

CLASS XII INTEGRALS CHAPTER 8

MISC. EX. SOLUTIONS

Question 1:

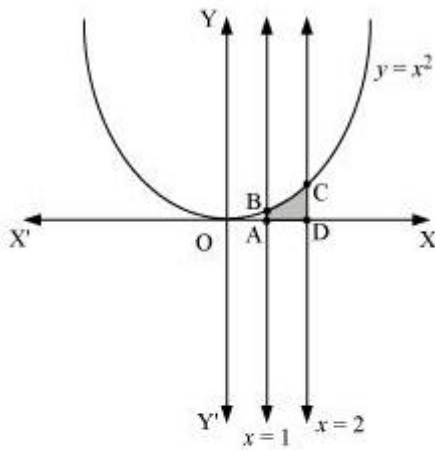
Find the area under the given curves and given lines:

(i) $y = x^2$, $x = 1$, $x = 2$ and x -axis

(ii) $y = x^4$, $x = 1$, $x = 5$ and x -axis

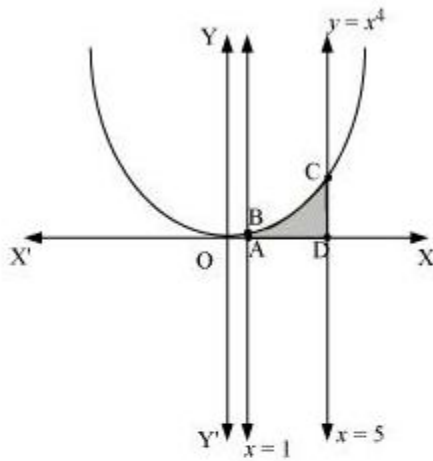
ANS :

i. The required area is represented by the shaded area ADCBA as



$$\begin{aligned}\text{Area ADCBA} &= \int_1^2 y dx \\ &= \int_1^2 x^2 dx \\ &= \left[\frac{x^3}{3} \right]_1^2 \\ &= \frac{8}{3} - \frac{1}{3} \\ &= \frac{7}{3} \text{ units}\end{aligned}$$

ii. The required area is represented by the shaded area ADCBA as



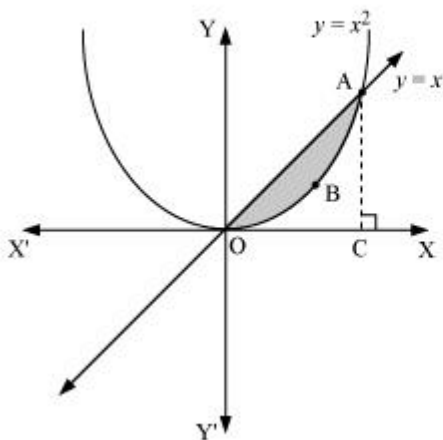
$$\begin{aligned}
 \text{Area ADCBA} &= \int_1^5 x^4 dx \\
 &= \left[\frac{x^5}{5} \right]_1^5 \\
 &= \frac{(5)^5}{5} - \frac{1}{5} \\
 &= (5)^4 - \frac{1}{5} \\
 &= 625 - \frac{1}{5} \\
 &= 624.8 \text{ units}
 \end{aligned}$$

Question 2:

Find the area between the curves $y = x$ and $y = x^2$

ANS :

The required area is represented by the shaded area OBAO as



The points of intersection of the curves, $y = x$ and $y = x^2$, is A (1, 1).

We draw AC perpendicular to x-axis.

\therefore Area (OBAO) = Area (Δ OCA) – Area (OCABO) ... (1)

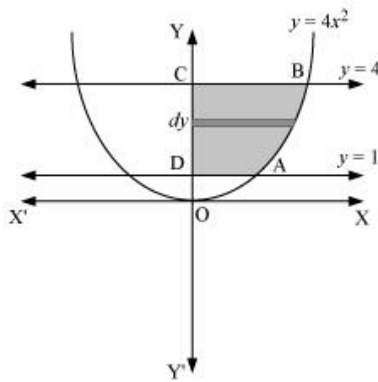
$$\begin{aligned} &= \int_0^1 x \, dx - \int_0^1 x^2 \, dx \\ &= \left[\frac{x^2}{2} \right]_0^1 - \left[\frac{x^3}{3} \right]_0^1 \\ &= \frac{1}{2} - \frac{1}{3} \\ &= \frac{1}{6} \text{ units} \end{aligned}$$

Question 3:

Find the area of the region lying in the first quadrant and bounded by $y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$

ANS :

