

# CLASS XII THREE DIMENSIONAL GEOMETRY CHAPTER 11

## MISC. EX. SOLUTIONS

Question 1:

Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points (3, 5, -1), (4, 3, -1).

ANS :

Let OA be the line joining the origin, O (0, 0, 0), and the point, A (2, 1, 1).

Also, let BC be the line joining the points, B (3, 5, -1) and C (4, 3, -1).

The direction ratios of OA are 2, 1, and 1 and of BC are (4 - 3) = 1, (3 - 5) = -2, and (-1 + 1) = 0

OA is perpendicular to BC, if  $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 2 \times 1 + 1(-2) + 1 \times 0 = 2 - 2 = 0$$

Thus, OA is perpendicular to BC.

Question 2:

If  $l_1, m_1, n_1$  and  $l_2, m_2, n_2$  are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are  $m_1n_2 - m_2n_1, n_1l_2 - n_2l_1, l_1m_2 - l_2m_1$ .

ANS :

It is given that  $l_1, m_1, n_1$  and  $l_2, m_2, n_2$  are the direction cosines of two mutually perpendicular lines. Therefore,

$$l_1l_2 + m_1m_2 + n_1n_2 = 0 \quad \dots(1)$$

$$l_1^2 + m_1^2 + n_1^2 = 1 \quad \dots(2)$$

$$l_2^2 + m_2^2 + n_2^2 = 1 \quad \dots(3)$$

Let  $l, m, n$  be the direction cosines of the line which is perpendicular to the line with direction cosines  $l_1, m_1, n_1$  and  $l_2, m_2, n_2$ .

$$\therefore ll_1 + mm_1 + nn_1 = 0$$

$$ll_2 + mm_2 + nn_2 = 0$$

$$\therefore \frac{l}{m_1n_2 - m_2n_1} = \frac{m}{n_1l_2 - n_2l_1} = \frac{n}{l_1m_2 - l_2m_1}$$

$$\Rightarrow \frac{l^2}{(m_1n_2 - m_2n_1)^2} = \frac{m^2}{(n_1l_2 - n_2l_1)^2} = \frac{n^2}{(l_1m_2 - l_2m_1)^2}$$

$$\Rightarrow \frac{l^2}{(m_1n_2 - m_2n_1)^2} = \frac{m^2}{(n_1l_2 - n_2l_1)^2} = \frac{n^2}{(l_1m_2 - l_2m_1)^2}$$

$$= \frac{l^2 + m^2 + n^2}{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2} \quad \dots(4)$$

$l, m, n$  are the direction cosines of the line.

$$\therefore l^2 + m^2 + n^2 = 1 \dots (5)$$

It is known that,

$$\begin{aligned} (l_1^2 + m_1^2 + n_1^2)(l_2^2 + m_2^2 + n_2^2) - (l_1l_2 + m_1m_2 + n_1n_2)^2 \\ = (m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2 \end{aligned}$$

From (1), (2), and (3), we obtain

$$\Rightarrow 1.1 - 0 = (m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2$$

$$\therefore (m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2 = 1 \quad \dots(6)$$

Substituting the values from equations (5) and (6) in equation (4), we obtain

$$\begin{aligned} \frac{l^2}{(m_1n_2 - m_2n_1)^2} = \frac{m^2}{(n_1l_2 - n_2l_1)^2} = \frac{n^2}{(l_1m_2 - l_2m_1)^2} = 1 \\ \Rightarrow l = m_1n_2 - m_2n_1, m = n_1l_2 - n_2l_1, n = l_1m_2 - l_2m_1 \end{aligned}$$

Thus, the direction cosines of the required line are  $m_1n_2 - m_2n_1, n_1l_2 - n_2l_1,$  and  $l_1m_2 - l_2m_1.$

### Question 3:

Find the angle between the lines whose direction ratios are  $a, b, c$  and  $b - c,$

$c - a, a - b.$

**ANS :**

The angle  $Q$  between the lines with direction cosines,  $a, b, c$  and  $b - c, c - a,$

$a - b,$  is given by,

$$\cos Q = \left| \frac{a(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2 + b^2 + c^2} + \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}} \right|$$

$$\Rightarrow \cos Q = 0$$

$$\Rightarrow Q = \cos^{-1} 0$$

$$\Rightarrow Q = 90^\circ$$

Thus, the angle between the lines is  $90^\circ.$

**Question 4:**

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Find the equation of a line parallel to  $x$ -axis and passing through the origin.

**ANS :**

The line parallel to  $x$ -axis and passing through the origin is  $x$ -axis itself.

Let A be a point on  $x$ -axis. Therefore, the coordinates of A are given by  $(a, 0, 0)$ , where  $a \in \mathbb{R}$ .

