ASSIGNMENT CLASS XII RELATIONS AND FUNCTIONS

Important Formulas

If A and B are finite sets containing m and n elements, then

1. Total number of relations from the set A to set B is 2^{mn}.
2. Total number of relations on the set A is 2^{m^2} .
3. Total number of functions from the set A to set B is n^m.
4. Total number of one-one functions from the set A to set B is ${}^nP_m$ if $n \geq m$, otherwise 0.
5. Total number of onto functions from set A to set B is $\sum_{r=1}^{n} (-1)^{n-r}\, {}^nC_r\, r^m$ if $m \geq n$, otherwise 0.
6. Total number of bijective functions from the set A to set B is $m!, if\ m = n$, otherwise 0.
7. Total number of binary operations on A is m^{m^2} .

Q1. Prove that the relation R on the set Z of all integers defined by $(a,b),(c,d) \in N \times N$ is divisible by n is an equivalence relation on Z .

Q2. Prove that the relation R on the set $N \times N$ defined by $(a,b)R(c,d) \Leftrightarrow a+d=b+c$ for all $(a,b),(c,d) \in N \times N$ is an equivalence relation.

Q3. Let N be the set of all natural numbers and let R be a relation on $N \times N$, defined by $(a,b)R(c,d) \Leftrightarrow ad = bc$ for all $(a,b),(c,d) \in N \times N$. Show that R is an equivalence relation on $N \times N$.

Q4. Let a relation R_1 on the set R of real numbers be defined as $(a,b) \in R_1 \Leftrightarrow 1+ab > 0$ for all $a,b \in R$. Show that R_1 is reflexive and symmetric but not transitive. R_1

Q5. Let R be a relation defined on the set of natural numbers N as $R = \{(x,y) : x, y \in N, 2x + y = 41\}$. Show that R is neither reflexive nor symmetric but it is transitive.

Q6. Let S be a relation on the set R of all the real numbers defined by $S = \{(a,b) \in R \times R : a^2 + b^2 = 1\}$. Prove that S is not an equivalence relation on R.

Q7. If $a \cong b (\text{mod}\, m)$ means that m divides $a - b$. Show that $\cong$ is an equivalence relation on Z .

Q8. Le N be the set of all natural numbers and let R be a relation in N , defined by $R = \{(a,b) : a$ is a factor of $b\}$. Show that R is reflexive and transitive but not symmetric.

Q9. Prove that the relation R on the set Z of all integers defined by $x\,R\,y \Leftrightarrow x^y = y^x$ for all $x, y \in Z$ is an equivalence relation.

Q10. Test whether the following relations R_1, R_2, R_3 are (i) reflexive (ii) symmetric (iii) transitive:

(i) R_1 on Q_0 defined by $(a,b) \in R_1 \Leftrightarrow a = 1/b$ (ii) R_2 on Z defined by $(a,b) \in R_2 \Leftrightarrow |a-b| \leq 5$

(iii) R_3 on R defined by $(a,b) \in R_3 \Leftrightarrow a^2 - 4ab + 3b^2 = 0$

Q11. Classify the following functions as injection, surjection or bijection:

(i) $f : R \to R, \quad f(x) = |x|$ (ii) $f : Z \to Z, \quad f(x) = x^2 + x$

(iii) $f : Z \to Z, \quad f(x) = x - 5$ (iv) $f : R \to R, \quad f(x) = \sin x$

(v) $f : R \to R, \quad f(x) = x^3 + 1$ (vi) $f : R \to R, \quad f(x) = x^3 - x$

(vii) $f : R \to R, \quad f(x) = \sin^2 x + \cos^2 x$ ($viii$) $f : Q - \{3\} \to Q, \quad f(x) = \dfrac{2x+3}{x-3}$

(ix) $f : Q \to Q, \quad f(x) = x^3 + 1$ (x) $f : R \to R, \quad f(x) = 5x^3 + 4$

Q12. Prove that the function $f : N \to N, \quad f(x) = x^2 + x + 1$ is one-one but not onto.

Q13. Let $f : R - \{2\} \to R - \{1\}$ is a mapping defined by $f(x) = \dfrac{x-1}{x-2}$, show that f is bijective.

Q14. Let $f : N - \{1\} \to N \quad f(n) =$ the highest prime factor of n. Show that f is neither one-one nor onto. Find the range of f also.

Q15. Find gof and fog when and $f:R\to R$ and $g:R\to R$ are defined by:

(i) $f(x)=2x+3$ and $g(x)=x^2+5$ (ii) $f(x)=2x+x^2$ and $g(x)=x^3$

(iii) $f(x)=x^2+8$ and $g(x)=3x^3+1$ (iv) $f(x)=x$ and $g(x)=|x|$

Q16. If $A=\{1,2,3,4,5\}$, $B=\{3,5,7,9\}$, $C=\{7,23,47,79\}$ and $f:A\to B$ and $g:B\to C$ be defined as $f(x)=2x+1$ and $g(x)=x^2-2$. Express as the sets of ordered pairs and verify $(gof)^{-1}=f^{-1}og^{-1}$.

Q17. Let f be the greatest integer function and g be the modulus function, then prove that:

(i) $gof\left(\frac{5}{3}\right)-fog\left(\frac{5}{3}\right)=0$ (ii) $gof\left(\frac{-5}{2}\right)-fog\left(\frac{-5}{2}\right)=1$ (iii) $fog\left(\frac{5}{2}\right)-gof\left(\frac{-5}{2}\right)=-1$

Q18. If $f(x)=\frac{x-1}{x+1}$ where $x\neq\pm1$. Show that fof^{-1} is an identity function.

Q19. Let $f:R\to R: f(x)=2x-3$ and $g:R\to R: g(x)=\frac{1}{2}(x+3)$. Show that $(fog)=I_R=(gof)$.

Q20. Show that the function $f:R\to R$ defined by $f(x)=x^3+3$ is invertible. Also find the inverse of f.

Q21. Determine whether or not each of the definition of $*$ given below gives a binary operation. In the event that $*$ is not a binary operation, give justification:

(i) On N, defined by $a*b=\frac{a+b}{2}$ (ii) On R^+, defined by $a*b=a^2+4ab$

(iii) On N, defined by $a*b=a+b-2$ (iv) On Q, defined by $a*b=\frac{a-1}{b+1}$

Q22. For each of binary operations $*$ defined below, determine whether $*$ is commutative and associative:

(i) On Q, defined by $a*b=\frac{a+b}{2}$ (ii) On R, defined by $a*b=2a+2b$

(iii) On Z, defined by $a*b=a+b-ab$ (iv) On Z, defined by $a*b=a-b+ab$

Q23. Let A be a set of all real numbers i.e. $A=R-\{-1\}$. Let $*$ be defined on A as $a*b=a+b+ab$.