The area in the first quadrant bounded by $y = 4x^2$, $x = 0$, $y = 1$, and $y = 4$ is represented by the shaded area ABCDA as



$$\begin{aligned} \therefore \text{Area ABCD} &= \int_1^4 x \, dx \\ &= \int_1^4 \frac{\sqrt{y}}{2} \, dy \\ &= \frac{1}{2} \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4 \\ &= \frac{1}{3} \left[(4)^{\frac{3}{2}} - 1 \right] \\ &= \frac{1}{3} [8 - 1] \\ &= \frac{7}{3} \text{ units} \end{aligned}$$

Question 4:

Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^0 |x+3| dx$

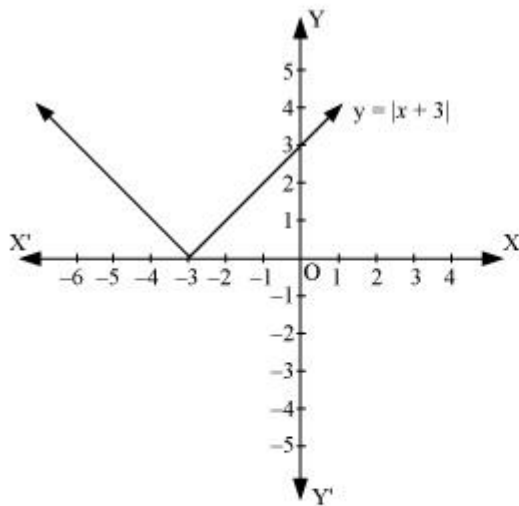
ANS :

The given equation is $y = |x+3|$

The corresponding values of x and y are given in the following table.

x	-6	-5	-4	-3	-2	-1	0
y	3	2	1	0	1	2	3

On plotting these points, we obtain the graph of $y = |x+3|$ as follows.



It is known that, $(x+3) \leq 0$ for $-6 \leq x \leq -3$ and $(x+3) \geq 0$ for $-3 \leq x \leq 0$

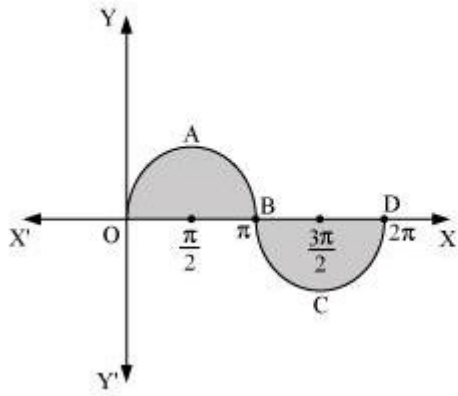
$$\begin{aligned} \therefore \int_{-6}^0 |x+3| dx &= -\int_{-6}^{-3} (x+3) dx + \int_{-3}^0 (x+3) dx \\ &= -\left[\frac{x^2}{2} + 3x \right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x \right]_{-3}^0 \\ &= -\left[\left(\frac{(-3)^2}{2} + 3(-3) \right) - \left(\frac{(-6)^2}{2} + 3(-6) \right) \right] + \left[0 - \left(\frac{(-3)^2}{2} + 3(-3) \right) \right] \\ &= -\left[-\frac{9}{2} \right] - \left[-\frac{9}{2} \right] \\ &= 9 \end{aligned}$$

Question 5:

Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$

ANS :

The graph of $y = \sin x$ can be drawn as



∴ Required area = Area OABO + Area BCDB

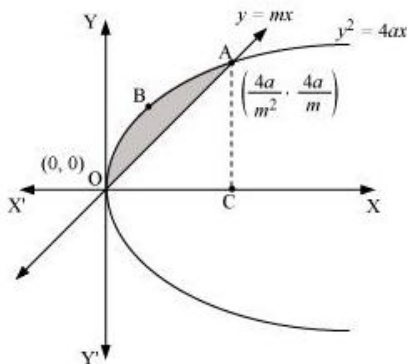
$$\begin{aligned} &= \int_0^{\pi} \sin x \, dx + \left| \int_{\pi}^{2\pi} \sin x \, dx \right| \\ &= [-\cos x]_0^{\pi} + \left| [-\cos x]_{\pi}^{2\pi} \right| \\ &= [-\cos \pi + \cos 0] + |-\cos 2\pi + \cos \pi| \\ &= 1 + 1 + |(-1 - 1)| \\ &= 2 + |-2| \\ &= 2 + 2 = 4 \text{ units} \end{aligned}$$

Question 6:

Find the area enclosed between the parabola $y^2 = 4ax$ and the line $y = mx$

ANS :

The area enclosed between the parabola, $y^2 = 4ax$, and the line, $y = mx$, is represented by the shaded area OABO as



The points of intersection of both the curves are $(0, 0)$ and $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$.

