ASSIGNMENT CLASS XII THREE DIMENSIONAL GEOMETRY

1. Find the coordinates of the foot of the perpendicular drawn from the point $(1,8,4)$ to the line joining $B(0,-1,3)$ and $C(2,-3,-1)$.

2. Find the vector equation of the line passing through the point $A(2,-1,1)$, and parallel to the line joining the points $B(-1,4,1)$ and $C(1,2,2)$. Also, find the Cartesian equation of the line.

3. The cartesian equations of a line are $6x-2=3y+1=2z-2$. Find (i) the direction ratios of the line, (ii) the cartesian equation of the line parallel to this line and passing through the point $(2,-1,-1)$.

4. Find the equations of the line passing through the point $(-1,3,-2)$ and perpendicular to each of the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$.

5. Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ intersect each other. Also, find point of their intersection.

6. Show that the lines $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5}$ and $\frac{x+2}{4}=\frac{y-1}{3}=\frac{z+1}{-2}$ do not intersect each other.

7. Find the foot of perpendicular drawn from the point $P(1,6,3)$ on the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also, find its distance from P.

8. Find the image of the point $(5,9,3)$ in the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$.

9. A perpendicular is drawn from the point $(0,2,7)$ to the line $\frac{x+2}{-1}=\frac{y-1}{3}=\frac{z-3}{-2}$. Find

(i) foot of the perpendicular (ii) length of the perpendicular (iii) image of the point in the line.

10. Find the coordinates of the point where the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}$ meets the plane $x+y+4z=6$.

11. Find the angle between the lines $\frac{x+1}{1}=\frac{2y-3}{3}=\frac{z-6}{2}$ and $\frac{x-4}{3}=\frac{y+3}{-2}, z=5$.

12. Find the value of k for which the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{6-z}{5}$ are perpendicular to each other.

13. Find the angles of a ΔABC whose vertices are $A(-1,3,2), B(2,3,5)$ and $C(3,5,-2)$.

14. Show that the angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

15. Find the shortest distance between the following pair of lines:

$(i)\ \frac{x-8}{3}=\frac{y+9}{-16}=\frac{10-z}{-7}$ and $\frac{x-15}{3}=\frac{58+2y}{-16}=\frac{z-5}{-5}$

$(ii)\ \vec{r}=(\lambda-1)\hat{i}+(\lambda+1)\hat{j}-(\lambda+1)k$ and $\vec{r}=(1-\mu)\hat{i}+2(2\mu-1)\hat{j}+(\mu+2)k$.

16. Find the shortest distance between the following pair of parallel lines:

$(i)\ \frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-3}{1}$ and $\frac{x-2}{-1}=\frac{y+1}{1}=\frac{z+1}{-1}$

$(ii)\ \vec{r}=\left(\hat{i}+\hat{j}\right)+\lambda\left(2\hat{i}-\hat{j}+k\right)$ and $\vec{r}=\left(2\hat{i}+\hat{j}-k\right)+\mu\left(4\hat{i}-2\hat{j}+2k\right)$.

17. Find the equation of the plane passing through the points $A(0,-1,-1)$, $B(4,5,1)$ and $C(3,9,4)$.

18. Show that the four points A, B, C, D with position vectors $4\hat{i}+5\hat{j}+k$, $-\left(\hat{j}+k\right)$, $3\hat{i}+9\hat{j}+4k$ and $4\left(-\hat{i}+\hat{j}+k\right)$ respectively are coplanar.

19. A plane meets the coordinate axes at A, B and C such that the centroid of ΔABC is $(3, 4, -6)$. Find the equation of the plane.

20. Reduce the equation of the plane $12x-3y+4z+52=0$ to the normal form, and hence find the length of the perpendicular from the origin to the plane. Write down the direction cosines of the normal to the plane.

21. The position vectors of two points A and B are $3\hat{i}+\hat{j}+2k$ and $\hat{i}-2\hat{j}-4k$ respectively. Find the vector equation of the plane passing through B and perpendicular to $\overrightarrow{AB}$.