(a) Prove that (i) $*$ is a binary operation on A. (ii) $*$ is commutative and associative.

(iii) 0 is the identity element. (iv) $\frac{-a}{1+a}$ is the inverse of a.

Q24. Show that the operation $*$ on Z, defined by $a*b=a+b+1$, satisfies

(i) closure property (ii) associative property (iii) commutative property.

Also find the identity element and inverse of an element $a\in A$.

Q25. Let $S=R_o\times R$. A binary operation $*$ is defined on S as follows:

$(a,b)*(c,d)=(ac,bc+d)\,\forall\,(a,b),(c,d)\in R_0\times R$. Find:

(i) identity element in S. (ii) invertible element in S.

ANSWERS

10. (i) R_1 is symmetric but neither reflexive nor transitive. (ii) R_2 is reflexive and symmetric but not transitive. (iii) $f:Z\to Z$ $f(x)=x-5$ is reflexive but neither symmetric nor transitive.

11. (i) Neither an injecton nor a surjection (ii) Neither an injecton nor a surjection (iii) bijective (iv) Neither an injecton nor a surjection (v) bijective (vi) surjective but not injective (vii) Neither an injecton nor a surjection $(viii)$ injective but not surjective (ix)injective (x) bijective 14. set of all prime numbers

15.(i) $4x^2+12x+14$, $2x^2+13$ (ii) $\left(x^2+2x\right)^3$, $2x^3+x^6$ (iii) $3\left(x^2+8\right)^3+1$, $9x^6+6x^3+9$ (iv) $|x|,|x|$ 16. $\{(7,1),(23,2),(47,3),(79,4)\}$ 20. $f^{-1}(x)=(x-3)^{\frac{1}{3}}$ 21. (i) No (ii) Yes (iii)No (iv) No

22. (i) commutative but not associative (ii) commutative but not associative (iii) commutative and associative (iv) neither commutative nor associative

24. identity is -1 and inverse of a is $-(2+a)$ 25. (i) $(1,0)$ (ii) $\left(\frac{1}{a},\frac{-b}{a}\right)$

ASSIGNMENT CLASS XII INVERSE TRIGONOMETRY

Q1. Find the principle value of the following:

(a) $\sin^{-1}(\frac{-1}{\sqrt{2}})$ (b) $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ (c) $\tan^{-1}(-\sqrt{3})$ (d) $\sec^{-1}(-2)$ (e) $\cos ec^{-1}(-\sqrt{2})$

(f) $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)$ (g) $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$ (h) $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$ (i) $\cos^{-1}\left(\cos\frac{8\pi}{7}\right)$ (j) $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)$

Q2. Evaluate the following:

(a) $\cos\left(\sin^{-1}\left(\frac{-3}{5}\right)\right)$ (b) $\cos ec\left(\cos^{-1}\left(\frac{-4}{5}\right)\right)$ (c) $\cos\left(\tan^{-1}\left(\frac{3}{4}\right)\right)$

(d) $\tan\left(\cos^{-1}\left(\frac{8}{17}\right)\right)$ (e) $\tan\left(2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right)$ (f) $\sin\left(2\cos^{-1}\left(\frac{-3}{5}\right)\right)$

Q3. Prove the following:

(a) $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$

(b) $\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65}=\frac{\pi}{2}$

(c) $2\tan^{-1}\frac{1}{5}+\sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)+2\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

(d) $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\tan^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{4}=\frac{\pi}{4}$

(e) $2\cot^{-1}5+\cot^{-1}7+2\cot^{-1}8=\frac{\pi}{4}$

(f) $2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$

(g) $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{63}{16}$

(h) $\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{3}{5}=\tan^{-1}\frac{27}{11}$

(i) $\tan^{-1}1+\cos^{-1}\left(\frac{-1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)=\frac{3\pi}{4}$

(j) $\sin\left[\tan^{-1}\left(\frac{1-x^2}{2x}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]=1$

Q4. Write the following in the simplest form:

(a) $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ (b) $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$ (c) $\tan^{-1}\left(\frac{\cos 2x}{1+\sin 2x}\right)$

(d) $\tan^{-1}\sqrt{\frac{a-x}{a+x}}$ (e) $\cos\left(\tan^{-1}\left(\sin\left(\cot^{-1}x\right)\right)\right)$ (f) $\cot^{-1}\left(\sqrt{1+x^2}-x\right)$

Q5. Solve the following equations:

(a) $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$

(b) $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{6}{17}$

(c) $\sin^{-1}x+\sin^{-1}2x=\frac{\pi}{3}$

(d) $\cos^{-1}x+\sin^{-1}\frac{x}{2}=\frac{\pi}{6}$

(e) $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$

(f) $4\sin^{-1}x+\cos^{-1}x=\pi$

ANSWERS

1.(a) $\frac{-\pi}{4}$ (b) $\frac{5\pi}{6}$ (c) $\frac{-\pi}{3}$ (d) $\frac{2\pi}{3}$ (e) $\frac{-\pi}{4}$ (f) $\frac{\pi}{4}$ (g) $\frac{3\pi}{4}$ (h) $\frac{-\pi}{3}$ (i) $\frac{6\pi}{7}$ (j) $\frac{-\pi}{6}$

2. (a) $\frac{4}{5}$ (b) $\frac{5}{3}$ (c) $\frac{4}{5}$ (d) $\frac{15}{17}$ (e) $\frac{-7}{17}$ (f) $\frac{-24}{25}$ 4.(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$ (c) $\frac{\pi}{4}-x$ (d) $\frac{1}{2}\cos^{-1}\frac{x}{a}$

e) $\sqrt{\frac{x^2+1}{x^2+2}}$ (f) $\frac{\pi}{4}+\frac{1}{2}\tan^{-1}x$ 5.(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}\sqrt{\frac{3}{7}}$ (d) ± 1 (e) $\frac{\sqrt{3}}{2}$ (f) $\frac{1}{2}$

ASSIGNMENT CLASS XII MATRICES

1. Construct a 2×3 matrix A, 3×2 matrix $\mathbf{B}$, whose elements are given by $a_{ij}=\dfrac{(i-2j)^2}{2}$.

2. If $A=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 3 & 5\\ 1 & -3 & -5\\ -1 & 3 & 5\end{bmatrix}$, show that $AB=BA=O_{3\times3}$.

3. If $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then (i) find λ,μ so that $A^2=\lambda A+\mu I$ (ii) prove $A^3-4A^2+A=O$.

4. (a) If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$, then show that $A^2=\begin{bmatrix}\cos2\alpha & \sin2\alpha\\ -\sin2\alpha & \cos2\alpha\end{bmatrix}$.