We draw AC perpendicular to x-axis.

\therefore Area OABO = Area OCABO – Area (Δ OCA)

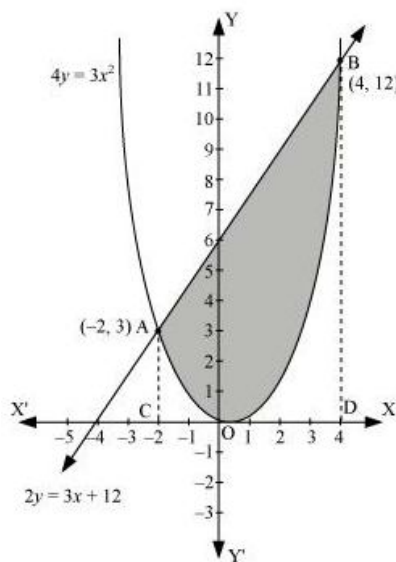
$$\begin{aligned}
 &= \int_0^{\frac{4a}{m^2}} 2\sqrt{ax} \, dx - \int_0^{\frac{4a}{m^2}} mx \, dx \\
 &= 2\sqrt{a} \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{4a}{m^2}} - m \left[\frac{x^2}{2} \right]_0^{\frac{4a}{m^2}} \\
 &= \frac{4}{3} \sqrt{a} \left(\frac{4a}{m^2} \right)^{\frac{3}{2}} - \frac{m}{2} \left[\left(\frac{4a}{m^2} \right)^2 \right] \\
 &= \frac{32a^2}{3m^3} - \frac{m}{2} \left(\frac{16a^2}{m^4} \right) \\
 &= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3} \\
 &= \frac{8a^2}{3m^3} \text{ units}
 \end{aligned}$$

Question 7:

Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$

ANS :

The area enclosed between the parabola, $4y = 3x^2$, and the line, $2y = 3x + 12$, is represented by the shaded area OBAO as



The points of intersection of the given curves are A (-2, 3) and (4, 12).

We draw AC and BD perpendicular to x-axis.

∴ Area OBAO = Area CDDBA - (Area ODBO + Area OACO)

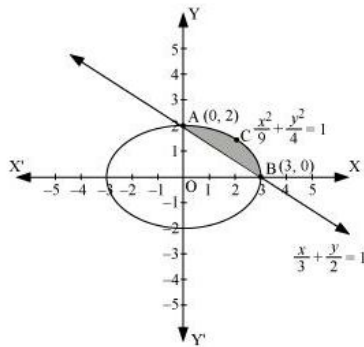
$$\begin{aligned}
 &= \int_{-2}^4 \frac{1}{2}(3x+12) dx - \int_{-2}^4 \frac{3x^2}{4} dx \\
 &= \frac{1}{2} \left[\frac{3x^2}{2} + 12x \right]_{-2}^4 - \frac{3}{4} \left[\frac{x^3}{3} \right]_{-2}^4 \\
 &= \frac{1}{2} [24 + 48 - 6 + 24] - \frac{1}{4} [64 + 8] \\
 &= \frac{1}{2} [90] - \frac{1}{4} [72] \\
 &= 45 - 18 \\
 &= 27 \text{ units}
 \end{aligned}$$

Question 8:

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$

ANS :

The area of the smaller region bounded by the ellipse, $\frac{x^2}{9} + \frac{y^2}{4} = 1$, and the line, $\frac{x}{3} + \frac{y}{2} = 1$, is represented by the shaded region BCAB as



∴ Area BCAB = Area (OBCAO) - Area (OBAO)

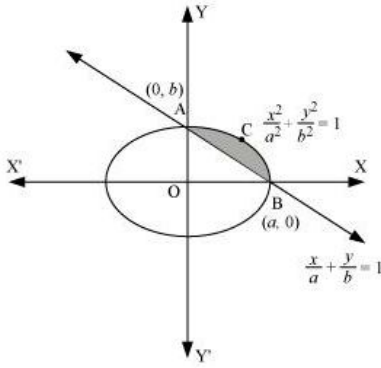
$$\begin{aligned}
 &= \int_0^3 2\sqrt{1-\frac{x^2}{9}} dx - \int_0^3 2\left(1-\frac{x}{3}\right) dx \\
 &= \frac{2}{3} \left[\int_0^3 \sqrt{9-x^2} dx \right] - \frac{2}{3} \int_0^3 (3-x) dx \\
 &= \frac{2}{3} \left[\frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_0^3 - \frac{2}{3} \left[3x - \frac{x^2}{2} \right]_0^3 \\
 &= \frac{2}{3} \left[\frac{9}{2} \left(\frac{\pi}{2} \right) \right] - \frac{2}{3} \left[9 - \frac{9}{2} \right] \\
 &= \frac{2}{3} \left[\frac{9\pi}{4} - \frac{9}{2} \right] \\
 &= \frac{2}{3} \times \frac{9}{4} (\pi - 2) \\
 &= \frac{3}{2} (\pi - 2) \text{ units}
 \end{aligned}$$