22. Find the vector equation of the plane passing through the point $(1,2,3)$ and perpendicular to the line with direction ratios $2,3,-4$.

23. Find the vector equation of the plane through the intersection of the planes $\vec{r}.\left(2\hat{i}+\hat{j}\right)+12=0$ and $\vec{r}.\left(3\hat{i}-\hat{j}+4k\right)=0$, which is at a unit distance from the origin.

24. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.\left(2\hat{i}-7\hat{j}+4k\right)=3$ and $\vec{r}.\left(3\hat{i}-5\hat{j}+4k\right)+11=0$, and passing through the point $(-2, 1, 3)$.

25. Find the equation of the plane passing through the intersection of the planes $2x+3y-z+1=0$ and $x+y-2z+3=0$, and perpendicular to plane $3x-y-2z-4=0$. Also find the inclination of this plane with xy - plane.

26. Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3$ and $5x-3y+4z+9=0$, and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.

27. Find the equation of the plane passing through the point (1,1,1) and perpendicular to each of the planes $x+2y+3z=7$ and $2x-3y+4z=0$.

28. Find λ for which the planes $\vec{r}.\left(2\hat{i}-\hat{j}+\lambda k\right)=7$ and $\vec{r}.\left(3\hat{i}+2\hat{j}+2k\right)=9$ are perpendicular to each other.

29. Find the equation of the plane passing through the point $P(1,-1,2)$ and $Q(2,-2,2)$ and perpendicular to the plane $6x-2y+2z=9$.

30. Show that the line $\vec{r}.\left(2\hat{i}-2\hat{j}+3k\right)+\lambda\left(\hat{i}-\hat{j}+4k\right)$ is parallel to the plane $\vec{r}.\left(\hat{i}+5\hat{j}+k\right)=5$. Also, find the distance between them.

31. Find the vector equation of a line passing through the point with position vector $\left(2\hat{i}-3\hat{j}-5k\right)$ and perpendicular to the plane $\vec{r}.\left(6\hat{i}-3\hat{j}+5k\right)+2=0$. Also, find the point of intersection of this line and the plane.

32. Find the angle between the line $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$ and the plane $3x+4y+z+5=0$.

33. Find the equation of the plane passing through the points $(1,2,3)$ and $(0,-1,0)$ and parallel to the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-3}$.

34. Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3$, $5x-3y+4z+9=0$, and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.

35. Find the distance of the point $(2,3,4)$ from the plane $\vec{r}.\left(3\hat{i}-6\hat{j}+2k\right)+11=0$.

36. Find the distance between the parallel planes $\vec{r}.\left(2\hat{i}-3\hat{j}+6k\right)=5$ and $\vec{r}.\left(6\hat{i}-9\hat{j}+18k\right)+20=0$.

37. Find the length and the foot of perpendicular from the point $(1,1,2)$ to the plane $\vec{r}.\left(2\hat{i}-2\hat{j}+4k\right)+5=0$.

38. Find the image of the point $P(1,\ 3,\ 4)$ in the plane $2x-y+z+3=0$.

39. Prove that the image of the point $(3,-2,1)$ in the plane $3x-y+4z=2$ lies on the plane $x+y+z+4=0$.

40. Find the distance of the point $(2,3,4)$ from the plane $3x+2y+2z+5=0$, measured parallel to the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$.

41. Find equation of the plane which contains two parallel lines $\frac{x-3}{3}=\frac{y+4}{2}=\frac{z-1}{1}$ and $\frac{x+1}{3}=\frac{y-2}{2}=\frac{z}{1}$.

42. Find the vector and cartesian forms of the equation of the plane containing two lines $\vec{r}=\left(\hat{i}+2\hat{j}-4k\right)+\lambda\left(2\hat{i}+3\hat{j}+6k\right)$ and $\vec{r}=\left(3\hat{i}+3\hat{j}-5k\right)+\mu\left(-2\hat{i}+3\hat{j}+8k\right)$.