   (b) If $A=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}$, then prove by mathematical induction that $A^n=\begin{bmatrix}\cos n\theta & i\sin n\theta\\ i\sin n\theta & \cos n\theta\end{bmatrix}, n\in N$.

   (c) If $A=\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}$, prove by mathematical induction that $A^n=\begin{bmatrix}1 & n & \dfrac{n(n+1)}{2}\\ 0 & 1 & n\\ 0 & 0 & 1\end{bmatrix}, n\in N$

5. Express the following matrices as the sum of symmetric and a skew-symmetric matrix:

   (a) $\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$ (b) $\begin{bmatrix}3 & -1 & 0\\ 2 & 0 & 3\\ 1 & -1 & 2\end{bmatrix}$ (c) $\begin{bmatrix}1 & 3 & 5\\ -6 & 8 & 3\\ -4 & 6 & 5\end{bmatrix}$ (d) $\begin{bmatrix}6 & 1 & -5\\ -2 & -5 & 4\\ -3 & 3 & -1\end{bmatrix}$

6.(a) Find the matrix C, such that $A+B+C$ is a zero matrix, where $A=\begin{bmatrix}2 & 0 & 1\\ 3 & -1 & 0\end{bmatrix}, B=\begin{bmatrix}2 & 1 & -1\\ 0 & 2 & 1\end{bmatrix}$.

   (b) Find a matrix X such that $2A+B+X=0$, where $A=\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}$.

7. From the following equation, find the values of x and y:

   (a) $2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & 4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 14\\ 15 & 14\end{bmatrix}$ (b) $\begin{bmatrix}x+10 & y^2+2y\\ 0 & -4\end{bmatrix}=\begin{bmatrix}3x+4 & 3\\ 0 & y^2-5y\end{bmatrix}$

8. If $A=\begin{bmatrix}1 & 2\\ -2 & 3\end{bmatrix}, B=\begin{bmatrix}2 & 1\\ 2 & 3\end{bmatrix}$ and $C=\begin{bmatrix}-3 & 1\\ 2 & 0\end{bmatrix}$, verify that :

   (i) $(A+B)'=A'+B'$ (ii) $(AB)'=B'A'$ (iii) $(AB)C=A(BC)$ (iv) $A(B+C)=AB+AC$

9. (a) If $A=\begin{bmatrix}-4 & 1\\ 3 & 2\end{bmatrix}$, find $f(A)$ if $f(x)=x^2-2x+3$. (b) $A=\begin{bmatrix}3 & 4\\ -4 & -3\end{bmatrix}$, find $f(A)$ *if* $f(x)=x^2-5x+7$

   (c) If $A=\begin{bmatrix}3 & -4\\ -1 & 2\end{bmatrix}$, find a matrix B such that $AB=I$. (d) If $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$, prove that $A^3-4A^2+A=0$.

   (e) If $A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$, find k such that $A^2-8A+kI=0$. (f) If $A=\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}$, prove that $A^2-5A-14I=0$.

10. Let $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}, B=\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}, C=\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$. Find a matrix D such that $CD-AB=0$.

11. (a) *If* $A=\begin{bmatrix}2 & 1 & 3\\ 4 & 1 & 0\end{bmatrix}$ *and* $B=\begin{bmatrix}1 & -1\\ 0 & 2\\ 5 & 0\end{bmatrix}$, *verify that* $(AB)'=B'A'$.

(b) *If* $A=\begin{bmatrix}2 & 4\\ 3 & 5\end{bmatrix}$ *and* $B=\begin{bmatrix}5 & 4\\ 3 & 2\end{bmatrix}$, *verify that* $(AB)'=B'A'$.

12. Using elementary transformations, find the inverse of the following matrices:

(a) $A=\begin{bmatrix}2 & 1\\ 4 & 7\end{bmatrix}$ (b) $A=\begin{bmatrix}2 & -3\\ 3 & 4\end{bmatrix}$ (c) $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 7\\ -2 & -4 & -5\end{bmatrix}$ (d) $A=\begin{bmatrix}2 & -1 & 4\\ 4 & 0 & 2\\ 3 & -2 & 7\end{bmatrix}$

ANSWERS

1. $\begin{bmatrix}\frac{1}{2} & \frac{9}{2} & \frac{25}{2}\\ 0 & 2 & 8\end{bmatrix}, \begin{bmatrix}\frac{1}{2} & \frac{9}{2}\\ 0 & 2\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$ 3. $\lambda=4, \mu=-1$ 5. (a) $\begin{bmatrix}3 & \frac{-3}{2}\\ \frac{-3}{2} & -1\end{bmatrix}+\begin{bmatrix}0 & \frac{-5}{2}\\ \frac{5}{2} & 0\end{bmatrix}$ (b) $\begin{bmatrix}3 & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & 0 & 1\\ \frac{1}{2} & 1 & 2\end{bmatrix}+\begin{bmatrix}0 & \frac{-3}{2} & \frac{-1}{2}\\ \frac{3}{2} & 0 & 2\\ \frac{1}{2} & -2 & 0\end{bmatrix}$

(c) $\begin{bmatrix}1 & \frac{-3}{2} & \frac{1}{2}\\ \frac{-3}{2} & 8 & \frac{9}{2}\\ \frac{1}{2} & \frac{9}{2} & 5\end{bmatrix}+\begin{bmatrix}0 & \frac{9}{2} & \frac{9}{2}\\ \frac{-9}{2} & 0 & \frac{-3}{2}\\ \frac{-9}{2} & \frac{3}{2} & 0\end{bmatrix}$ (d) $\begin{bmatrix}6 & \frac{-1}{2} & -4\\ \frac{-1}{2} & -5 & \frac{7}{2}\\ -4 & \frac{7}{2} & -1\end{bmatrix}+\begin{bmatrix}0 & \frac{3}{2} & -1\\ \frac{-3}{2} & 0 & \frac{1}{2}\\ 1 & \frac{-1}{2} & 0\end{bmatrix}$

6.(a) $\begin{bmatrix}-4 & -1 & 0\\ -3 & -1 & -1\end{bmatrix}$ (b) $X=\begin{bmatrix}-1 & -2\\ -7 & -13\end{bmatrix}$ 7. (a) $x=2, y=9$ (b) $x=3, y=1$

9.(a) $f(A)=\begin{bmatrix}30 & -4\\ -12 & 6\end{bmatrix}$ (b) $f(A)=\begin{bmatrix}-15 & -20\\ 20 & 15\end{bmatrix}$ (c) $B=\begin{bmatrix}1 & 2\\ \frac{1}{2} & \frac{3}{2}\end{bmatrix}$ (e) $k=7$