Question 9:

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$

ANS :

The area of the smaller region bounded by the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and the line, $\frac{x}{a} + \frac{y}{b} = 1$, is represented by the shaded region BCAB as



\therefore Area BCAB = Area (OBCAO) – Area (OBAO)

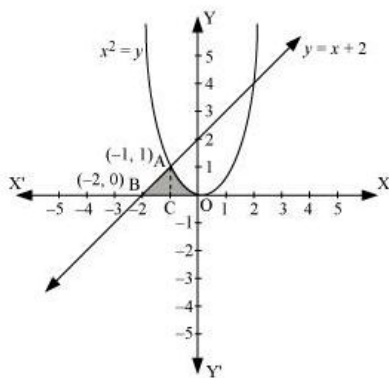
$$\begin{aligned}
 &= \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} dx - \int_0^a b \left(1 - \frac{x}{a}\right) dx \\
 &= \frac{b}{a} \int_0^a \sqrt{a^2 - x^2} dx - \frac{b}{a} \int_0^a (a - x) dx \\
 &= \frac{b}{a} \left[\left\{ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right\}_0^a - \left\{ ax - \frac{x^2}{2} \right\}_0^a \right] \\
 &= \frac{b}{a} \left[\left\{ \frac{a^2}{2} \left(\frac{\pi}{2} \right) \right\} - \left\{ a^2 - \frac{a^2}{2} \right\} \right] \\
 &= \frac{b}{a} \left[\frac{a^2 \pi}{4} - \frac{a^2}{2} \right] \\
 &= \frac{ba^2}{2a} \left[\frac{\pi}{2} - 1 \right] \\
 &= \frac{ab}{2} \left[\frac{\pi}{2} - 1 \right] \\
 &= \frac{ab}{4} (\pi - 2)
 \end{aligned}$$

Question 10:

Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and x-axis

ANS :

The area of the region enclosed by the parabola, $x^2 = y$, the line, $y = x + 2$, and x-axis is represented by the shaded region OABCO as



The point of intersection of the parabola, $x^2 = y$, and the line, $y = x + 2$, is A $(-1, 1)$.

\therefore Area OABCO = Area (BCA) + Area COAC

$$\begin{aligned}
 &= \int_{-2}^{-1} (x+2) dx + \int_{-1}^0 x^2 dx \\
 &= \left[\frac{x^2}{2} + 2x \right]_{-2}^{-1} + \left[\frac{x^3}{3} \right]_{-1}^0 \\
 &= \left[\frac{(-1)^2}{2} + 2(-1) - \frac{(-2)^2}{2} - 2(-2) \right] + \left[-\frac{(-1)^3}{3} \right] \\
 &= \left[\frac{1}{2} - 2 - 2 + 4 + \frac{1}{3} \right] \\
 &= \frac{5}{6} \text{ units}
 \end{aligned}$$

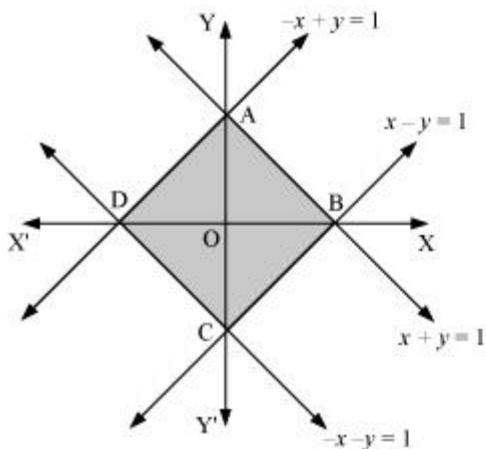
Question 11:

Using the method of integration find the area bounded by the curve $|x| + |y| = 1$

[Hint: the required region is bounded by lines $x + y = 1$, $x - y = 1$, $-x + y = 1$ and $-x - y = 1$]

ANS :

The area bounded by the curve, $|x| + |y| = 1$, is represented by the shaded region ADCB as



The curve intersects the axes at points A $(0, 1)$, B $(1, 0)$, C $(0, -1)$, and D $(-1, 0)$.

