\cos\phi}{6-\cos^2\phi-4\sin\phi}$
41. $\dfrac{x+2}{2x^2+6x+5}$
42. $\dfrac{x^2}{x^2+6x+12}$
43. $\sqrt{\dfrac{a-x}{a+x}}$
44. $\dfrac{3x+1}{\sqrt{5-2x-x^2}}$
45. $\dfrac{1}{a^2\sin^2 x+b^2\cos^2 x}$
46. $\dfrac{\sin x}{\sin 3x}$
47. $\dfrac{1}{2-3\cos 2x}$
48. $\dfrac{\sin 2x}{\sin^4 x+\cos^4 x}$
49. $\dfrac{1}{1-2\sin x}$
50. $\dfrac{1}{5+4\cos x}$
51. $\dfrac{1}{3+2\sin x+\cos x}$
52. $\dfrac{3\sin x+2\cos x}{3\cos x+2\sin x}$
53. $(\log x)^2$
54. $\sin^{-1} x$
55. $\dfrac{x-\sin x}{1-\cos x}$
56. $\sec^3 x$

57. $\dfrac{\sin^{-1} x}{\left(1-x^2\right)^{3/2}}$

58. $x^2 \tan^{-1} x$

59. $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$

60. $\dfrac{\sin^{-1} x}{x^2}$

61. $e^x\left(\dfrac{1-\sin x}{1-\cos x}\right)$

62. $\dfrac{\log x}{\left(1+\log x\right)^2}$

63. $\dfrac{2-x}{\left(1-x\right)^2}e^x$

64. $\dfrac{1}{\log x}-\dfrac{1}{\left(\log x\right)^2}$

65. $e^{ax}\cos bx$

66. $\sqrt{7x-10-x^2}$

67. $\dfrac{\sqrt{16+\left(\log x\right)^2}}{x}$

68. $\left(3x-2\right)\sqrt{x^2+x+1}$

69. $\left(x+1\right)\sqrt{1-x-x^2}$

70. $\dfrac{2x-1}{\left(x-1\right)\left(x+2\right)\left(x-3\right)}$

71. $\dfrac{1}{\sin x-\sin 2x}$

72. $\dfrac{3x+1}{\left(x-2\right)^2\left(x+2\right)}$

73. $\dfrac{8}{\left(x+2\right)\left(x^2+4\right)}$

74. $\dfrac{x^2}{\left(x^2+1\right)\left(x^2+4\right)}$

75. $\dfrac{\tan\theta+\tan^3\theta}{1+\tan^3\theta}$

76. $\dfrac{\sin 2x}{\left(1+\sin x\right)\left(2+\sin x\right)}$

77. $\dfrac{1}{x\left(x^5+1\right)}$

78. $\dfrac{x^2-1}{x^4+x^2+1}$

79. $\dfrac{x^2+4}{x^4+16}$

80. $\dfrac{1}{x^4+1}$

81. $\sqrt{\tan x}$

82. $\sqrt{\cot x}$

83. $\dfrac{1}{\sin^4 x+\cos^4 x}$

84. $\dfrac{x^2-1}{x^4+1}$

85. $\dfrac{1}{\left(x-3\right)\sqrt{x+1}}$

86. $\dfrac{1}{\left(x^2-4\right)\sqrt{x+1}}$

87. $\dfrac{1}{\left(x+1\right)\sqrt{x^2-1}}$

88. $\dfrac{1}{x^2\sqrt{1+x^2}}$

89. $\dfrac{x}{x^3+x^2+x+1}$

90. $\dfrac{\sin\left(x-\alpha\right)}{\sin\left(x+\alpha\right)}$

91. $\dfrac{\left(x+1\right)e^x}{\left(x+2\right)^2}$

92. $\dfrac{1-\tan x}{x+\log\left(\cos x\right)}$

93. $\dfrac{\sqrt{x^2+a^2}}{x}$

94. $\dfrac{e^x}{\sqrt{5-4e^x-e^{2x}}}$

95. $\dfrac{1}{x^4-5x^2+16}$

96. $\sqrt{1+2\tan x\left(\sec x+\tan x\right)}$

97. $e^{\sqrt{x}}\dfrac{\cos e^{\sqrt{x}}}{\sqrt{x}}$

98. $\cos\left(\log x\right)$

99. $\dfrac{2\sin 2\theta-\cos\theta}{6-\cos^2\theta-4\sin\theta}$

100. $\tan^{-1}\left(\sqrt{\dfrac{1-\cos 2x}{1+\cos 2x}}\right)$

ANSWERS (INDEFINITE INTEGRALS) (add a constant c to every answer)

1. $\frac{a^x}{\log a}+\frac{x^{a+1}}{a+1}+a^a x$ 2. $-\cot x+\operatorname{cosec} x$ 3. $\sec x-\tan x$ 4. $\frac{\pi x}{4}+\frac{x^2}{4}$ 5. $\tan x-\cot x-3x$

6. $\frac{\pi x}{4}-\frac{x^2}{2}$ 7. $\frac{2}{27}\left\{(3x+4)^{3/2}+(3x+1)^{3/2}\right\}$ 8. $\frac{x^3}{3}-x^2+4x-8\log|x+2|$ 9. $\frac{1}{32}\left[\frac{-3}{2}\cos 2x+\frac{1}{6}\cos 6x\right]$

10. $\frac{1}{8}\left[3x+2\sin 2x+\frac{\sin 4x}{4}\right]$ 11. $\frac{1}{2}\left[\frac{\sin 6x}{6}+\frac{\sin 2x}{2}\right]$ 12. $\frac{x^3}{3}-x+4\tan^{-1}x$ 13. $\frac{-1}{22}\cos 11x+\frac{1}{6}\cos 3x$

14. $-2\cot\frac{x}{2}-x$ 15. $\log\left|e^x+e^{-x}\right|$ 16. $\frac{1}{(a^2-b^2)}\log\left|a^2\sin^2 x+b^2\cos^2 x\right|$

17. $\sin a\log\left|\sin(x-a)\right|+(x-a)\cos a$ 18. $\operatorname{cosec}(a-b).\log\left|\frac{\sin(x-a)}{\sin(x-b)}\right|$ 19. $-\log\left|\cos x+\sin x\right|$

20. $-\log\left|1+e^{-x}\right|$ 21. $2\log\left|\sqrt{x}+1\right|$ 22. $\frac{-2}{b^2}\left[\log\left|a+b\cos x\right|+\frac{a}{a+b\cos x}\right]$ 23. $\frac{1}{2}\tan\left(2\tan^{-1}x\right)$

24. $\frac{\sin^3 x}{3}-\frac{2\sin^5 x}{5}+\frac{\sin^7 x}{7}$ 25. $-\left[\cos x+\cos^3 x-\frac{3\cos^5 x}{5}+\frac{1}{7}\cos^7 x\right]$ 26. $\frac{1}{2}\tan^2 x-\log\left|\sec x\right|$

27. $\frac{1}{3}(x+\log x)^3$ 28. $\frac{1}{4}\sin^4 x-\sin^2 x+\log\left|\sin x\right|$ 29. $\frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right)$ 30. $\frac{x^3}{3}-x+2\tan^{-1}x$

31. $\frac{1}{4}\log\left|\frac{x+1}{3-x}\right|$ 32. $\frac{1}{2}\tan^{-1}\left(\frac{x+4}{2}\right)$ 33. $\frac{1}{4}\log\left|\frac{e^x+1}{e^x+5}\right|$ 34. $\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|$ 35. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2-1}{\sqrt{3}}\right)$

36. $\sin^{-1}\left(\frac{x-4}{5}\right)$ 37. $\sin^{-1}\left(\frac{x+3}{5}\right)$ 38. $\sin^{-1}\left(\frac{2x^2+1}{\sqrt{5}}\right)$ 39. $\log\left|x^2+3x-18\right|-\frac{2}{3}\log\left|\frac{x-3}{x+6}\right|$

40. $2\log\left|\sin^2\phi-4\sin\phi+5\right|+7\tan^{-1}(\sin\phi-2)$ 41. $\frac{1}{4}\log\left|2x^2+6x+5\right|+\frac{1}{2}\tan^{-1}(2x+3)$

42. $x-3\log\left|x^2+6x+12\right|2\sqrt{3}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right)$ 43. $a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{a^2-x^2}$ 44. $-3\sqrt{5-2x-x^2}-2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)$

45. $\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)$ 46. $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right|$ 47. $\frac{1}{2\sqrt{5}}\log\left|\frac{\sqrt{5}\tan x-1}{\sqrt{5}\tan x+1}\right|$ 48. $\tan^{-1}\left(\tan^2 x\right)$

49. $\frac{1}{\sqrt{3}}\log\left|\frac{\tan(x/2)-2-\sqrt{3}}{\tan(x/2)-2+\sqrt{3}}\right|$ 50. $\frac{2}{3}\tan^{-1}\left(\frac{\tan x/2}{3}\right)$ 51. $\tan^{-1}\left(1+\tan\frac{x}{2}\right)$