10. $\begin{bmatrix}-191 & -110\\ 77 & 44\end{bmatrix}$ 12.(a) $A^{-1}=\begin{bmatrix}\frac{7}{10} & \frac{-1}{10}\\ \frac{-2}{5} & \frac{1}{5}\end{bmatrix}$ (b) $A^{-1}=\frac{1}{17}\begin{bmatrix}4 & 3\\ -3 & 2\end{bmatrix}$ (c) $A^{-1}=\begin{bmatrix}3 & -2 & 1\\ -4 & 1 & -1\\ 2 & 0 & 1\end{bmatrix}$

(d) $A^{-1}=\begin{bmatrix}-2 & \frac{1}{2} & 1\\ 11 & -1 & -6\\ 4 & \frac{-1}{2} & -2\end{bmatrix}$

ASSIGNMENT CLASS XII DETERMINANTS

1. Find the values of x , if (i) $\begin{vmatrix}2&4\\5&1\end{vmatrix}=\begin{vmatrix}2x&4\\6&x\end{vmatrix}$ (ii) $\begin{vmatrix}2&3\\4&5\end{vmatrix}=\begin{vmatrix}x&3\\2x&5\end{vmatrix}$ (iii) $\begin{vmatrix}x-2&-3\\3x&2x\end{vmatrix}=3$

2. Using properties of determinants, show that:

(i) $\begin{vmatrix}a-b-c&2a&2a\\2b&b-c-a&2b\\2c&2c&c-a-b\end{vmatrix}=(a+b+c)^3$ (ii) $\begin{vmatrix}a&b&c\\a^2&b^2&c^2\\bc&ca&ab\end{vmatrix}=(a-b)(b-c)(c-a)(ab+bc+ca)$

(iii) $\begin{vmatrix}b+c&c+a&a+b\\c+a&a+b&b+c\\a+b&b+c&c+a\end{vmatrix}=2(a+b+c)(ab+bc+ca-a^2-b^2-c^2)$ (iv) $\begin{vmatrix}1&a&a^2-bc\\1&b&b^2-ca\\1&c&c^2-ab\end{vmatrix}=0$

(v) $\begin{vmatrix}x&y&z\\x^2&y^2&z^2\\x^3&y^3&z^3\end{vmatrix}=xyz(x-y)(y-z)(z-x)$ (vi) $\begin{vmatrix}(b+c)^2&a^2&a^2\\b^2&(c+a)^2&b^2\\c^2&c^2&(a+b)^2\end{vmatrix}=2abc(a+b+c)^3$

(vii) $\begin{vmatrix}x&y&z\\x^2&y^2&z^2\\y+z&z+x&x+y\end{vmatrix}=(x-y)(y-z)(z-x)(x+y+z)$ (viii) $\begin{vmatrix}a^2+2a&2a+1&1\\2a+1&a+2&1\\3&3&1\end{vmatrix}=(a-1)^3$

(ix) $\begin{vmatrix}1&x+y&x^2+y^2\\1&y+z&y^2+z^2\\1&z+x&z^2+x^2\end{vmatrix}=(x-y)(y-z)(z-x)$ (x) $\begin{vmatrix}a&a+b&a+2b\\a+2b&a&a+b\\a+b&a+2b&a\end{vmatrix}=9b^2(a+b)$

(xi) $\begin{vmatrix}1&a^2+bc&a^3\\1&b^2+ca&b^3\\1&c^2+ab&c^3\end{vmatrix}=(a-b)(b-c)(a-c)(a^2+b^2+c^2)$ (xii) $\begin{vmatrix}-\alpha^2&\alpha\beta&\gamma\alpha\\\alpha\beta&-\beta^2&\beta\gamma\\\gamma\alpha&\beta\gamma&-\gamma^2\end{vmatrix}=4\alpha^2\beta^2\gamma^2$

(xiii) $\begin{vmatrix}a&b&c\\a-b&b-c&c-a\\b+c&c+a&a+b\end{vmatrix}=a^3+b^3+c^3-3abc$ (xiv) $\begin{vmatrix}b+c&a&b\\c+a&c&a\\a+b&b&c\end{vmatrix}=(a+b+c)(a-c)^2$

(xv) $\begin{vmatrix}1+a&1&1\\1&1+b&1\\1&1&1+c\end{vmatrix}=abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=abc+bc+ca+ab$ (xvi) $\begin{vmatrix}0&b^2a&c^2a\\a^2b&0&c^2b\\a^2c&b^2c&0\end{vmatrix}=2a^3b^3c^3$

3. Without expanding the determinants, show that:

(i) $\begin{vmatrix}1&a&bc\\1&b&ca\\1&c&ab\end{vmatrix}=\begin{vmatrix}1&a&a^2\\1&b&b^2\\1&c&c^2\end{vmatrix}$ (ii) $\begin{vmatrix}a&a^2&bc\\b&b^2&ca\\c&c^2&ab\end{vmatrix}=\begin{vmatrix}1&a^2&a^3\\1&b^2&b^3\\1&c^2&c^3\end{vmatrix}$ (iii) $\begin{vmatrix}a+b&b+c&c+a\\b+c&c+a&a+b\\c+a&a+b&b+c\end{vmatrix}=2\begin{vmatrix}a&b&c\\b&c&a\\c&a&b\end{vmatrix}$

4. Using properties of determinants, solve for x:

(i) $\begin{vmatrix}a+x&a-x&a-x\\a-x&a+x&a-x\\a-x&a-x&a+x\end{vmatrix}=0$ (ii) $\begin{vmatrix}3x-8&3&3\\3&3x-8&3\\3&3&3x-8\end{vmatrix}=0$ (iii) $\begin{vmatrix}x&-6&-1\\2&-3x&x-3\\-3&2x&x+2\end{vmatrix}=0$

5. Using determinants, find the area of the triangle whose vertices are $(-2,4),(2,-6)$ and $(5,4)$. Are the given points collinear?

6. Find the equation of the line joining $A(1,3)$ and $B(0,0)$ using determinants and find k if $D(k,0)$ is a point such that $ar(\Delta ABD)$ is 3 sq. units.

7. Find the value of x if the area of the triangle with vertices $(x,4),(2,-6)$ and $(5,4)$ be 70 sq. cm.

8. Find the value of λ so that the points $(\lambda,2-2\lambda),(-\lambda+1,2\lambda)$ and $(-4-\lambda,6-2\lambda)$ are collinear?