It can be observed that the given curve is symmetrical about x -axis and y -axis.

$$\therefore \text{Area ADCB} = 4 \times \text{Area OBAO}$$

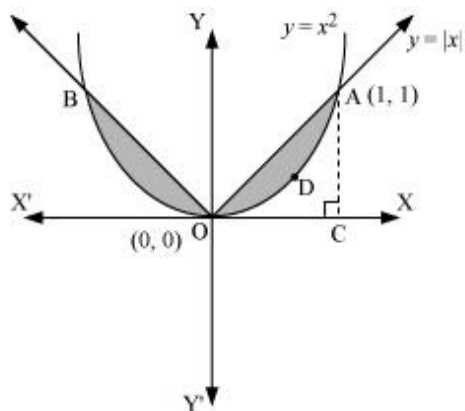
$$\begin{aligned} &= 4 \int_0^1 (1-x) dx \\ &= 4 \left(x - \frac{x^2}{2} \right)_0^1 \\ &= 4 \left[1 - \frac{1}{2} \right] \\ &= 4 \left(\frac{1}{2} \right) \\ &= 2 \text{ units} \end{aligned}$$

Question 12:

Find the area bounded by curves $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$

ANS :

The area bounded by the curves, $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$, is represented by the shaded region as



It can be observed that the required area is symmetrical about y -axis.

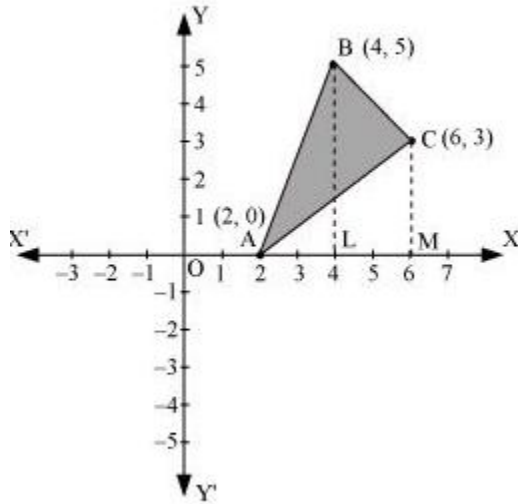
$$\begin{aligned} \text{Required area} &= 2 \left[\text{Area (OCAO)} - \text{Area (OCADO)} \right] \\ &= 2 \left[\int_0^1 x dx - \int_0^1 x^2 dx \right] \\ &= 2 \left[\left[\frac{x^2}{2} \right]_0^1 - \left[\frac{x^3}{3} \right]_0^1 \right] \\ &= 2 \left[\frac{1}{2} - \frac{1}{3} \right] \\ &= 2 \left[\frac{1}{6} \right] = \frac{1}{3} \text{ units} \end{aligned}$$

Question 13:

Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are A (2, 0), B (4, 5) and C (6, 3)

ANS :

The vertices of $\triangle ABC$ are A (2, 0), B (4, 5), and C (6, 3).



Equation of line segment AB is

$$y - 0 = \frac{5 - 0}{4 - 2}(x - 2)$$

$$2y = 5x - 10$$

$$y = \frac{5}{2}(x - 2) \quad \dots(1)$$

Equation of line segment BC is

$$y - 5 = \frac{3 - 5}{6 - 4}(x - 4)$$

$$2y - 10 = -2x + 8$$

$$2y = -2x + 18$$

$$y = -x + 9 \quad \dots(2)$$

Equation of line segment CA is

$$y - 3 = \frac{0 - 3}{2 - 6}(x - 6)$$

$$-4y + 12 = -3x + 18$$

$$4y = 3x - 6$$

$$y = \frac{3}{4}(x - 2) \quad \dots(3)$$

Area ($\triangle ABC$) = Area (ABLA) + Area (BLMCB) - Area (ACMA)

$$= \int_2^4 \frac{5}{2}(x - 2) dx + \int_4^6 (-x + 9) dx - \int_2^6 \frac{3}{4}(x - 2) dx$$

$$= \frac{5}{2} \left[\frac{x^2}{2} - 2x \right]_2^4 + \left[\frac{-x^2}{2} + 9x \right]_4^6 - \frac{3}{4} \left[\frac{x^2}{2} - 2x \right]_2^6$$

$$= \frac{5}{2} [8 - 8 - 2 + 4] + [-18 + 54 + 8 - 36] - \frac{3}{4} [18 - 12 - 2 + 4]$$

$$= 5 + 8 - \frac{3}{4}(8)$$

$$= 13 - 6$$

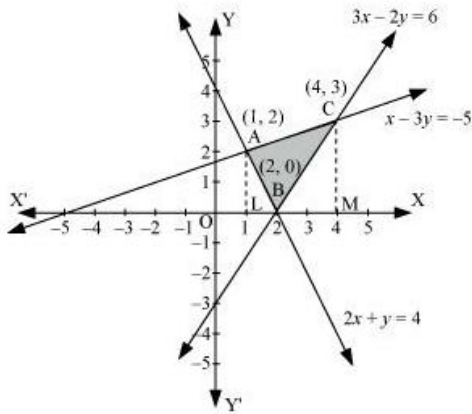
$$= 7 \text{ units}$$

Question 14:

Using the method of integration find the area of the region bounded by lines:

$$2x + y = 4, 3x - 2y = 6 \text{ and } x - 3y + 5 = 0$$

ANS :



The area of the region bounded by the lines is the area of $\triangle ABC$. AL and CM are the perpendiculars on x -axis.

$$\text{Area}(\triangle ABC) = \text{Area}(\triangle ALMCA) - \text{Area}(\triangle ALB) - \text{Area}(\triangle CMB)$$

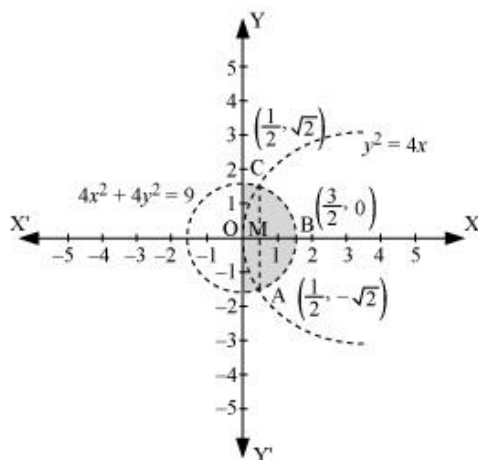
$$\begin{aligned} &= \int_1^4 \left(\frac{x+5}{3} \right) dx - \int_1^2 (4-2x) dx - \int_2^4 \left(\frac{3x-6}{2} \right) dx \\ &= \frac{1}{3} \left[\frac{x^2}{2} + 5x \right]_1^4 - \left[4x - x^2 \right]_1^2 - \frac{1}{2} \left[\frac{3x^2}{2} - 6x \right]_2^4 \\ &= \frac{1}{3} \left[8 + 20 - \frac{1}{2} - 5 \right] - [8 - 4 - 4 + 1] - \frac{1}{2} [24 - 24 - 6 + 12] \\ &= \left(\frac{1}{3} \times \frac{45}{2} \right) - (1) - \frac{1}{2} (6) \\ &= \frac{15}{2} - 1 - 3 \\ &= \frac{15}{2} - 4 = \frac{15-8}{2} = \frac{7}{2} \text{ units} \end{aligned}$$

Question 15:

Find the area of the region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

ANS :

The area bounded by the curves, $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$, is represented as



The points of intersection of both the curves are $\left(\frac{1}{2}, \sqrt{2}\right)$ and $\left(\frac{1}{2}, -\sqrt{2}\right)$.

The required area is given by OABCO.

It can be observed that area OABCO is symmetrical about x -axis.

$$\therefore \text{Area OABCO} = 2 \times \text{Area OBC}$$

$$\text{Area OBCO} = \text{Area OMC} + \text{Area MBC}$$

$$\begin{aligned} &= \int_0^{\frac{1}{2}} 2\sqrt{x} \, dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2} \sqrt{9-4x^2} \, dx \\ &= \int_0^{\frac{1}{2}} 2\sqrt{x} \, dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2} \sqrt{(3)^2 - (2x)^2} \, dx \end{aligned}$$