52. $\frac{-5x}{13}+\frac{12}{13}\log\left|3\cos x+2\sin x\right|$ 53. $x(\log x)^2-2(x\log x-x)$ 54. $x\sin^{-1}x+\sqrt{1-x^2}$

55. $-x\cot\frac{x}{2}$ 56. $\frac{1}{2}\sec x\tan x+\frac{1}{2}\log\left|\sec x+\tan x\right|$ 57. $\frac{x}{\sqrt{1-x^2}}\sin^{-1}x|+\frac{1}{2}\log\left|1-x^2\right|$

58. $\frac{x^3}{3}\tan^{-1}x-\frac{1}{6}x^2+\frac{1}{6}\log\left|x^2+1\right|$ 59. $2x\tan^{-1}x-\log\left|1+x^2\right|$ 60. $-\frac{\sin^{-1}x}{x}+\log\left|\frac{1-\sqrt{1-x^2}}{x}\right|$

61. $-e^x\cot\frac{x}{2}$ 62. $\frac{x}{(\log x+1)}$ 63. $\frac{e^x}{1-x}$ 64. $\frac{x}{\log x}$ 65. $\frac{e^{ax}}{a^2+b^2}(a\cos bx+b\sin bx)$

66. $\frac{1}{4}(2x-7)\sqrt{7x-10-x^2}+\frac{9}{8}\sin^{-1}\left(\frac{2x-7}{3}\right)$ 67. $\frac{1}{2}\log x\sqrt{(\log x)^2+16}+8\log\left|\log x+\sqrt{(\log x)^2+16}\right|$

68. $\left(x^2+x+1\right)^{3/2}-\frac{7}{2}\left[\left(x+\frac{1}{2}\right)\sqrt{x^2+x+1}+\frac{3}{8}\log\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|\right]$ 69. $-\frac{1}{3}\left(1-x-x^2\right)^{3/2}+\frac{1}{8}(2x+1)\sqrt{1-x-x^2}+\frac{5}{16}\sin^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)$

70. $-\frac{1}{6}\log\left|x-1\right|\left|1+\cos x\right|-\frac{1}{3}\log\left|x+2\right|+\frac{1}{2}\log\left|x-3\right|$ 71. $-\frac{1}{2}\log\left|1-\cos x\right|-\frac{1}{6}\log\left|1+\cos x\right|+\frac{2}{3}\log\left|1-2\cos x\right|$

72. $\frac{5}{16}\log\left|x-2\right|-\frac{7}{4(x-2)}-\frac{5}{16}\log\left|x+2\right|$ 73. $\log\left|x+2\right|-\frac{1}{2}\log\left|x^2+4\right|+\tan^{-1}\frac{x}{2}$ 74. $-\frac{1}{3}\tan^{-1}x+\frac{2}{3}\tan^{-1}\left(\frac{x}{2}\right)$

75. $-\frac{1}{3}\log\left|1+\tan\theta\right|+\frac{1}{6}\log\left|\tan^2\theta-\tan\theta+1\right|+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan\theta-1}{\sqrt{3}}\right)$ 76. $\log\left|\frac{(2+\sin x)^4}{(1+\sin x)^2}\right|$

77. $\frac{1}{5}\log\left|\frac{x^5}{x^5+1}\right|$ 78. $\frac{1}{2}\log\left|\frac{x^2-x+1}{x^2+x+1}\right|$ 79. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-4}{2\sqrt{2}\,x}\right)$ 80. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}\,x}\right)-\frac{1}{4\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}\,x+1}{x^2+\sqrt{2}\,x+1}\right|$

81. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan\theta-1}{\sqrt{2\tan\theta}}\right)+\frac{1}{2\sqrt{2}}\log\left|\frac{\tan\theta-\sqrt{2\tan\theta}+1}{\tan\theta+\sqrt{2\tan\theta}+1}\right|$ 82. $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\cot\theta-1}{\sqrt{2\cot\theta}}\right)-\frac{1}{2\sqrt{2}}\log\left|\frac{\cot\theta-\sqrt{2\cot\theta}+1}{\cot\theta+\sqrt{2\cot\theta}+1}\right|$

83. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2x-1}{\sqrt{2}\tan x}\right)$ 84. $\frac{1}{2\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}\,x+1}{x^2+\sqrt{2}\,x+1}\right|$ 85. $2\left[\sqrt{x}-\tan^{-1}\sqrt{x}\right]$ 86. $\frac{1}{4\sqrt{3}}\log\left|\frac{\sqrt{x+1}-\sqrt{3}}{\sqrt{x+1}+\sqrt{3}}\right|-\frac{1}{2}\tan^{-1}\left(\sqrt{x+1}\right)$

87. $\sqrt{\frac{x-1}{x+1}}$ 88. $-\frac{\sqrt{1+x^2}}{x}$ 89. $\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\tan^{-1}x-\frac{1}{2}\log\left|x+1\right|$ 90. $(x+\alpha)\cos2\alpha-\sin2\alpha.\log\left|\sin\left(x+\alpha\right)\right|$

91. $\frac{e^x}{x+2}$ 92. $\log\left|x+\log\left(\cos x\right)\right|$ 93. $\sqrt{x^2+a^2}-a\log\left(\frac{a+\sqrt{x^2+x}}{x}\right)$ 94. $\sin^{-1}\left(\frac{e^x+2}{3}\right)$

95. $\frac{1}{8\sqrt{3}}\tan^{-1}\left(\frac{x^2-4}{\sqrt{3}\,x}\right)-\frac{1}{16\sqrt{3}}\log\left|\frac{x^2-\sqrt{13}\,x+4}{x^2+\sqrt{13}\,x+4}\right|$ 96. $\log\left|\sec^2x+\sec x\tan x\right|$ 97. $2\sin\left(e^{\sqrt{x}}\right)$

98. $\frac{x}{2}\left[\sin\left(\log x\right)+\cos\left(\log x\right)\right]$ 99. $2\log\left|\sin^2\theta-4\sin\theta+5\right|+7\tan^{-1}\left(\sin\theta-2\right)$ 100. $\frac{x^2}{2}$

ASSIGNMENT CLASS XII DEFINITE INTEGRALS

Evaluate the following:

1. $\int_0^{\pi/2} \cos^3 x\,dx$
2. $\int_0^{\pi/4} \sqrt{1-\sin 2x}\,dx$
3. $\int_0^4 \frac{1}{\sqrt{x^2+2x+3}}\,dx$
4. $\int_0^1 \frac{2x}{5x^2+1}\,dx$
5. $\int_1^2 \frac{\log x}{x^2}\,dx$
6. $\int_1^2 \frac{1}{x\left(1+x^2\right)}\,dx$
7. $\int_{\pi/4}^{\pi/2} \cos 2x \log \sin x\,dx$
8. $\int_1^2 \left(\frac{x-1}{x^2}\right)e^x\,dx$
9. $\int_0^{\pi/2} \frac{\cos\theta}{(1+\sin\theta)(2+\sin\theta)}\,dx$
10. $\int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{\left(1-x^2\right)^{3/2}}\,dx$
11. $\int_0^{\pi/2} \cos^4 x\,dx$
12. $\int_0^{\pi/2} \left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$
13. $\int_0^{\pi} \frac{1}{5+4\cos x}\,dx$
14. $\int_0^{\pi/2} \frac{1}{2\cos x+4\sin x}\,dx$
15. $\int_0^{\pi/2} \frac{\cos x}{3\cos x+\sin x}\,dx$
16. $\int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x+\cos^4 x}\,dx$
17. $\int_0^1 \frac{e^x}{1+e^{2x}}\,dx$
18. $\int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2}\,dx$
19. $\int_0^{\pi/4} \sec^4 x\,dx$
20. $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$
21. $\int_1^2 \frac{1}{x\left(1+\log x\right)^2}\,dx$
22. $\int_0^{\pi} |\cos x|\,dx$
23. $\int_{-1}^1 e^{|x|}\,dx$
24. $\int_{-1}^1 f(x)\,dx$, where $f(x)=\begin{cases}1-2x & x\le 0\\ 1+2x & x\ge 0\end{cases}$
25. $\int_0^3 [x]\,dx$
26. $\int_0^2 \left[x^2\right]dx$
27. $\int_{-1}^1 |2x+1|\,dx$
28. $\int_{-\pi/2}^{\pi/2} \left(\sin|x|+\cos|x|\right)dx$
29. $\int_{-\pi/4}^{\pi/4} |\sin x|\,dx$
30. $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}}\,dx$
31. $\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\,dx$
32. $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x}\,dx$
33. $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x+\cos^n x}\,dx$
34. $\int_0^{\pi/2} \sin 2x \log(\tan x)\,dx$
35. $\int_{-\pi/4}^{\pi/4} x^3 \sin^4 x\,dx$
36. $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\,dx$
37. $\int_0^{\pi} \frac{x\tan x}{\sec x \operatorname{cosec} x}\,dx$
38. $\int_0^1 \cot^{-1}\left(1-x+x^2\right)dx$
39. $\int_{-1}^1 \log\left(\frac{2-x}{2+x}\right)dx$
40. $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$
41. $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$
42. $\int_0^1 \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$
43. $\int_0^{\pi/2} \frac{1}{1+\sqrt{\cot x}}\,dx$
44. $\int_0^{\infty} \frac{x\left(\tan^{-1} x\right)^2}{\left(1+x^2\right)^{3/2}}$
45. $\int_0^{\pi/2} x^2 \cos 2x\,dx$
46. $\int_{-\infty}^{\infty} \frac{1}{9+x^2}\,dx$
47. $\int_0^1 x\sqrt{\frac{1-x^2}{1+x^2}}\,dx$
48. $\int_0^{\pi/2} \frac{1}{1+\tan^3 x}\,dx$
49. $\int_0^1 x\left(1-x\right)^5\,dx$
50. $\int_0^a \frac{1}{x+\sqrt{a^2-x^2}}\,dx$
51. $\int_0^{\infty} \frac{1}{\left(x^2+a^2\right)\left(x^2+b^2\right)}\,dx$
52. $\int_0^{2\pi} e^x \cos\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$