9. Let $A=\begin{bmatrix}1 & -2 & 1\\ -2 & 3 & 1\\ 1 & 1 & 5\end{bmatrix}$, verify that: $(i)\ A(adjA)=(adjA)A=|A|I$ $\quad (ii)\ (adjA)^{-1}=adj(A^{-1})$ $\quad (iii)\ (A^{-1})^{-1}=A$

$(iv)\ (A^T)^{-1}=(A^{-1})^T$ $\quad (v)\ adjA^T=(adjA)^T$

10. If $A=\begin{bmatrix}2 & 2 & 1\\ -2 & 1 & 2\\ 1 & -2 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 3 & 2\\ 1 & 1 & 1\\ 2 & -3 & -1\end{bmatrix}$, verify that: $(i)\ (AB)^{-1}=B^{-1}A^{-1}$ $\quad (ii)\ adj(AB)=(adjB)(adjA)$

11. Given $A=\begin{bmatrix}5 & 0 & 4\\ 2 & 3 & 2\\ 1 & 2 & 1\end{bmatrix}$ and $B^{-1}=\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$. Compute $(AB)^{-1}$. Ans: $(AB)^{-1}=\begin{bmatrix}-2 & 19 & -27\\ -2 & 18 & -25\\ -3 & 29 & -42\end{bmatrix}$

12. If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, prove that $A^2-4A-5I=0$. Hence find A^{-1}. Ans: $A^{-1}=\frac{1}{5}\begin{bmatrix}-3 & 2 & 2\\ 2 & -3 & 2\\ 2 & 2 & -3\end{bmatrix}$

13. Find A, so that $(i)\ A\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}=\begin{bmatrix}14 & 7\\ 7 & 7\end{bmatrix}$ $(ii)\ \begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $(iii)\ \begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}A=\begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$

Ans: $A=\begin{bmatrix}3 & 1\\ 1 & -2\end{bmatrix}$ $\quad A=\begin{bmatrix}25 & 15\\ -37 & -22\end{bmatrix}$ $\quad A=\begin{bmatrix}1 & -2 & -5\\ 3 & 4 & 0\end{bmatrix}$

14. Find whether the following system of equations is consistent or not, find the solution of the system also:

(i) $3x-y+2z=3$, $x-2y-z=1$, $2x+y+3z=5$

(ii) $5x-7y+z=11$, $6x-8y-z=15$, $3x+2y-6z=7$

(iii) $x+y+z=6$, $x+2y+3z=14$, $x+4y+7z=30$

Ans: inconsistent $\quad$ consistent $x=1, y=-1, z=-1$ $\quad$ consistent, $x=k-2, y=8-2k, z=k$

15. Using matrix method, solve the following system of linear equations:

(i) $4x+2y+3z=2$, $x+y+z=1$, $3x+y-2z=5$

(ii) $x+2y+z=7$, $x+3z=11$, $2x-3y=1$

(iii) $x-y+z=2$, $2x-y=0$, $2y-z=1$

(iv) $x+y-z=1$, $3x+y-2z=3$, $x-y-z=-1$

(v) $x-y=3$, $2x+3y+4z=17$, $y+2z=7$

Ans: $x=\frac{12}{2}, y=\frac{3}{2}, z=-1$ $\quad x=2, y=1, z=3$ $\quad x=1, y=2, z=3$ $\quad x=2, y=1, z=2$ $\quad x=2, y=-1, z=4$

16. Find the product of matrices $A=\begin{bmatrix}-5 & 1 & 3\\ 7 & 1 & -5\\ 1 & -1 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 1 & 2\\ 3 & 2 & 1\\ 2 & 1 & 3\end{bmatrix}$ and use it for solving the equations $x+y+2z=1, 3x+2y+z=7, 2x+y+3z=2$. Ans: $AB=4I, x=2, y=1, z=-1$

17. If $A=\begin{bmatrix}1 & 2 & 5\\ 1 & -1 & -1\\ 2 & 3 & -1\end{bmatrix}$, find A^{-1}. Hence solve the following: $x+2y+5z=10, x-y-z=-2, 2x+3y-z=-11$.

Ans: $x=-1, y=-2, z=3$

ASSIGNMENT CLASS XII CONTINUITY AND DIFFERENTIABILITY

Important Formulas

1. A function $f(x)$ is continuous at $x=a$ iff $\lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)=f(a)$.

2. A function $f(x)$ is differentiable at $x=a$ iff $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ exists finitely ie.

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}.$$

1. Show that the function $f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0 & x=0 \end{cases}$ is continuous at $x=0$.

2. Show that the function $f(x)=\begin{cases} \frac{\sin x}{x}+\cos x, & x\neq 0 \\ 2 & x=0 \end{cases}$ is continuous at $x=0$.

3. Show that the function $f(x)=\begin{cases} 5x-4 & when\ 0<x\leq 1 \\ 4x^3-3x & when\ 1<x<2 \end{cases}$ is continuous at $x=1$.

4. Show that the function $f(x)=2x-|x|$ is continuous at $x=0$.

5. Show that the function $f(x)=\begin{cases} \frac{x-|x|}{x}, & x\neq 0 \\ 2 & x=0 \end{cases}$ is discontinuous at $x=0$.

6. If f is defined as $f(x)=\begin{cases} \frac{|x-4|}{x-4}, & x\neq 4 \\ 0 & x=4 \end{cases}$. Show that f is everywhere continous except at $x=4$.

7. Show that the function $f(x)=\begin{cases} \frac{|\sin x|}{x}, & x\neq 0 \\ 1 & x=0 \end{cases}$ is discontinuous at $x=0$.

8. Find the value of k so that the function f is continous at the indicated point:

(a) $f(x)=\begin{cases} \frac{1-\cos 2x}{2x^2}, & x\neq 0 \\ k & x=0 \end{cases}$ at $x=0$. (b) $f(x)=\begin{cases} \frac{x^2-25}{x-5}, & x\neq 5 \\ k & x=5 \end{cases}$ at $x=5$.

9. Show that the function $f(x)=\begin{cases} x-1 & if\ x<2 \\ 2x-3 & if\ x\geq 2 \end{cases}$ is not differentiable at $x=2$.