Direction ratios of OA are  $(a - 0) = a, 0, 0$

The equation of OA is given by,

$$\frac{x-0}{a} = \frac{y-0}{0} = \frac{z-0}{0}$$
$$\Rightarrow \frac{x}{1} = \frac{y}{0} = \frac{z}{0} = a$$

Thus, the equation of line parallel to  $x$ -axis and passing through origin is

$$\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$$

**Question 5:**

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If the coordinates of the points A, B, C, D be  $(1, 2, 3)$ ,  $(4, 5, 7)$ ,  $(-4, 3, -6)$  and  $(2, 9, 2)$  respectively, then find the angle between the lines AB and CD.

**ANS :**

The coordinates of A, B, C, and D are  $(1, 2, 3)$ ,  $(4, 5, 7)$ ,  $(-4, 3, -6)$ , and  $(2, 9, 2)$  respectively.

The direction ratios of AB are  $(4 - 1) = 3$ ,  $(5 - 2) = 3$ , and  $(7 - 3) = 4$

The direction ratios of CD are  $(2 - (-4)) = 6$ ,  $(9 - 3) = 6$ , and  $(2 - (-6)) = 8$

It can be seen that,  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

Therefore, AB is parallel to CD.

Thus, the angle between AB and CD is either  $0^\circ$  or  $180^\circ$ .

**Question 6:**

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If the lines  $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$  and  $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$  are perpendicular, find the value of  $k$ .

**ANS :**

The direction of ratios of the lines,  $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$  and  $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ , are  $-3, 2k, 2$  and  $3k, 1, -5$  respectively.

It is known that two lines with direction ratios,  $a_1, b_1, c_1$  and  $a_2, b_2, c_2$ , are perpendicular, if  $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\therefore -3(3k) + 2k \times 1 + 2(-5) = 0$$

$$\Rightarrow -9k + 2k - 10 = 0$$

$$\Rightarrow 7k = -10$$

$$\Rightarrow k = \frac{-10}{7}$$

Therefore, for  $k = -\frac{10}{7}$ , the given lines are perpendicular to each other.

**Question 7:**

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Find the vector equation of the plane passing through  $(1, 2, 3)$  and perpendicular to the plane  $\vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$

**ANS:**

The position vector of the point  $(1, 2, 3)$  is  $\vec{r}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$

The direction ratios of the normal to the plane,  $\vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$ , are 1, 2, and  $-5$  and the normal vector is  $\vec{N} = \hat{i} + 2\hat{j} - 5\hat{k}$

The equation of a line passing through a point and perpendicular to the given plane is given by,  $\vec{l} = \vec{r} + \lambda\vec{N}$ ,  $\lambda \in R$

$$\Rightarrow \vec{l} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 5\hat{k})$$

**Question 8:**

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Find the equation of the plane passing through  $(a, b, c)$  and parallel to the plane  $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$

**ANS:**

Any plane parallel to the plane,  $\vec{r}_1 \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$ , is of the form  $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = \lambda$  ... (1)

The plane passes through the point  $(a, b, c)$ . Therefore, the position vector  $\vec{r}$  of this point is  $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$

Therefore, equation (1) becomes

$$(a\hat{i} + b\hat{j} + c\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = \lambda$$

$$\Rightarrow a + b + c = \lambda$$

Substituting  $\lambda = a + b + c$  in equation (1), we obtain

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = a + b + c \quad \dots(2)$$

This is the vector equation of the required plane.

Substituting  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  in equation (2), we obtain

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = a + b + c$$

$$\Rightarrow x + y + z = a + b + c$$

**Question 9:**

Find the shortest distance between lines  $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$

and  $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$ .

**ANS:**

The given lines are

$$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}) \quad \dots(1)$$

$$\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}) \quad \dots(2)$$

It is known that the shortest distance between two lines,  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$ , is given by

$$d = \frac{|\left(\vec{b}_1 \times \vec{b}_2\right) \cdot (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}_1 \times \vec{b}_2|} \quad \dots(3)$$

Comparing  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  to equations (1) and (2), we obtain

$$\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{a}_2 = -4\hat{i} - \hat{k}$$

$$\vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$$

$$\Rightarrow \vec{a}_2 - \vec{a}_1 = (-4\hat{i} - \hat{k}) - (6\hat{i} + 2\hat{j} + 2\hat{k}) = -10\hat{i} - 2\hat{j} - 3\hat{k}$$

$$\Rightarrow \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} = (4+4)\hat{i} - (-2-6)\hat{j} + (-2+6)\hat{k} = 8\hat{i} + 8\hat{j} + 4\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{(8)^2 + (8)^2 + (4)^2} = 12$$

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (8\hat{i} + 8\hat{j} + 4\hat{k}) \cdot (-10\hat{i} - 2\hat{j} - 3\hat{k}) = -80 - 16 - 12 = -108$$