53. $\int_{-5}^{0}\left(|x|+|x+2|+|x+5|\right)dx$ 54. If $\int_{0}^{a}\sqrt{x}\,dx=2a\int_{0}^{\pi/2}\sin^3 x\,dx$, find the value of $\int_{a}^{a+1}x\,dx$.

Evaluate the following integrals as limit of sums:

55. $\int_{0}^{2}(2x+1)\,dx$ 56. $\int_{2}^{4}(2x-1)\,dx$ 57. $\int_{0}^{2}(x^2+3)\,dx$ 58. $\int_{1}^{3}(2x^2+5)\,dx$ 59. $\int_{1}^{3}(x^2+x)\,dx$

60. $\int_{2}^{3}(2x^2+1)\,dx$ 61. $\int_{0}^{3}(2x^2+3x+5)\,dx$ 62. $\int_{a}^{b}e^x\,dx$

ANSWERS

1. $\frac{2}{3}$ 2. $\sqrt{2}-1$ 3. $\log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$ 4. $\frac{1}{5}\log 6$ 5. $\frac{1}{2}\log\left(\frac{e}{2}\right)$ 6. $\frac{3}{2}\log 2-\frac{1}{2}\log 5$

7. $\frac{1}{4}\log 2-\frac{\pi}{8}+\frac{1}{4}$ 8. $\frac{e^2}{2}-e$ 9. $\log\left(\frac{4}{3}\right)$ 10. $\frac{\pi}{4}-\frac{1}{2}\log 2$ 11. $\frac{3\pi}{16}$ 12. $\sqrt{2}\,\pi$

13. $\frac{\pi}{3}$ 14. $\frac{1}{\sqrt{5}}\log\left(\frac{3+\sqrt{5}}{2}\right)$ 15. $\frac{3\pi}{20}-\frac{1}{10}\log 3$ 16. $\frac{\pi}{2}$ 17. $\tan^{-1}e-\frac{\pi}{4}$

18. $\frac{1}{12}\pi^{3/2}$ 19. $\frac{4}{3}$ 20. $\frac{\pi}{2}-1$ 21. $\frac{\log 2}{1+\log 2}$ 22. 2 23. $2e-2$ 24. 4

25. 3 26. $5-\sqrt{2}-\sqrt{3}$ 27. $\frac{5}{2}$ 28. 4 29. $2-\sqrt{2}$ 30. $\frac{1}{2}$ 31. $\frac{\pi}{4}$

32. $-\frac{1}{\sqrt{2}}\log\left(\sqrt{2}-1\right)$ 33. $\frac{\pi}{4}$ 34. 0 35. 0 36. $a\pi$ 37. $\frac{\pi^2}{4}$

38. $\frac{\pi}{2}-\log 2$ 39. 0 40. $\frac{\pi}{2}-\log 2$ 41. $\frac{\pi}{8}\log 2$ 42. $\frac{\pi}{2}-\log 2$

43. $\frac{\pi}{4}$ 44. $\pi-2$ 45. $-\frac{\pi}{4}$ 46. $\frac{\pi}{3}$ 47. $\frac{\pi}{4}-\frac{1}{2}$ 48. $\frac{\pi}{4}$ 49. $\frac{1}{42}$

50. $\frac{\pi}{4}$ 51. $\frac{\pi}{2ab(a+b)}$ 52. $\frac{-3\sqrt{2}}{5}\left(e^{2\pi}+1\right)$ 53. $\frac{63}{2}$ 54. $\frac{1}{2}$ or $\frac{9}{2}$ 55. 6

56. 10 57. $\frac{26}{3}$ 58. $\frac{82}{3}$ 59. $\frac{38}{3}$ 60. $\frac{41}{3}$ 61. $\frac{93}{2}$ 62. e^b-e^a

ASSIGNMENT CLASS XII AREAS OF BOUNDED REGIONS

1. Sketch the region bounded by $y=2x-x^2$ and x - axis and find its area.
2. Find the area of the region included between the parabolas $y^2=4ax$ and $x^2=4ay$, where $a>0$.
3. Find the smaller area bounded by the curves $x^2+y^2=8$ and $y=|x|$.
4. Find the area of the region $\{(x,y): x^2 \le y \le x\}$.
5. Find the area of the region $\{(x,y): x^2 \le y \le |x|\}$.
6. Find the area bounded by the curves $y^2=4ax$ and the lines $y=2a$ and $y-$axis .
7. Find the area of the region $\{(x,y): x^2+y^2 \le 1 \le x+y\}$.
8. Find the area bounded by the curves $y=x, y=x^3$.
9. Using integration, find area of ΔABC whose vertices have the coordinates:
   (i) $A(2,5), B(4,7)$ and $C(6,2)$ (ii) $A(3,0), B(4,5)$ and $C(5,1)$
10. Find the area of the region bounded by the following curves after making a rough sketch:
    $y=1+|x+1|, x=-3, x=3, y=0$
11. Sketch the graph of $y=|x+1|$. Evaluate $\int_{-3}^{1} |x+1|\,dx$. What does this value represent on the graph?
12. Sketch the region common to the circle $x^2+y^2=16$ and the parabola $x^2=6y$. Also, find the area of the region using integration.
13. Find the area bounded by the lines :
    (i) $y=4x+5, y=5-x, 4y=x+5$ (ii) $x+2y=2, y-x=1, 2x+y=7$
14. Sketch the graph of $f(x)=\begin{cases} |x-2|+2 & \text{when } x\le 2 \\ x^2-2 & \text{when } x>2 \end{cases}$. Evaluate $\int_0^4 f(x)\,dx$. What does this value represent on the graph?
15. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and the line $\frac{x}{4}+\frac{y}{3}=1$.
16. Find the area of the region enclosed between the circles $x^2+y^2=16$ and $(x+4)^2+y^2=16$.
17. Draw the rough sketch of $y^2=x+1$ and $y^2=-x+1$ and determine the area enclosed by them.
18. Find the area of the region bounded by the curve $y=\sqrt{1-x^2}$, line $y=x$ and the positive $x-axis$.

ANSWERS

1. $\frac{4}{3}$ sq. units 2. $\frac{16}{3}a^2$ sq. units 3. 2π sq. units 4. $\frac{1}{6}$ sq. units 5. $\frac{1}{3}$ sq. units
6. $\frac{2}{3}a^2$ sq. units 7. $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ sq. units 8. $\frac{1}{2}$ sq. units 9 (i). 7 sq. units (ii) $\frac{9}{2}$ sq. units
10. 16 sq. units 11. 4 12. $\left(\frac{4\sqrt{3}}{3}+\frac{16\pi}{3}\right)$ sq. units 13 (i). $\frac{15}{2}$ sq. units (ii) 6 sq. units
14. $\frac{62}{3}$ sq. units , This value represents the area of the region bounded by the given curve and x-axis between $x=0$ to 4. 15. $3(\pi-2)$ sq. units 16. $8\left(\frac{4\pi}{3}-\sqrt{3}\right)$ sq. units 17. $\frac{8}{3}$ sq. units 18. $\frac{\pi+1}{8}$ sq. units

ASSIGNMENT CLASS XII DIFFERENTIAL EQUATIONS

1. Determine the order and degree of each of the following differential equations:

$(i)\ \frac{1}{x^2}\left(\frac{d^2y}{dx^2}\right)+9y=-4e^{-x}$ $(ii)\ x\sqrt{1-y^2}\,dx+y\sqrt{1-x^2}\,dy=0$ $(iii)\ y=x\frac{dy}{dx}+a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

$(iv)\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^3=0$ $(v)\ \frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2=x^2\log\left(\frac{d^2y}{dx^2}\right)$ $(vi)\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2=x\sin\left(\frac{d^2y}{dx^2}\right)$

2. Form the differential equations from the following family of curves:

$(i)\ y=c(x-c)^2$ $(ii)\ y^2=a(b-x)(b+x)$ $(iii)\ y^2-2ay+x^2=a^2$

$(iv)\ (x+a)^2-2y^2=a^2$ $(v)\ y=a\sin(x+b)$ $(vi)\ xy=Ae^x+Be^{-x}+x^2$

3. Find the differential equation of all the circles in the first quadrant which touch coordinate axes.

4. Show that $y=ae^{2x}+be^{-x}$ is a solution of the differential equation $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0$.