10. Discuss the continuity and differentiability of $f(x)=|x-1|+|x-2|$.

ANSWERS

8.(a) 1 (b) 10 **10**. continous but not differentiable at $x=1,2$

ASSIGNMENT CLASS XII DIFFERENTIATION

1. Find $\frac{dy}{dx}$ for the following:

(a) $y=\frac{1}{\sqrt{a^2-x^2}}$ (b) $y=\frac{5x}{\sqrt[3]{1-x^2}}+\sin^2(2x+3)$ (c) $y=\frac{\cos x+\sin x}{\cos x-\sin x}$

(d) $y=\log\sqrt{\frac{1+\cos^2 x}{1-e^{2x}}}$ (e) $y=\log(x+\sqrt{1+x^2})$ (f) $y=\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$

2. Show that $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]=\sqrt{a^2-x^2}$.

3. If $y=\sqrt{\frac{1-x}{1+x}}$, prove that $(1-x^2)\frac{dy}{dx}+y=0$.

4. If $y=\left(x+\sqrt{x^2+a^2}\right)^n$, prove that $\frac{dy}{dx}=\frac{ny}{\sqrt{x^2+a^2}}$.

5. Find $\frac{dy}{dx}$ for the following:

(a) $\sin^{-1}(\cos x)+\cos^{-1}(\sin x)$ (b) $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ (c) $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

(d) $\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$ (e) $\tan^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

6. Find $\frac{dy}{dx}$ for the following:

(a) $\cos^{-1}(4x^3-3x)$ (b) $\cot^{-1}\left(\frac{1-x}{1+x}\right)$ (c) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

(d) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ (e) $\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$

7. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

8. If $y=\sqrt{x}+\frac{1}{\sqrt{x}}$, prove that $2x\frac{dy}{dx}+y=2\sqrt{x}$.

9. If $y\sqrt{x^2+1}=\log\left(\sqrt{x^2+1}-x\right)$, show that $(x^2+1)\frac{dy}{dx}+xy+1=0$.

10. If $y\log x = x - y$, prove that $\dfrac{dy}{dx}=\dfrac{\log x}{(1+\log x)^2}$.

11. If $\log\left(\sqrt{x^2+y^2}\right)=\tan^{-1}\dfrac{y}{x}$, prove that $\dfrac{dy}{dx}=\dfrac{x+y}{x-y}$.

12. If $y=\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$, prove that $\left(1-x^2\right)\dfrac{dy}{dx}=xy+1$.

13. If $y=x^{\cos x}+\cos x^{\sin x}$, find $\dfrac{dy}{dx}$.

14. If $x^a y^b=\left(x+y\right)^{(a+b)}$, prove that $\dfrac{dy}{dx}=\dfrac{y}{x}$.

15. If $f(x)=\left(\dfrac{3+x}{1+x}\right)^{2+3x}$, find $f'(0)$.

16. Differentiate $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$.

17. If $x=a\sin 2t\left(1+\cos 2t\right), y=b\cos 2t\left(1-\cos 2t\right)$, show that $\left(\dfrac{dy}{dx}\right)_{at\, t=\frac{\pi}{4}}=\dfrac{b}{a}$.

18. If $x=a\left(\dfrac{1+t^2}{1-t^2}\right), y=\dfrac{2t}{1-t^2}$, show that $\dfrac{dy}{dx}=\dfrac{1+t^2}{2at}$.

19. If $x=2\cos\theta-\cos 2\theta$ and $y=2\sin\theta-\sin 2\theta$, find $\left(\dfrac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{2}}$.

20. If $y=A\cos nx+B\sin nx$, prove that $\dfrac{d^2y}{dx^2}+n^2y=0$.

21. If $y=e^x\left(\sin x+\cos x\right)$, prove that $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+2y=0$.

22. If $y=\tan^{-1}x$, show that $\left(1+x^2\right)\dfrac{d^2y}{dx^2}+2x\dfrac{dy}{dx}=0$.

ANSWERS

1.(a) $\dfrac{x}{(a^2-x^2)^{3/2}}$ (b) $\dfrac{15-5x^2}{3(1-x^2)^{4/3}}+2\sin(4x+6)$ (c) $\sec^2(x+\dfrac{\pi}{4})$ (d) $\dfrac{-\sin x\cos x}{1+\cos^2 x}+\dfrac{e^{2x}}{1-e^{2x}}$ (e) $\dfrac{1}{\sqrt{1+x^2}}$

(f) $-\sec^2(\dfrac{\pi}{4}-x)$ **5.** (a) -2 (b) $\dfrac{1}{2}$ (c) -1 (d) $\dfrac{1}{2}$ (e) $-\dfrac{1}{2}$ **6.**(a) $\dfrac{-3}{\sqrt{1-x^2}}$ (b) $\dfrac{1}{1+x^2}$ (c) $\dfrac{1}{2(1+x^2)}$

(d) $\dfrac{x}{\sqrt{1-x^4}}$ (e) $\dfrac{-1}{\sqrt{1-x^2}}$ **13.** $x^{\cos x}\left\{\dfrac{\cos x}{x}-\left(\log x\right)\sin x\right\}+\cos x^{\sin x}\left\{-\sin x\tan x+\cos x\log\left(\cos x\right)\right\}$

15. $27\log 3-12$ **16.** 1 **19.** $\dfrac{-3}{2}$

ASSIGNMENT CLASS XII APPLICATION OF DERIVATIVES

Questions based on Mean Value Theorem

1. Verify Rolle's Theorem for each of the following functions:

(a) $f(x)=x^2-x-6$ in $[-2,3]$ (b) $f(x)=x^3-7x^2+16x-12$ in $[2,3]$

(c) $f(x)=(x-1)(x-2)^2$ in $[1,2]$ (d) $f(x)=\sin x+\cos x$ *in* $\left[0,\frac{\pi}{2}\right]$

2. Discuss the applicability of the Rolle's Theorem for the following functions on the indicated intervals:

(a) $f(x)=x^{2/3}$ *on* $[-1,1]$ (b) $f(x)=|x|$ *on* $[-1,1]$

(c) $f(x)=[x]$ *on* $[-1,1]$ (d) $f(x)=\begin{cases} x^2+1 & when\ 0<x\le 1 \\ 3-x & when\ 1<x\le 2 \end{cases}$ on $[0,2]$

3. Using Rolle's Theorem, find the points on the curve $y=16-x^2$, $x\in[-1,1]$, where the tangent is parallel to x-axis.

4. Verify Lagrange's Mean Value Theorem for each of the following functions:

(a) $f(x)=\sqrt{x^2-1}$ in $[2,4]$ (b) $f(x)=x^2+x-1$ in $[0,4]$

(c) $f(x)=x^2-2x+4$ on $[1,5]$ (d) $f(x)=x^3+x^2-6x$ *on* $[-1,4]$

5. Find a point on the parabola $y=(x-3)^2$, where the tangent is parallel to chord joining $(3,0)$ and $(4,1)$.

6. Use Lagrange's Mean Value Theorem to determine a point P on the curve $f(x)=\sqrt{x-2}$ definded on $[2,3]$, where the tangent is parallel to the chord joining the end points of the curve.

Questions based on Rate of Change of Quantities

7. Find the points on the curve $y^2=8x$ for which abscissa and ordinate change at the same rate.

8. A partical moves along the curve $y=\frac{2}{3}x^3+1$. Find the points on the curve at which y-coordinate is changing twice as fast as x-coordinate.