$$\text{Put } 2x = t \Rightarrow dx = \frac{dt}{2}$$

$$\text{When } x = \frac{3}{2}, t = 3 \text{ and when } x = \frac{1}{2}, t = 1$$

$$\begin{aligned} &= \int_0^{\frac{1}{2}} 2\sqrt{x} \, dx + \frac{1}{4} \int_1^3 \sqrt{(3)^2 - (t)^2} \, dt \\ &= 2 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{1}{2}} + \frac{1}{4} \left[\frac{t}{2} \sqrt{9-t^2} + \frac{9}{2} \sin^{-1} \left(\frac{t}{3} \right) \right]_1^3 \\ &= 2 \left[\frac{2}{3} \left(\frac{1}{2} \right)^{\frac{3}{2}} \right] + \frac{1}{4} \left[\left\{ \frac{3}{2} \sqrt{9-(3)^2} + \frac{9}{2} \sin^{-1} \left(\frac{3}{3} \right) \right\} - \left\{ \frac{1}{2} \sqrt{9-(1)^2} + \frac{9}{2} \sin^{-1} \left(\frac{1}{3} \right) \right\} \right] \\ &= \frac{2}{3\sqrt{2}} + \frac{1}{4} \left[\left\{ 0 + \frac{9}{2} \sin^{-1} (1) \right\} - \left\{ \frac{1}{2} \sqrt{8} + \frac{9}{2} \sin^{-1} \left(\frac{1}{3} \right) \right\} \right] \\ &= \frac{\sqrt{2}}{3} + \frac{1}{4} \left[\frac{9\pi}{4} - \sqrt{2} - \frac{9}{2} \sin^{-1} \left(\frac{1}{3} \right) \right] \\ &= \frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8} \sin^{-1} \left(\frac{1}{3} \right) \\ &= \frac{9\pi}{16} - \frac{9}{8} \sin^{-1} \left(\frac{1}{3} \right) + \frac{\sqrt{2}}{12} \end{aligned}$$

$$\text{Therefore, the required area is } \left[2 \times \left(\frac{9\pi}{16} - \frac{9}{8} \sin^{-1} \left(\frac{1}{3} \right) + \frac{\sqrt{2}}{12} \right) \right] = \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) + \frac{1}{3\sqrt{2}} \text{ units}$$

Question 16:

Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

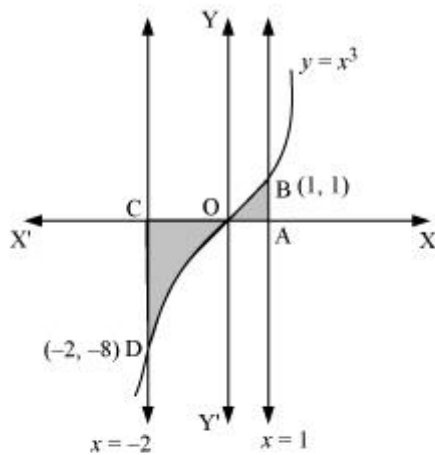
A. -9

B. $-\frac{15}{4}$

C. $\frac{15}{4}$

D. $\frac{17}{4}$

ANS :



$$\text{Required area} = \int_{-2}^1 y dx$$

$$= \int_{-2}^1 x^3 dx$$

$$= \left[\frac{x^4}{4} \right]_{-2}^1$$

$$= \left[\frac{1}{4} - \frac{(-2)^4}{4} \right]$$

$$= \left(\frac{1}{4} - 4 \right) = -\frac{15}{4} \text{ units}$$

Thus, the correct answer is B.

Question 17:

The area bounded by the curve $y = x|x|$, x -axis and the ordinates $x = -1$ and $x = 1$ is given by

[Hint: $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$]

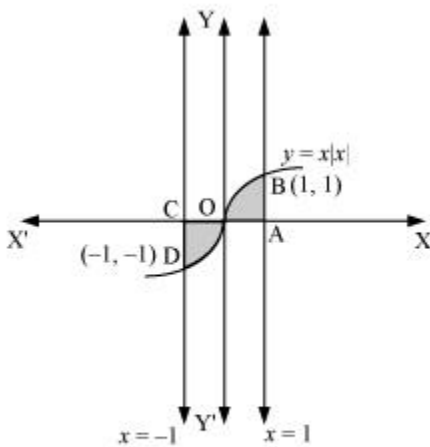
A. 0

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D.

ANS :



$$\text{Required area} = \int_{-1}^1 y dx$$

$$= \int_{-1}^1 x|x| dx$$

$$= \int_{-1}^0 x^2 dx + \int_0^1 x^2 dx$$

$$= \left[\frac{x^3}{3} \right]_{-1}^0 + \left[\frac{x^3}{3} \right]_0^1$$

$$= -\left(-\frac{1}{3}\right) + \frac{1}{3}$$

$$= \frac{2}{3} \text{ units}$$

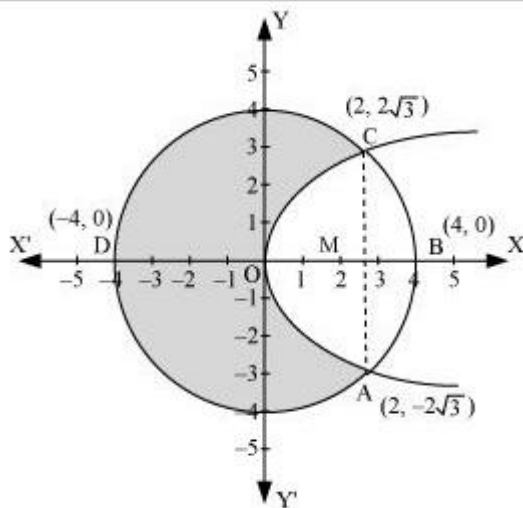
Thus, the correct answer is C.