43. Find the equation of the plane containing two lines $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - k\right)$ and $\vec{r} = \left(\hat{i} + \hat{j}\right) + \mu\left(-\hat{i} + \hat{j} - 2k\right)$. Find the distance of this plane from the origin and also from the point $(1, 1, 1)$.

44. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also, find the equation of the plane containing these two lines.

ANSWERS

1. $D\left(\frac{-5}{3}, \frac{2}{3}, \frac{19}{3}\right)$ 2. $\vec{r} = \left(2\hat{i} - \hat{j} + k\right) + \lambda\left(2\hat{i} - 2\hat{j} + k\right)$, $\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-1}{1}$ 3. $(i) 1, 2, 3$ $(ii) \frac{x-2}{1} = \frac{y+1}{2} = \frac{z-1}{3}$

4. $\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$ 5. $(1, 3, 2)$ 7. $N(1, 3, 5); \sqrt{13}$ units 8. $(1, 1, 11)$ 9. $(i) \left(\frac{-3}{2}, \frac{-1}{2}, 4\right)$

$(ii) \frac{\sqrt{70}}{2}$ units $(iii) (-3, -3, 1)$ 10. $(1, 1, 1)$ 11. $\frac{\pi}{2}$ 12. $k = \frac{-10}{7}$

13. $\angle A = 90^0, \angle B = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right), \angle C = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$ 15. $(i) 1.4$ units $(ii) \frac{5\sqrt{2}}{2}$ units 16. $(i) \sqrt{26}$ units

$(ii) \frac{1}{6}\sqrt{66}$ units 17. $5x - 7y + 11z + 4 = 0$ 19. $4x + 3y - 2z = 36$ 20. $4, \left(\frac{-12}{13}, \frac{3}{13}, \frac{-4}{13}\right)$

21. $\vec{r}.\left(2\hat{i} + 3\hat{j} + 6k\right) + 28 = 0$ 22. $\vec{r}.\left(2\hat{i} + 3\hat{j} - 4k\right) + 4 = 0$ 23. $\vec{r}.\left(2\hat{i} + \hat{j} + 2k\right) + 3 = 0, \vec{r}.\left(-\hat{i} + 2\hat{j} - 2k\right) + 3 = 0$

24. $\vec{r}.\left(15\hat{i} - 47\hat{j} + 28k\right) = 7$ 25. $7x + 13y + 4z = 9$, $\theta = \cos^{-1}\left(\frac{4}{\sqrt{234}}\right)$ 26. $7x + 9y - 10z = 27$

27. $17x + 2y - 7z - 12 = 0$ 28. $\lambda = 2$ 29. $x + y - 2z + 4 = 0$ 30. $\frac{10}{3\sqrt{3}}$ units

31. $\vec{r} = \left(2\hat{i} - 3\hat{j} - 5k\right) + \lambda\left(6\hat{i} - 3\hat{j} + 5k\right)$, $\left(\frac{76}{35}, \frac{-108}{35}, \frac{-170}{35}\right)$ 32. $\sin^{-1}\left(\sqrt{\frac{7}{52}}\right)$ 33. $6x - 3y + z = 3$

34. $7x + 9y - 10z - 27 = 0$ 35. 1 unit 36. $\frac{5}{3}$ units 37. $\frac{13\sqrt{6}}{12}$ units, $\left(\frac{-1}{12}, \frac{25}{12}, \frac{-1}{6}\right)$

38. $Q(-3, 5, 2)$ 40. 7 units 41. $8x + y - 26z + 6 = 0$ 42. $\vec{r}.\left(6\hat{i} - 28\hat{j} + 12k\right) + 98 = 0$, $3x - 14y + 6z + 49 = 0$

43. $x - y - z = 0$; 0 units, $\frac{1}{\sqrt{3}}$ units 44. $x - 2y + z = 0$