9. The volume of spherical balloon is increasing at the rate of $25\,cm^3/\sec$. Find the rate of change of its surface area at the instant when its radius is 5 cm.

10. The surface area of a spherical bubble is increasing at the rate of $2\,cm^2/\sec$. Find the rate at which the volume of the bubble is increasing at the instant its radius is 6 cm.

11. Water is leaking from a conical funnel at the rate of $5\,cm^3/\sec$. If the radius of the base of funnel is 5 cm and its altitude is 10 cm, find the rate at which water level is dropping when it is 2.5 cm from top.

12. Water is running into a conical vessel, 15 cm deep and 5 cm in radius, at the rate of $0.1\,cm^3/\sec$. When the water is 6 cm deep, find at what rate is:

(a) water level rising? (b) water surface area increasing? (c)wetted surface of vessel increasing?

Questions based on Increasing and Decreasing Functions

13. Show that the function $f(x)=x^3-6x^2+12x-18$ is an increasing function on R.

14. Show that the function $f(x)=\cos^2 x$ is strictly decreasing on $\left(0,\frac{\pi}{2}\right)$.

15. Find the intervals on which the following functions are (i) increasing (ii) decreasing:

(a) $f(x)=2x^3-15x^2+36x+6$ (b) $f(x)=5+36x+3x^2-2x^3$

(c) $f(x)=\frac{4x^2+1}{x}$, $x\ne 0$ (d) $f(x)=\frac{x}{x^2+1}$ (e) $f(x)=(x+2)e^{-x}$

(f) $f(x)=(x-1)^3(x-2)^2$ (g) $f(x)=\frac{x}{2}+\frac{2}{x}, x\ne 0$ (h) $f(x)=\sin^4 x+\cos^4 x$ in $\left[0,\frac{\pi}{2}\right]$

Questions based on Tangent and Normal

16. Prove that the tangents to the curves $y = x^2 - 5x + 6$ at the points $(2, 0)$ and $(3, 0)$ are at right angles.

17. Find points on the curve $\frac{x^2}{9} - \frac{y^2}{16} = 1$ at which the tangents are parallel to (a) x-axis (b)y-axis.

18. Find the equation of the tangent to the curve $y = \sqrt{5x-3} - 2$, which is parallel to the line $4x - 2y + 3 = 0$.

19. Find the equation of the normals to the curve $3x^2 - y^2 = 8$, which is parallel to the line $x + 3y = 4$.

20. Find the equation of the tangent to the curve $x^2 + 3y = 3$, which is parallel to the line $y - 4x + 5 = 0$.

21. At what points on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$, is the tangent parallel to the y-axis.

22. Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

23. Find the equation of the tangent to the curve $x = \theta + \sin\theta, y = 1 + \cos\theta$ at $\theta = \frac{\pi}{4}$.

24. Find the points on the curve $4x^2 + 9y^2 = 1$, where the tangents are perpendicular to the line $2y + x = 0$.

25. If the tangent to the curve $y = x^3 + ax + b$ at $(1, -6)$ is parallel to the line $x - y + 5 = 0$, find a and b.

26. Find equation of the tangent to the curve $y = 4x^3 - 3x + 4$, which are perpendicular to $9y + x + 3 = 0$.

27. Show that the curves $2x = y^2$ and $2xy = k$ cut at right angles if $k^2 = 8$.

28. Find the equations of the tangent and the normal to the following curves at the indicated points:

(a) $y = 2x^2 - 3x - 1$ at $(1, -2)$ (b) $y = x^2 + 4x + 1$ at $x = 3$

(c) $y^2 = \frac{x^3}{4-x}$ at $(2, -2)$ (d) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a\cos\theta, b\sin\theta)$

Questions based on Approximations

29. Using differentials, find the approximate value of the following:

(a) $\sqrt{0.037}$ (b) $\sqrt{0.48}$ (c) $\sqrt[3]{29}$ (d) $(33.1)^{1/5}$

30. If $y = x^4 - 12$ and if x chnges from 2 to 1.99, what is the approximate change in the y?

31. Find approximate change in volume V of a cube of side x meters caused by increasing the side by 2%.

32. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, find approximating error in calculating its volume.

Questions based on Maxima and Minima

33. Find the points of local maxima or local minima and the corresponding local maximum and minimum values of each of the following functions:

(a) $f(x) = 2x^3 - 21x^2 + 36x - 20$ (b) $f(x) = x^4 - 62x^2 + 120x + 9$

(c) $f(x) = (x-1)(x+2)^2$ (d) $f(x) = \frac{-3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$

(e) $f(x) = \sin 2x - x$, where $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$ (f) $f(x) = 2\cos x + x$, where $0 < x < \pi$

34. Find the absolute maximum value and absolute minimum value of the following functions:

(a) $f(x) = 4x - \frac{x^2}{2}$ in $[-2, 4.5]$ (b) $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 1$ in $[1, 4]$

35. Find the maximum and minimum values of the following functions:

(a) $f(x) = -x + 2\sin x$ *on* $[0, 2\pi]$ (b) $f(x) = \left(\sin x + \frac{1}{2}\cos x\right)$ on $0 \le x \le \frac{\pi}{2}$

36. Show that $f(x) = \sin x(1 + \cos x)$ is maximum at $x = \frac{\pi}{3}$ in the interval $[0, \pi]$.

__Questions based on Maxima and Minima(Word Problems)__

37. Show that of all the rectangles of the given area, the square has the smallest perimeter.

38. Find the point on the curve $y^2 = 4x$ which is nearest to the point $(2,1)$.

39. A rectangle is inscribed in a semicircle of radius r with one of its sides on the diameter of the semi-circle. Find the dimensions of the rectangle, so that its area is maximum. Also find the maximum area.

40. A right circular cylinder is inscribed in a given cone. Show that the curved surface area of the cylinder is maximum when diameter of cylinder is equal to the radius of the base of cone.

41. An open tank with the square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be least when the depth of the tank is half of its width.

42. Show that a closed right circular cylinder of given surface area and maximum volume is such that its height is equal to the diameter of the base.

43. Show that the height of the right circular cylinder of maximum volume that can be inscribed in a given right circular cone of height h is $\frac{h}{3}$.

44. Find the largest possible area of a right angled triangle whose hypotenuse is 5 cm long.

45. A closed circular cylinder has a volume of 2156 cu. Cm. What will be the radius of its base so that its total surface area is minimum?

46. Of all the rectangles each of which has a perimeter 40 m, find one which has maximum area. Find the maximum area also.

47. A wire of length 20 m is to be cut into two pieces. One of the pieces will be bent into shape of a square and the other into shape of an equilateral triangle. Where the wire should be cut so thet the sum of the areas of the square and triangle is minimum?