5. Show that $y=A\cos nx+B\sin nx$ is a solution of the differential equation $\frac{d^2y}{dx^2}+n^2y=0$.

6. Show that $y=e^{m\cos^{-1}x}$ is a solution of the differential equation $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-m^2y=0$.

7. Show that $y=Ax+\frac{B}{x}$, $x\neq 0$ is a solution of the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$.

8. Show that $y=e^x+e^{2x}$ is a solution of the differential equation $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=0$, $y(0)=1, y'(0)=3$.

9. Solve the following differential equations:

$(i)\ \sqrt{1-x^2}\left(\frac{dy}{dx}-3x^2\right)=6-\cos^{-1}x$ $(ii)\ \frac{dy}{dx}=e^{x-y}+x^2e^{-y}$ $(iii)\ \frac{dy}{dx}=1+x+y+xy$

$(iv)\ \cos x(1+\cos y)\,dx-\sin y(1+\sin x)\,dy=0$ $(v)\ x\cos y\,dy=\left(xe^x\log x+e^x\right)dx$

$(vi)\ \frac{dy}{dx}=\frac{x+2y-1}{x+2y+1}$ $(vii)\ x\sqrt{1-y^2}\,dx+y\sqrt{1-x^2}\,dy=0$ $(viii)\ y(1-x^2)\frac{dy}{dx}=x(1+y^2)$

$(ix)\ y-x\frac{dy}{dx}=a\left(y^2+\frac{dy}{dx}\right)$ $(x)\ \cos^{-1}\left(\frac{dx}{dy}\right)=x+y$

$(xi)\ x(1+y^2)\,dy-y(1+x^2)\,dy=0$, given that $y=0$, when $x=1$

$(xii)\ \frac{dy}{dx}=y\sin 2x$, given that $y(0)=1$ $(xiii)\ (1+y^2)(1+\log x)\,dx+x\,dy=0$, given that when $x=1, y=1$

10. Solve the following differential equations:

$(i)\ \frac{dy}{dx}=\frac{3x+2y}{2x-3y}$ $(ii)\ x^2\frac{dy}{dx}=2xy+y^2$ $(iii)\ (x^3+y^3)\,dy-x^2y\,dx=0$ $(iv)\ x\frac{dy}{dx}=y-x\tan\frac{y}{x}$

$(v)\ (3yx+y^2)\,dx=(x^2+xy)\,dy$ $(vi)\ 2xy\,dx+(x^2+2y^2)\,dy=0$ $(vii)\ x\,dy-y\,dx=\sqrt{x^2-y^2}\,dx$

$(viii)\ x\frac{dy}{dx}=y(\log y-\log x+1)$ $(ix)\ 2xy+y^2-2x^2\frac{dy}{dx}=0$, $y(1)=2$

$(x)\ x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)+x-y\sin\left(\frac{y}{x}\right)$, $y(1)=\frac{\pi}{2}$

11. Solve the following differential equations:

$(i)\ 4\frac{dy}{dx}+8y=5e^{-3x}$ $(ii)\ x\frac{dy}{dx}-y-2x^3=0$ $(iii)\left(1+x^2\right)\frac{dy}{dx}-2xy=\left(x^2+2\right)\left(x^2+1\right)$

$(iv)\left(1+x^2\right)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$ $(v)\left(x^2-1\right)\frac{dy}{dx}+2xy=\frac{2}{x^2-1}$ $(vi)\sin x\frac{dy}{dx}+y\cos x=\sin^2 x\cos x$

$(vii)\frac{dy}{dx}=-\frac{x+y\cos x}{1+\sin x}$ $(viii)\ x\frac{dy}{dx}+y=x\cos x+\sin x\ ,\ y\left(\frac{\pi}{2}\right)=1$

$(ix)\frac{dy}{dx}+y\cot x=2x+x^2\cot x\ ,\ y\left(\frac{\pi}{2}\right)=0$ $(x)\frac{dy}{dx}+2y=e^{-2x}\sin x\ ,\ y(0)=0$

ANSWERS

1. $(i)\ 2,1$ $(ii)\ 1,1$ $(iii)\ 1,2$ $(iv)\ 2,2$ (v) undefined, undefined $(vi)\ 2$, undefined

2. $(i)\left(\frac{dy}{dx}\right)^3=4y\left(x\frac{dy}{dx}-2y\right)$ $(ii)\ xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$ $(iii)\left(x^2-2y^2\right)\left(\frac{dy}{dx}\right)^2-4xy\frac{dy}{dx}-x^2=0$

$(iv)\ x^2+2y^2=4xy\frac{dy}{dx}$ $(v)\frac{d^2y}{dx^2}+y=0$ $(vi)\ xy-x^2+2=x\frac{d^2y}{dx^2}+2\frac{dy}{dx}$

3. $\left(x-y\right)^2\left(1+\left(\frac{dy}{dx}\right)^2\right)=\left(x+y\frac{dy}{dx}\right)^2$ 9. $(i)\ y=x^3+6\sin^{-1}x+\frac{1}{2}\left(\cos^{-1}x\right)^2+c$ $(ii)\ e^y=e^x+\frac{x^3}{3}+c$

$(iii)\log\left|1+y\right|=x+\frac{x^2}{2}+c$ $(iv)\left(1+\sin x\right)\left(1+\cos y\right)=c$ $(v)\sin y=e^x\log x+c$

$(vi)\ 2\left(y-x\right)+\frac{4}{3}\log\left|3x+6y-1\right|+c$ $(vii)\ \sqrt{1-x^2}+\sqrt{1-y^2}=c$ $(viii)\left(1-x^2\right)\left(1+y^2\right)=c$

$(ix)\left(x+a\right)\left(1-ay\right)=cy$ $(x)\ y=\tan\left(\frac{x+y}{2}\right)+c$ $(xi)\ 1+x^2=2\left(1+y^2\right)$ $(xii)\log\left|y\right|=\frac{1}{2}\left(1-\cos 2x\right)$

$(xiii)\ y=\tan\left\{\frac{\pi}{4}+\frac{1}{2}-\frac{1}{2}\left(1+\log x\right)\right\}$ 10. $(i)\ 3\log\left(x^2+y^2\right)=4\tan^{-1}\left(\frac{y}{x}\right)+c$ $(ii)\ y=cx\left(x+y\right)$

$(iii)-\frac{x^3}{3y^3}+\log y=c$ $(iv)\ x\sin\left(\frac{y}{x}\right)=c$ $(v)\log\left|y\right|+\frac{y}{x}=3\log\left|x\right|+c$ $(vi)\ 3x^2y+2y^3=c$

$(vii)\sin^{-1}\left(\frac{y}{x}\right)=\log\left|x\right|+c$ $(viii)\log\left|y\right|=\log\left|x\right|+cx$ $(ix)\ y=\frac{x}{1-\log\left|x\right|},\ x\neq 0,\pm e$ $(x)\log\left|x\right|=\cos\left(\frac{y}{x}\right)$

11. $(i)\ y=-\frac{5}{4}e^{-3x}+ce^{-2x}$ $(ii)\ y=x^3+cx$ $(iii)\ y=x\left(1+x^2\right)+\tan^{-1}x\left(1+x^2\right)+c\left(1+x^2\right)$

$(iv)\ y\left(1+x^2\right)=\frac{x\sqrt{x^2+4}}{2}+2\log\left|x+\sqrt{x^2+4}\right|+c$ $(v)\ y\left(x^2-1\right)=\log\left|\frac{x-1}{x+1}\right|+c$ $(vi)\ y\sin x=\frac{1}{3}\sin^3 x+c$

$(vii)\ y=\frac{2c-x^2}{2\left(1+\sin x\right)}$ $(viii)\ y=\sin x$ $(ix)\ y=x^2-\frac{\pi^2}{4\sin x}$ $(x)\ ye^{2x}=1-\cos x$

ASSIGNMENT CLASS XII VECTOR ALGEBRA

1. In a regular hexagon ABCDEF, if $\overrightarrow{AB}=\vec{a}$ and $\overrightarrow{BC}=\vec{b}$, then express $\overrightarrow{CD},\overrightarrow{DE},\overrightarrow{EF},\overrightarrow{FA},\overrightarrow{AC},\overrightarrow{AD},\overrightarrow{AE}$ and $\overrightarrow{CF}$ in terms of $\vec{a}$ *and* $\vec{b}$.