Question 18:

The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is

- A. $\frac{4}{3}(4\pi - \sqrt{3})$
- B. $\frac{4}{3}(4\pi + \sqrt{3})$
- C. $\frac{4}{3}(8\pi - \sqrt{3})$
- D. $\frac{4}{3}(4\pi + \sqrt{3})$

ANS :



Area bounded by the circle and parabola

$$\begin{aligned}
 &= 2 \left[\text{Area}(\text{OADO}) + \text{Area}(\text{ADBA}) \right] \\
 &= 2 \left[\int_0^2 \sqrt{16x} dx + \int_2^4 \sqrt{16-x^2} dx \right] \\
 &= 2 \left[\sqrt{6} \left\{ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right\}_0^2 \right] + 2 \left[\frac{x}{2} \sqrt{16-x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_2^4 \\
 &= 2\sqrt{6} \times \frac{2}{3} \left[x^{\frac{3}{2}} \right]_0^2 + 2 \left[8 \cdot \frac{\pi}{2} - \sqrt{16-4} - 8 \sin^{-1} \left(\frac{1}{2} \right) \right] \\
 &= \frac{4\sqrt{6}}{3} (2\sqrt{2}) + 2 \left[4\pi - \sqrt{12} - 8 \frac{\pi}{6} \right]
 \end{aligned}$$

$$\begin{aligned}
&= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{8}{3}\pi \\
&= \frac{4}{3}[4\sqrt{3} + 6\pi - 3\sqrt{3} - 2\pi] \\
&= \frac{4}{3}[\sqrt{3} + 4\pi] \\
&= \frac{4}{3}[4\pi + \sqrt{3}] \text{ units}
\end{aligned}$$

$$\text{Area of circle} = \pi (r)^2$$

$$= \pi (4)^2$$

$$= 16\pi \text{ units}$$

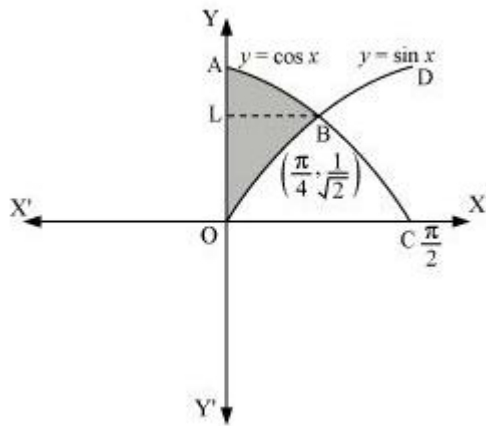
$$\begin{aligned}
\therefore \text{Required area} &= 16\pi - \frac{4}{3}[4\pi + \sqrt{3}] \\
&= \frac{4}{3}[4 \times 3\pi - 4\pi - \sqrt{3}] \\
&= \frac{4}{3}(8\pi - \sqrt{3}) \text{ units}
\end{aligned}$$

Thus, the correct answer is C.

Question 19:

The area bounded by the y -axis, $y = \cos x$ and $y = \sin x$ when $0 \leq x \leq \frac{\pi}{2}$

- A. $2(\sqrt{2} - 1)$
- B. $\sqrt{2} - 1$
- C. $\sqrt{2} + 1$
- D. $\sqrt{2}$



Required area = Area (ABLA) + area (OBLO)

$$= \int_{\frac{1}{\sqrt{2}}}^1 x dy + \int_0^{\frac{1}{\sqrt{2}}} x dy$$

$$= \int_{\frac{1}{\sqrt{2}}}^1 \cos^{-1} y dy + \int_0^{\frac{1}{\sqrt{2}}} \sin^{-1} x dy$$

Integrating by parts, we obtain

$$\begin{aligned}
 &= \left[y \cos^{-1} y - \sqrt{1-y^2} \right]_{\frac{1}{\sqrt{2}}}^1 + \left[x \sin^{-1} x + \sqrt{1-x^2} \right]_0^{\frac{1}{\sqrt{2}}} \\
 &= \left[\cos^{-1}(1) - \frac{1}{\sqrt{2}} \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} \right] + \left[\frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} - 1 \right] \\
 &= \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \\
 &= \frac{2}{\sqrt{2}} - 1 \\
 &= \sqrt{2} - 1 \text{ units}
 \end{aligned}$$