48. An open box with a square base id to be made out of a given quantity of sheet of area a^2. Show that the maximum volume of the box is $\frac{a^3}{6\sqrt{3}}$.

49. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle 30^0 is $\frac{4}{81}\pi h^3$.

50. Show that a right triangle of given hypotenuse has maximum area when it is an isosceles triangle.

51. A window is in the form of an rectangle above which there is a semi-circle. If the perimeter of the window is p cm, show that the window will allow the maximum possible light only when the radius of the semi-circle is $\frac{p}{\pi+4}$.

52. Show that the rectangle of maximum area that can be inscribed in a circle of radius r is a square of side $\sqrt{2}\,r$.

53. Two sides of a triangle have lengths a and b and the angle between them is θ. What value of θ will maximize the area of the triangle? Find the maximum area of the triangle also.

54. A rectangular window is surmounted by an equilateral triangle. Given that the perimeter is $16\,m$, find the width of the window so that the maximum amount of light may enter.

55. Show that a right circular cylinder which is open at top, and has a given surface area, will have greatest volume if its height is equal to the radius of its base.

56. Show that the maximum volume of the cylinder which can be inscribed in the sphere of radius $5\sqrt{3}\,cm$ is $500\pi\, cm^3$.

57. Show that the right circular cylinder of given volume open at the top has minimum total surface area, provided its height is equal to the radius of its base.

58. Prove that the surface area of the solid cuboid, of square base and given volume, is minimum when it is a cube.

59. Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius $12\,cm$ is $16\,cm$.

60. A jet of an enemy is flying along the curve $y = x^2 + 2$. A soldier is placed at the point $(3,2)$. What is the nearest distance between the soldier and the jet?

ANSWERS (*APPLICATION OF DERIVATIVES*)

1.(a) $c=\frac{1}{2}$ (b) $c=\frac{8}{3}$ (c) $\frac{4}{3}$ (d) $c=\frac{\pi}{4}$ 2. Not applicable in any case 3. $(0,16)$

4.(a) $c=\sqrt{6}$ (b) $c=2$ (c) $c=3$ (d) $c=\frac{\sqrt{31}-1}{3}$ 5. $\left(\frac{7}{2},\frac{1}{4}\right)$ 6. $\left(\frac{9}{4},\frac{1}{2}\right)$ 7. $(2,4)$

8. $\left(1,\frac{5}{3}\right);\left(-1,\frac{1}{3}\right)$ 9. $10\,cm^2$ / sec 10. $6\,cm^3$ / sec 11. $\frac{-16}{45\pi}cm$ / sec 12. (a) $\frac{1}{40\pi}cm$ / sec

(b) $\frac{1}{30}cm^2$ / sec (c) $\frac{\sqrt{10}}{30}cm^2$ / sec 15. (a) inc. on $(-\infty,2]\cup[3,\infty)$, dec. on $[2,3]$

(b) inc. on $[-2,3]$, dec. on $(-\infty,-2]\cup[3,\infty)$ (c) inc. on $\left(-\infty,\frac{-1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$, dec. on $\left[\frac{-1}{2},\frac{1}{2}\right]$

(d) inc. on $[-1,1]$, dec. on $(-\infty,-1]\cup[1,\infty)$ (e) inc. on $[-\infty,-1]$, dec. on $[-1,\infty)$

(f) inc. on $\left(-\infty,\frac{8}{5}\right)\cup(2,\infty)$, dec. on $\left(\frac{8}{5},2\right)$ (g) inc. on $(-\infty,-2)\cup(2,\infty)$, dec. on $(-2,0)\cup(0,2)$

(h) inc. on $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$, dec. on $\left[0,\frac{\pi}{4}\right]$ 17. (a) No point (b) $(3,0)$ and $(-3,0)$

18. $80x-40y-103=0$ 19. $x+3y+8=0$ 20. $4x-y+13=0$ 21. $(-1,2)$ and $(3,2)$

23. $y=\left(1-\sqrt{2}\right)x+\frac{\left(\sqrt{2}-1\right)\pi}{4}+2$ 24. $\left(\frac{3}{2\sqrt{10}},\frac{-1}{3\sqrt{10}}\right)$ and $\left(\frac{-3}{2\sqrt{10}},\frac{1}{3\sqrt{10}}\right)$ 25. $a=-2,b=-5$ 26. $9x-y-3=0$ and $9x-y-13=0$ 28. (a) $x-y-3=0,x+y+1=0$ (b) $10x-y-8=0,x+10y-223=0$

(c) $2x+y-2=0,x-2y-6=0$ (d) $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1, ax\sec\theta-by\,\text{cosec}\theta=a^2-b^2$

29. (a) 0.1925 (b) 0.693 (c) 3.074 (d) 2.01375

30. –0.32 31. $0.06x^3$ m^3 32. 9.72π cm^3

33.(a)Local max. value is -3 at $x=1$ and Local min. value is -128 at $x=6$

(b)Local max. value is 68 at $x=1$ and Local min. value is -1647 at $x=-6$ and -316 at $x=5$

(c)Local max. value is 0 at $x=-2$ and Local min. value is -4 at $x=0$

(d) Local max. value is 105 at $x=0$ and $\frac{295}{4}$ at $x=-5$ and Local min. value is $\frac{231}{4}$ at $x=-3$

(e) Local max. value is $\frac{\sqrt{3}}{2}-\frac{\pi}{6}$ at $x=\frac{\pi}{6}$ and Local min. value is $-\left(\frac{\sqrt{3}}{2}+\frac{\pi}{6}\right)$ at $x=\frac{-\pi}{6}$

34.(a)Abs max. is 8 at $x=4$, Abs. min. is -10 at $x=-2$

(b)Abs max. is 257 at $x=4$, Abs. min. is -63 at $x=2$

35. (a) max. value is $\left(-\frac{\pi}{3}+\sqrt{3}\right)$ at $x=\frac{\pi}{3}$ and min. value is $-\left(\frac{5\pi}{3}+\sqrt{3}\right)$ at $x=\frac{5\pi}{3}$

(b) max. value is $\frac{3}{4}$ at $x=\frac{\pi}{6}$ and min. value is $\frac{1}{2}$ at $x=\frac{\pi}{2}$ 38. $(1,2)$ 39. $\frac{r}{\sqrt{2}}$ units, $\sqrt{2}\,r$ units; r^2 units

44. $\frac{25}{4}cm^2$ 45. 7 cm 46. square, $100\,m^2$ 47. $\frac{20\sqrt{3}}{9+4\sqrt{3}},\frac{60}{9+4\sqrt{3}}$ 53. $90^0,\frac{1}{2}ab$

54. $3.46\,m$ 60. $\sqrt{5}$