2. If $\vec{a}=\hat{i}+\hat{j}$, $\vec{b}=\hat{j}+k$, $\vec{c}=k+\hat{i}$, find a unit vector in the direction of $\vec{a}+\vec{b}+\vec{c}$.

3. The position vectors of the points P, Q and R are $\hat{i}+2\hat{j}+3k$, $-2\hat{i}+3\hat{j}+5k$, $7\hat{i}-k$ respectively. Prove that P, Q and R are collinear.

4. If $\vec{a}=\hat{i}+2\hat{j}+3k$, $\vec{b}=2\hat{i}+4\hat{j}-5k$ represents two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

5. Prove that the points $\hat{i}-\hat{j}$, $4\hat{i}+3\hat{j}+k$, $2\hat{i}-4\hat{j}+5k$ are the vertices of a right angled triangle.

6. If the position vectors of the vertices of a triangle ABC are $\hat{i}+2\hat{j}+3k$, $2\hat{i}+3\hat{j}+k$, $3\hat{i}+\hat{j}+2k$, prove that ΔABC is an equilateral triangle.

7. Write the position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ externally in the ratio 1: 4, where $\vec{a}=2\hat{i}+3\hat{j}+4k$ and $\vec{b}=-\hat{i}+\hat{j}+k$.

8. Find the projection of $\vec{b}+\vec{c}$ on $\vec{a}$, where $\vec{a}=2\hat{i}-2\hat{j}+k$, $\vec{b}=\hat{i}+2\hat{j}-2k$ and $\vec{c}=2\hat{i}-\hat{j}+4k$.

9. If $\vec{a}=\hat{i}+\hat{j}+2k$ and $\vec{b}=3\hat{i}+2\hat{j}-k$, find the value of $\left(\vec{a}+3\vec{b}\right).\left(2\vec{a}-\vec{b}\right)$.

10. Find a vector whose magnitude is 3 units and which is perpendicular to each of the vectors $\vec{a}=3\hat{i}+\hat{j}-4k$ and $\vec{b}=6\hat{i}+5\hat{j}-2k$.

11. If $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $\left|\vec{a}\right|=3$, $\left|\vec{b}\right|=5$, $\left|\vec{c}\right|=7$, find angle between $\vec{a}$ and $\vec{b}$.

12. If $\vec{a}$ and $\vec{b}$ are vectors such that $\left|\vec{a}\right|=2$, $\left|\vec{b}\right|=3$ and $\vec{a}.\vec{b}=4$, find $\left|\vec{a}-\vec{b}\right|$ and $\left|\vec{a}+\vec{b}\right|$.

13. If a and $\hat{b}$ are unit vectors and θ is the angle between them, prove that $\sin\frac{\theta}{2}=\frac{1}{2}\left|a-\hat{b}\right|$ and $\cos\frac{\theta}{2}=\frac{1}{2}\left|a+\hat{b}\right|$.

14. Show that the points A,B and C with position vectors $2\hat{i}-\hat{j}+k$, $\hat{i}-3\hat{j}-5k$, $3\hat{i}-4\hat{j}-4k$ respectively, are the vertices of the right triangle. Also, find the remaining angles of the triangle.

15. If $\vec{a}=\hat{i}+2\hat{j}-3k$ and $\vec{b}=3\hat{i}-\hat{j}+2k$, then show that $\vec{a}+\vec{b}$ is perpendicular to $\vec{a}-\vec{b}$.

16. Find the angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, if $\vec{a}=2\hat{i}-\hat{j}+3k$ and $\vec{b}=3\hat{i}+\hat{j}-2k$.

17. Express the vectors $\vec{a}=5\hat{i}-2\hat{j}+5k$ as sum of two vectors such that one is parallel to the vector $\vec{b}=3\hat{i}+k$ and the other is perpendicular to $\vec{b}$.

18. The dot products of a vector with the vectors $\hat{i}+\hat{j}-3k$, $\hat{i}+3\hat{j}-2k$ and $2\hat{i}+\hat{j}+4k$ are 0, 5, 8 respectively. Find the vector.

19. Find a unit vector perpendicular to each of the vectors $\vec{a}=4\hat{i}-\hat{j}+3k$ and $\vec{b}=2\hat{i}+2\hat{j}-k$.

20. If $|\vec{a}|=\sqrt{26}$, $|\vec{b}|=7$ and $|\vec{a}\times\vec{b}|=35$, find $\vec{a}.\vec{b}$.

21. Find the area of the triangle whose adjacent sides are determined by the vectors $\vec{a}=-2\hat{i}-5k$ and $\vec{b}=\hat{i}-2\hat{j}-k$.

22. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=3\hat{i}+\hat{j}-2k$ and $\vec{b}=\hat{i}-3\hat{j}+4k$.

23. Find the area of the parallelogram whose diagonals are determined by the vectors $\vec{a}=2\hat{i}+3\hat{j}-6k$ and $\vec{b}=3\hat{i}-4\hat{j}-k$.

24. Show that points whose position vectors are $\vec{a}=5\hat{i}+6\hat{j}+7k$, $\vec{b}=7\hat{i}-8\hat{j}-9k$, $\vec{c}=3\hat{i}+20\hat{j}+5k$ are collinear.

25. Let $\vec{a}=\hat{i}-\hat{j}$, $\vec{b}=3\hat{j}-k$, $\vec{c}=7\hat{i}-k$. Find a vector $\vec{d}$ such that it is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d}=1$

26. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the position vectors of the vertices A, B and C of a ΔABC respectively, find an expression for the area of ΔABC and hence deduce the condition for the points A, B and C to be collinear.

27. If $\vec{a}=\hat{i}+\hat{j}+k$, $\vec{c}=\hat{j}-k$ are given vectors, then find a vector $\vec{b}$ satisfying equations $\vec{a}\times\vec{b}=\vec{c}$ and $\vec{a}.\vec{b}=3$.

28. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, then prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

29. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}, \vec{b}\neq\vec{c}$.

30. If $\vec{a}.\vec{b}=\vec{a}.\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ and $\vec{a}\neq 0$, then show that $\vec{b}=\vec{c}$.

ANSWERS

1. $\overrightarrow{CD}=\vec{b}-\vec{a}$, $\overrightarrow{DE}=-\vec{a}$, $\overrightarrow{EF}=-\vec{b}$, $\overrightarrow{FA}=\vec{a}-\vec{b}$, $\overrightarrow{AC}=\vec{a}+\vec{b}$, $\overrightarrow{AD}=2\vec{b}$, $\overrightarrow{AE}=2\vec{b}-\vec{a}$, $\overrightarrow{CF}=-2\vec{a}$

2. $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+k\right)$ 4. $\frac{1}{7}\left(3\hat{i}+6\hat{j}-2k\right)$, $\frac{1}{\sqrt{69}}\left(\hat{i}+2\hat{j}-8k\right)$ 7. $3\hat{i}+\frac{11}{3}\hat{j}+5k$ 8. 2 9. –15

10. $2\hat{i}-2\hat{j}+k$ 11. 60^0 12. $\sqrt{5}$, $\sqrt{21}$ 14. $\cos^{-1}\left(\sqrt{\frac{35}{41}}\right), \cos^{-1}\left(\sqrt{\frac{6}{41}}\right)$ 16. $\frac{\pi}{2}$

17. $6\hat{i}+2k, -\hat{i}-2\hat{j}+k$ 18. $\hat{i}+2\hat{j}+k$ 19. $\frac{1}{3}\left(-\hat{i}+2\hat{j}+2k\right)$ 20. 7 21. $\frac{1}{2}\sqrt{165}$ sq.units

22. $10\sqrt{3}$ sq.units 23. $\frac{1}{2}\sqrt{1274}$ sq.units 25. $\frac{1}{4}\left(\hat{i}+\hat{j}+3k\right)$ 26.

$ar(\Delta ABC)=\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$; $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$ 27. $\vec{b}=\frac{1}{3}\left(5\hat{i}+2\hat{j}+2k\right)$

ASSIGNMENT CLASS XII THREE DIMENSIONAL GEOMETRY

1. Find the coordinates of the foot of the perpendicular drawn from the point to the line joining $B(0,-1,3)$ and $C(2,-3,-1)$.

2. Find the vector equation of the line passing through the point $A(2,-1,1)$, and parallel to the line joining the points $B(-1,4,1)$ and $C(1,2,2)$. Also, find the Cartesian equation of the line.

3. The cartesian equations of a line are $6x-2=3y+1=2z-2$. Find (i) the direction ratios of the line, (ii) the cartesian equation of the line parallel to this line and passing through the point $(2,-1,-1)$.

4. Find the equations of the line passing through the point $(-1, 3,-2)$ and perpendicular to each of the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$.

5. Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ intersect each other. Also, find point of their intersection.

6. Show that the lines $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5}$ and $\frac{x+2}{4}=\frac{y-1}{3}=\frac{z+1}{-2}$ do not intersect each other.

7. Find the foot of perpendicular drawn from the point $P(1,6,3)$ on the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also, find its distance from P.

8. Find the image of the point $(5,9,3)$ in the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$.

9. A perpendicular is drawn from the point $(0,2,7)$ to the line $\frac{x+2}{-1}=\frac{y-1}{3}=\frac{z-3}{-2}$. Find

(i) foot of the perpendicular (ii) length of the perpendicular (iii) image of the point in the line.

10. Find the coordinates of the point where the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}$ meets the plane $x+y+4z=6$.

11. Find the angle between the lines $\frac{x+1}{1}=\frac{2y-3}{3}=\frac{z-6}{2}$ and $\frac{x-4}{3}=\frac{y+3}{-2}, z=5$.

12. Find the value of k for which the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{6-z}{5}$ are perpendicular to each other.

13. Find the angles of a ΔABC whose vertices are $A(-1,3,2), B(2,3,5)$ and $C(3,5,-2)$.

14. Show that the angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

15. Find the shortest distance between the following pair of lines:

$(i)\ \dfrac{x-8}{3}=\dfrac{y+9}{-16}=\dfrac{10-z}{-7}$ and $\dfrac{x-15}{3}=\dfrac{58+2y}{-16}=\dfrac{z-5}{-5}$

$(ii)\ \vec{r}=(\lambda-1)\hat{i}+(\lambda+1)\hat{j}-(\lambda+1)k$ and $\vec{r}=(1-\mu)\hat{i}+2(2\mu-1)\hat{j}+(\mu+2)k$.

16. Find the shortest distance between the following pair of parallel lines:

$(i)\ \dfrac{x-1}{1}=\dfrac{y-2}{-1}=\dfrac{z-3}{1}$ and $\dfrac{x-2}{-1}=\dfrac{y+1}{1}=\dfrac{z+1}{-1}$

$(ii)\ \vec{r}=\left(\hat{i}+\hat{j}\right)+\lambda\left(2\hat{i}-\hat{j}+k\right)$ and $\vec{r}=\left(2\hat{i}+\hat{j}-k\right)+\mu\left(4\hat{i}-2\hat{j}+2k\right)$.

17. Find the equation of the plane passing through the points $A(0,-1,-1), B(4,5,1)$ and $C(3,9,4)$.

18. Show that the four points A, B, C, D with position vectors $4\hat{i}+5\hat{j}+k$, $-\left(\hat{j}+k\right)$, $3\hat{i}+9\hat{j}+4k$ and $4\left(-\hat{i}+\hat{j}+k\right)$ respectively are coplanar.

19. A plane meets the coordinate axes at A, B and C such that the centroid of ΔABC is $(3, 4, -6)$. Find the equation of the plane.

20. Reduce the equation of the plane $12x-3y+4z+52=0$ to the normal form, and hence find the length of the perpendicular from the origin to the plane. Write down the direction cosines of the normal to the plane.

21. The position vectors of two points A and B are $3\hat{i}+\hat{j}+2k$ and $\hat{i}-2\hat{j}-4k$ respectively. Find the vector equation of the plane passing through B and perpendicular to $\overrightarrow{AB}$.

22. Find the vector equation of the plane passing through the point $(1,2,3)$ and perpendicular to the line with direction ratios $2, 3, -4$.

23. Find the vector equation of the plane through the intersection of the planes $\vec{r}.\left(2\hat{i}+\hat{j}\right)+12=0$ and $\vec{r}.\left(3\hat{i}-\hat{j}+4k\right)=0$, which is at a unit distance from the origin.

24. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.\left(2\hat{i}-7\hat{j}+4k\right)=3$ and $\vec{r}.\left(3\hat{i}-5\hat{j}+4k\right)+11=0$, and passing through the point $(-2, 1, 3)$.

25. Find the equation of the plane passing through the intersection of the planes $2x+3y-z+1=0$ and $x+y-2z+3=0$, and perpendicular to plane $3x-y-2z-4=0$. Also find the inclination of this plane with xy- plane.

26. Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3$ and $5x-3y+4z+9=0$, and parallel to the line $\dfrac{x-1}{2}=\dfrac{y-3}{4}=\dfrac{z-5}{5}$.

27. Find the equation of the plane passing through the point $(1,1,1)$ and perpendicular to each of the planes $x+2y+3z=7$ and $2x-3y+4z=0$.

28. Find λ for which the planes $\vec{r}.\left(2\hat{i}-\hat{j}+\lambda k\right)=7$ and $\vec{r}.\left(3\hat{i}+2\hat{j}+2k\right)=9$ are perpendicular to each other.

29. Find the equation of the plane passing through the point $P(1,-1,2)$ and $Q(2,-2,2)$ and perpendicular to the plane $6x-2y+2z=9$.

30. Show that the line $\vec{r}.\left(2\hat{i}-2\hat{j}+3k\right)+\lambda\left(\hat{i}-\hat{j}+4k\right)$ is parallel to the plane $\vec{r}.\left(\hat{i}+5\hat{j}+k\right)=5$. Also, find the distance between them.

31. Find the vector equation of a line passing through the point with position vector $\left(2\hat{i}-3\hat{j}-5k\right)$ and perpendicular to the plane $\vec{r}.\left(6\hat{i}-3\hat{j}+5k\right)+2=0$. Also, find the point of intersection of this line and the plane.

32. Find the angle between the line $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$ and the plane $3x+4y+z+5=0$.

33. Find the equation of the plane passing through the points $(1,2,3)$ and $(0,-1,0)$ and parallel to the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-3}$.

34. Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3$, $5x-3y+4z+9=0$, and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.

35. Find the distance of the point $(2,3,4)$ from the plane $\vec{r}.\left(3\hat{i}-6\hat{j}+2k\right)+11=0$.

36. Find the distance between the parallel planes $\vec{r}.\left(2\hat{i}-3\hat{j}+6k\right)=5$ and $\vec{r}.\left(6\hat{i}-9\hat{j}+18k\right)+20=0$.

37. Find the length and the foot of perpendicular from the point $(1,1,2)$ to the plane $\vec{r}.\left(2\hat{i}-2\hat{j}+4k\right)+5=0$.

38. Find the image of the point $P(1,\ 3,\ 4)$ in the plane $2x-y+z+3=0$.

39. Prove that the image of the point $(3,-2,1)$ in the plane $3x-y+4z=2$ lies on the plane $x+y+z+4=0$.

40. Find the distance of the point $(2,3,4)$ from the plane $3x+2y+2z+5=0$, measured parallel to the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$.

41. Find equation of the plane which contains two parallel lines $\frac{x-3}{3}=\frac{y+4}{2}=\frac{z-1}{1}$ and $\frac{x+1}{3}=\frac{y-2}{2}=\frac{z}{1}$.

42. Find the vector and cartesian forms of the equation of the plane containing two lines $\vec{r}=\left(\hat{i}+2\hat{j}-4k\right)+\lambda\left(2\hat{i}+3\hat{j}+6k\right)$ and $\vec{r}=\left(3\hat{i}+3\hat{j}-5k\right)+\mu\left(-2\hat{i}+3\hat{j}+8k\right)$.

43. Find the equation of the plane containing two lines $\vec{r}=\left(\hat{i}+\hat{j}\right)+\lambda\left(\hat{i}+2\hat{j}-k\right)$ and $\vec{r}=\left(\hat{i}+\hat{j}\right)+\mu\left(-\hat{i}+\hat{j}-2k\right)$. Find the distance of this plane from the origin and also from the point $(1, 1, 1)$.

44. Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar. Also, find the equation of the plane containing these two lines.

ANSWERS

1. $D\left(\frac{-5}{3},\frac{2}{3},\frac{19}{3}\right)$ 2. $\vec{r}=\left(2\hat{i}-\hat{j}+k\right)+\lambda\left(2\hat{i}-2\hat{j}+k\right)$, $\frac{x-2}{2}=\frac{y+1}{-2}+\frac{z-1}{1}$ 3. $(i)1,2,2$ $(ii)\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-1}{3}$

4. $\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$ 5. $(1,3,2)$ 7. $N(1,3,5);\sqrt{13}$ units 8. $(1, 1, 11)$ 9. $(i)\left(\frac{-3}{2},\frac{-1}{2},4\right)$

$(ii)\frac{\sqrt{70}}{2}$ units $(iii)(-3,-3,1)$ 10. $(1,1,1)$ 11. $\frac{\pi}{2}$ 12. $k=\frac{-10}{7}$

13. $\angle A=90^0,\angle B=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right),\angle C=\cos^{-1}\left(\frac{2}{3}\right)$ 15. $(i)1.4$ units $(ii)\frac{5\sqrt{2}}{2}$ units 16. $(i)\sqrt{26}$ units

$(ii)\frac{1}{6}\sqrt{66}$ units 17. $5x-7y+11z+4=0$ 19. $4x+3y-2z=36$ 20. $4,\left(\frac{-12}{13},\frac{3}{13},\frac{-4}{13}\right)$

21. $\vec{r}.\left(2\hat{i}+3\hat{j}+6k\right)+28=0$ 22. $\vec{r}.\left(2\hat{i}+3\hat{j}-4k\right)+4=0$ 23. $\vec{r}.\left(2\hat{i}+\hat{j}+2k\right)+3=0,\vec{r}.\left(-\hat{i}+2\hat{j}-2k\right)+3=0$

24. $\vec{r}.\left(15\hat{i}-47\hat{j}+28k\right)=7$ 25. $7x+13y+4z=9$, $\theta=\cos^{-1}\left(\frac{4}{\sqrt{234}}\right)$ 26. $7x+9y-10z=27$

27. $17x+2y-7z-12=0$ 28. $\lambda=2$ 29. $x+y-2z+4=0$ 30. $\frac{10}{3\sqrt{3}}$ units

31. $\vec{r}=\left(2\hat{i}-3\hat{j}-5k\right)+\lambda\left(6\hat{i}-3\hat{j}+5k\right)$, $\left(\frac{76}{35},\frac{-108}{35},\frac{-170}{35}\right)$ 32. $\sin^{-1}\left(\sqrt{\frac{7}{52}}\right)$ 33. $6x-3y+z=3$

34. $7x+9y-10z-27=0$ 35. 1 unit 36. $\frac{5}{3}$ units 37. $\frac{13\sqrt{6}}{12}$ units, $\left(\frac{-1}{12},\frac{25}{12},\frac{-1}{6}\right)$

38. $Q(-3,5,2)$ 40. 7 units 41. $8x+y-26z+6=0$ 42. $\vec{r}.\left(6\hat{i}-28\hat{j}+12k\right)+98=0$,
$3x-14y+6z+49=0$

43. $x-y-z=0$; 0 units , $\frac{1}{\sqrt{3}}$ units 44. $x-2y+z=0$

ASSIGNMENT CLASS XII LINEAR PROGRAMMING

1. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foos A and B are available at the cost of Rs 5 and Rs 4 perpendicular unit respectively. One unit of the food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories, while one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the foods A and B should be used to have least cost.

2. Every gram of wheat provides 0.1 gm of proteins and 0.25 gm of carbohydrates. The corresponding values for rice are 0.05 gm and 0.5 gm respectively. Wheat costs Rs. 4 per kg and rice Rs. 6 per kg. The minimum daily requirements of proteins and carbohydrates for an average child are 50 gms and 200 gms respectively. In what quantities should wheat and rice be mixed in the daily diet to provide minimum daily requirements of proteins and carbohydrates at minimum cost. Frame an L.P.P. and solve it graphically.

3. A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair is Rs. 30 while by selling one table the profit is Rs. 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit? Formulate the problem as L.P.P. and solve it graphically.

4. A factory owner purchases two types of machines, A and B for his factory. The requirements and the limitations for the machines are as follows:

Machine	Area occupied (in m^2)	Labour force	Daily output (in units)
A	1000	12 men	60
B	1200	8 men	40

He has maximum area of 9000 m^2 available, and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximize the daily output?

5. An oil company requires 12000, 20000 and 15000 barrels of high-grade, medium-grade and low-grade oil, respectively. Refinery A produces 100, 300 and 200 barrels perpendicular day of high-grade, medium-grade and low-grade oil, respectively, while refinery B produces 200, 400 and 100 barrels perpendicular day of high-grade, medium-grade and low-grade oil, respectively. If the refinery A costs Rs 400 per day and refinery B costs Rs 300 per day to operate, how many days should each be run to minimize costs.

6. A manufacturer produces two types of steel trunks. He has two machines A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type of trunk requires 3 hours on machine A and 2 hours on machine B. Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of Rs 30 and Rs 25 per trunk of the first type and second type respectively. How many trunks of each type must he make each day to make the maximum profit?

7. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 5760 to invest and has space for at most 20 items. A fan costs him Rs 360 and a sewing machine costs him Rs 240. He expects to sell a fan a profit of Rs 22 and a sewing machine at a profit of Rs 18. Assuming that he can sell all the items that he buys, how should he invest his money to maximize the profit? what is the maximum profit?

8. If a young man drives his vehicle at 25 km/hr, he has to spend Rs 2 perpendicular km on petrol. If he rides at a faster speed of 40 km/hr, the petrol cost increases at Rs 5 perpendicular km. He has Rs 100 to spend on petrol and wishes to find what is the maximum distance he can travel in one hour. Express this as an LPP and solve it graphically.

9. A catering agency has two kitchens to prepare food at two places A and B. From these places 'Mid-day Meal' is to be supplied to three different schools situated at P, Q, R. The monthly requirements of the schools are respectively 40, 40 and 50 food packets. A packet contains lunch for 1000 students. Preparing capacity of kitchens A and B are 60 and 70 packets per month respectively. The transportation cost per packet from the kitchens to schools is given below:

Transportation cost per packet (in rupees)		
To	**From**	
	A	**B**
P	5	4
Q	4	2
R	3	5

How many packets from each kitchen should be transported to school so that the cost of transportation is minimum ? Also find the minimum cost.

10. A company has factories located at each of the two places P and Q. From these locations, a certain commodity is delivered to each of the depots situated at A, B and C. The weekly requirements of the depots are respectively 7, 6 and 4 units of the commodity while the weekly production capacities of the factories at P and Q are respectively 9 and 8 units. The cost of transportation per unit is given below:

To / From	Cost(in Rs)		
	A	B	C
P	16	10	15
Q	10	12	10

How many units should be transported from each factory to each depot in order that the transportation cost is minimum. Formulate the above LPP mathematically and the solve it.

ANSWERS

1. 5 units of A and 30 units of B; minimum cost is Rs 145
2. 400 g of wheat and 200 g of rice; minimum cost is Rs 2.80
3. 15 chairs and 25 tables; maximum profit is Rs 1,950
4. 6 machines of type A and no machine of type B OR 2 machines of type A and 6 machines of type B
5. A for 60 days, B for 30 days
6. 3 trunks of each type; maximum profit is Rs 165
7. 8 fans and 12 sewing machines; maximum profit is Rs 392
8. at 25 km/hr – 50/3 km; at 40 km/hr – 40/3 km; maximum distance is 30 km
9. 10, 0, 50 packets from A and 30, 40, 0 packets from B; minimum cost is Rs 400
10. 0, 6 and 3 unis from P and 7, 0 and 1 unis from Q

ASSIGNMENT CLASS XII PROBABILITY

1. If A and B are two events such that:

   $(i)\ P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$ and $P(A\cup B)=\frac{5}{12}$, find $P(A|B)$ and $P(B|A)$

   $(ii)\ P(A)=\frac{7}{13}, P(B)=\frac{9}{13}$ and $P(A\cap B)=\frac{4}{13}$, find $P(A'|B)$.

2. A pair of dice is thrown. Find the probability of getting 7 as the sum, if it is known that the second die always exhibits an odd number.

3. The probability that a student selected at random from a class will pass im mathematics is $\frac{4}{5}$, and the probability that he will pass in mathematics and economics is $\frac{1}{2}$. What is the probability that he will pass in economics if it is known that he has passed in mathematics?

4. A pair of dice is thrown. If the two numbers appearing on them are different, find the probability tha:
   (i) the sum of the numbers is 6 (ii) the sum of the numbers is 4 or less.

5. Find the probability of drqwing a diamond card in each of the two consecutive draws from a well shuffled pack of cards, if the card drawn is not replaced after the first draw.

6. A bag contains 19 tickets, numbered from 1 to 19. A ticket is drawn and then another ticket is drawn without replacement. Find the probability that both tickets will show even numbers.

7. Two cards are drawn without replacement from a pack of 52 cards. Find the probability that:
   (i) both are kings (ii) the first is a king and the second is an ace.

8. A bag contains 10 white and 15 black balls. Two balls are drawn succession without replacement. Find the probability that the first is white and the second is black ball?

9. A die is rolled. If the outcome is an odd numbers, what is the probability that it is a prime?

10. A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared atleast once?

11. Given two independent events A and B such that $P(A)=0.3$ and $P(B)=0.6$, find:
    $(i)\ P(A\cap B)$ $(ii)\ P(A\cap \overline{B})$ $(iii)\ P(\overline{A}\cap B)$ $(iv)\ P(\overline{A}\cap \overline{B})$ $(v)\ P(A\cup B)$ $(vi)\ P(A|B)$ $(vii)\ P(B|A)$

12. A coin is tossed thrice. Let the event E be 'the first throw results in a hea', and the event F be 'the last throw results in tail'. Find whether the events E and F are independent?

13. Sumit and Nishu appear for an interview for two vacancies in a company. The probabilities of their selection are respectively $\frac{1}{5}$ and $\frac{1}{6}$. what is the probability that:

    (i) both of them are selected (ii) only one of them is selected (iii) none of them is selected?

14. A and B appear for an interview for two posts. The probabilities of their selection are respectively $\frac{1}{3}$ and $\frac{2}{5}$. What is the probability that only one of them will be selected?
15. A can solve 90% of the problems given in a book, and B can solve 70%. What is the probability that atleast one of them will solve a problem selected at random from the book?

16. A speaks the truth in 60% of the cases, and B in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

17. A problem in mathematics is given to three students whose chances of solving it correctly are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that only one of them solves it correctly?

18. An urn contains 4 red and 7 black balls. Two balls are drawn at random with replacement. Find the probability of getting: (*i*) 2 red balls (*ii*) 2 blue balls (*iii*) one red and one blue ball.

19. A can hit a target 4 times in 5 shots, B 3 times in 4 shots, and C 2 times in 3 shots. Find the probability that: (*i*) A, B, C all may hit (*ii*) B, C may hit and A may lose
(*iii*) any two of A, B, and C will hit the target (*iv*) none of them will hit the target?

20. Two persons A nad B throw a coin alternately till one of them gets a 'head' and wins the game. Find their respective probabilities of winning if A starts first.

21. A speaks the truth 8 times out of 10 times. A die is thrown. He reports that it was 5. What is the probability that it was actually 5?

22. In a bulb factory, machines A, B and C manufatures 60%, 30% and 10% bulbs respectively. 1%, 2% and 3% of the bulbs produced respectively by A, B and C are found to be defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by the machine A.

23. A candidate has to reach the examination centre in time. Probability of him going by bus or scooter or by other means of transport are $\frac{3}{10}, \frac{1}{10}, \frac{3}{5}$ respectively. The probability that he will be late is $\frac{1}{4}$ and $\frac{1}{3}$ respectively, if he travels by bus or scooter. But he reaches in time if he uses any other mode of transport. He reached late at the centre. Find the probability that he travelled by bus.

24. Two bags A and B contain 4 white 3 black balls and 2 white and 2 black balls respectively. From bag A two balls are transferred to bag B. Find the probability of drawing:
(*i*) 2 white balls from bag B ? (*ii*) 2 black balls from bag B ? (*iii*) 1 white & 1 black ball from bag B?

25. In a bolt factory machines, A, B and C manufacture respectively 25%, 35% and 40% of the total bolts. Of their output 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from product.
(*i*) What is the probability that the bolt drawn is defective ?
(*ii*) If the bolt is found to be defective find the probability that it is a product of machine B.

26. A letter is known to have come from either TATANAGAR or CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probabiity that the letter has come from:
(*i*) Tata nagar (*ii*) Calcutta

27. Bag A contains 2 white and 3 red balls, and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find probability that it was drawn from bag B.

28. Three urns A, B and C contain 6 red and 4 white; 2 red and 6 whit; and 1 red and 5 white balls respectively. An urn is chosen at random and a ball is drawn. If the drawn ball is found to be red, Find the probability that the ball was drawn from the urn A.

29. A company has two plants to manufacture bicycles. The first plant manufatures 60% of the bicycles and the second plant, 40%. Also, 80% of the bicycles are rated of standard quality at the first plant and 90% of standard quality at the second plant. A bicycle is picked at random and found to be of standard quality. Find the probability that it comes from the second plant.

30. A factory has three machines, X, Y and Z, producing 1000, 2000 and 3000 bolts perpendicular day respectively. The machine X produces 1% defective bolts, machine Y produces 1.5% defective bolts and machine Z produces 2% defective bolts. At the end of the day, a bolt is drawn at random and it is found to be defective. Find the probability that this defective bolt has been produced by the machine X?

31. A random variable X has the following probability distribution:

x_i	-2	-1	0	1	2	3
p_i	0.1	k	0.2	$2k$	0.3	k

(*i*) Find the value of k (*ii*) Find mean of X (*iii*) Find variance of X.

32. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes.

33. A football match may be either won, drawn or lost by the host country's team. So there are three ways of forecasting the result of any one match, one correct and two incorrect. Find the probability of forecasting at least three correct results for four matches.

34. Let X denote the number of colleges where you will apply after your results and P(X = *x*) denotes your probability of getting admission in *x* number of colleges. It is given that:

$$P(X=x)=\begin{cases} kx & \text{if } x=0 \text{ or } 1 \\ 2kx & \text{if } x=2 \\ 5(5-x) & \text{if } x=3 \text{ or } 4 \end{cases} \qquad k \text{ is a positive integer.}$$

(*i*) Find the value of k (*ii*) What is the probability that you will get admission in exactly 2 colleges? (*iii*) Find the mean and variance of the probability distribution.

35. Find the probability distribution of the number of white balls drawn in a random draw of 3 balls without replacement from a bag containing 4 white and 6 red balls. Also find the mean and variance of the distribution.

36. A fair die is tossed twice. If the number appearing on the top is less than 3, it is a success. Find the probability distribution of number of successes.

37. The probability of hitting a target by A is $\frac{1}{5}$. If he fires 5 times, find the probability that he will hit atleast two times.

38. Two cards are drawn successively with replacement from a pack of 52 cards. Find the mean and variance of the number of kings.

39. A coin is tossed 4 times. Let X denote the number of heads. Find the mean and variance of X.

40. 3 defective bulbs are mixed with 7 good ones. Let X be the number of defective bulbs when 3 bulbs are drawn at random. Find the mean and variance of X.

41. An unbiased coin is tossed 8 times. Find, by using binomial distribution, the probability of getting atleast 3 heads.

42. The probability of a man hitting a target is 1/4. How many times must he fire so that the probability of his hitting the target at least once is greater than 2/3?

43. Six coins are tossed simultaneously. Find the probability of getting:
(*i*) 3 heads (*ii*) no head (*iii*) atleast one head (*iv*) not more than 3 heads

44. The probability that a student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university:

(i) none will graduate (ii) only one will graduate (iii) all will graduate

45. A pair of dice is thrown 7 times If getting a total of 7 is considered as success, Find the probability of:

(i) no success (ii) 6 successes (iii) atleast 6 successes (iv) atmost 6 successes

46. Find the binomial distribution for which the mean and variance are 12 and 3 respectively.

ANSWERS

1. (i) $\frac{2}{3}, \frac{1}{2}$ (ii) $\frac{5}{9}$ 2. $\frac{1}{6}$ 3. $\frac{5}{8}$ 4. (i) $\frac{2}{15}$ (ii) $\frac{2}{15}$ 5. $\frac{1}{17}$ 6. $\frac{4}{19}$

7. (i) $\frac{1}{221}$ (ii) $\frac{4}{663}$ 8. $\frac{1}{4}$ 9. $\frac{2}{3}$ 10. $\frac{2}{5}$ 11. (i) 0.18 (ii) 0.12 (iii) 0.42

(iv) 0.28 (v) 0.72 (vi) 0.3 (vii) 0.6 12. yes 13. (i) $\frac{1}{30}$ (ii) $\frac{3}{10}$ (iii) $\frac{2}{3}$ 14. $\frac{7}{15}$

15. 0.97 16. 42% 17. $\frac{11}{24}$ 18. (i) $\frac{16}{121}$ (ii) $\frac{49}{121}$ (iii) $\frac{56}{121}$ 19. (i) $\frac{2}{5}$ (ii) $\frac{1}{10}$ (iii) $\frac{13}{30}$ (iv) $\frac{1}{60}$

20. $\frac{2}{3}, \frac{1}{3}$ 21. $\frac{4}{9}$ 22. $\frac{2}{5}$ 23. $\frac{9}{13}$ 24. (i) $\frac{5}{21}$ (ii) $\frac{4}{21}$ (iii) $\frac{4}{7}$ 25. (i) 0.0345 (ii) $\frac{28}{69}$

26. (i) $\frac{7}{11}$ (ii) $\frac{4}{11}$ 27. $\frac{25}{52}$ 28. $\frac{36}{61}$ 29. $\frac{3}{7}$ 30. 0.1 31. (i) 0.1 (ii) 0.8 (iii) 2.16

32.

X	0	1	2	3	4
P(X)	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$

33. $\frac{1}{9}$ 34. (i) $k = \frac{1}{8}$ (ii) $\frac{1}{2}$ (iii) $\frac{19}{8}, \frac{47}{64}$ 35.

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

1.20, 0.56 36.

X	0	1	2
P(X)	4/9	4/9	1/9

37. $\frac{821}{3125}$ 38. $\frac{24}{169}$ 39. 2, 1

40. 0.9, 0.49 41. $\frac{219}{256}$ 42. 4 times 43. (i) $\frac{5}{16}$ (ii) $\frac{1}{64}$ (iii) $\frac{63}{64}$ (i) $\frac{21}{32}$ 44. (i) 0.216 (ii) 0.432 (iii) 0.064

45. (i) $\left(\frac{5}{6}\right)^7$ (ii) $35\left(\frac{1}{6}\right)^7$ (iii) $\left(\frac{1}{6}\right)^5$ (iv) $1-\left(\frac{1}{6}\right)^7$

46. $P(X=r) = {}^{16}C_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{16-r}$, where $r = 0,1,2,...,15$