Substituting all the values in equation (3), we obtain

$$d = \left| \frac{-108}{12} \right| = 9$$

Therefore, the shortest distance between the two given lines is 9 units

**Question 10:**

Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ-plane

**ANS:**

It is known that the equation of the line passing through the points,  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$ , is  $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

The line passing through the points, (5, 1, 6) and (3, 4, 1), is given by,

$$\begin{aligned}\frac{x-5}{3-5} &= \frac{y-1}{4-1} = \frac{z-6}{1-6} \\ \Rightarrow \frac{x-5}{-2} &= \frac{y-1}{3} = \frac{z-6}{-5} = k \text{ (say)} \\ \Rightarrow x &= 5-2k, y = 3k+1, z = 6-5k\end{aligned}$$

Any point on the line is of the form  $(5-2k, 3k+1, 6-5k)$ .

The equation of YZ-plane is  $x = 0$

Since the line passes through YZ-plane,

$$5-2k=0$$

$$\Rightarrow k = \frac{5}{2}$$

$$\Rightarrow 3k+1 = 3 \times \frac{5}{2} + 1 = \frac{17}{2}$$

$$6-5k = 6-5 \times \frac{5}{2} = \frac{-13}{2}$$

Therefore, the required point is  $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$ .

**Question 11:**

Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the ZX-plane.

**ANS:**

It is known that the equation of the line passing through the points,  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$ , is  $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

The line passing through the points, (5, 1, 6) and (3, 4, 1), is given by,

$$\begin{aligned}\frac{x-5}{3-5} &= \frac{y-1}{4-1} = \frac{z-6}{1-6} \\ \Rightarrow \frac{x-5}{-2} &= \frac{y-1}{3} = \frac{z-6}{-5} = k \text{ (say)} \\ \Rightarrow x &= 5-2k, y = 3k+1, z = 6-5k\end{aligned}$$

Any point on the line is of the form  $(5 - 2k, 3k + 1, 6 - 5k)$ .

Since the line passes through ZX-plane,

$$3k + 1 = 0$$

$$\Rightarrow k = -\frac{1}{3}$$

$$\Rightarrow 5 - 2k = 5 - 2\left(-\frac{1}{3}\right) = \frac{17}{3}$$

$$6 - 5k = 6 - 5\left(-\frac{1}{3}\right) = \frac{23}{3}$$

Therefore, the required point is  $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$ .

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**Question 12:**

Find the coordinates of the point where the line through  $(3, -4, -5)$  and  $(2, -3, 1)$  crosses the plane  $2x + y + z = 7$ .

**ANS:**

It is known that the equation of the line through the points,  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$ , is  $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

Since the line passes through the points,  $(3, -4, -5)$  and  $(2, -3, 1)$ , its equation is given by,

$$\frac{x-3}{2-3} = \frac{y+4}{-3+4} = \frac{z+5}{1+5}$$

$$\Rightarrow \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = k \text{ (say)}$$

$$\Rightarrow x = 3 - k, y = k - 4, z = 6k - 5$$

Therefore, any point on the line is of the form  $(3 - k, k - 4, 6k - 5)$ .

This point lies on the plane,  $2x + y + z = 7$

$$\therefore 2(3 - k) + (k - 4) + (6k - 5) = 7$$

$$\Rightarrow 5k - 3 = 7$$

$$\Rightarrow k = 2$$

Hence, the coordinates of the required point are  $(3 - 2, 2 - 4, 6 \times 2 - 5)$  i.e.,

$$(1, -2, 7).$$

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**Question 13:**

Find the equation of the plane passing through the point  $(-1, 3, 2)$  and perpendicular to each of the planes  $x + 2y + 3z = 5$  and  $3x + 3y + z = 0$ .

**ANS:**

The equation of the plane passing through the point  $(-1, 3, 2)$  is

$$a(x + 1) + b(y - 3) + c(z - 2) = 0 \dots (1)$$

where,  $a, b, c$  are the direction ratios of normal to the plane.

It is known that two planes,  $a_1x + b_1y + c_1z + d_1 = 0$  and  $a_2x + b_2y + c_2z + d_2 = 0$ , are perpendicular, if  $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Plane (1) is perpendicular to the plane,  $x + 2y + 3z = 5$

$$\begin{aligned} \therefore a \cdot 1 + b \cdot 2 + c \cdot 3 &= 0 \\ \Rightarrow a + 2b + 3c &= 0 \end{aligned} \dots(2)$$

Also, plane (1) is perpendicular to the plane,  $3x + 3y + z = 0$

$$\begin{aligned} \therefore a \cdot 3 + b \cdot 3 + c \cdot 1 &= 0 \\ \Rightarrow 3a + 3b + c &= 0 \end{aligned} \dots(3)$$

From equations (2) and (3), we obtain

$$\begin{aligned} \frac{a}{2 \times 1 - 3 \times 3} &= \frac{b}{3 \times 3 - 1 \times 1} = \frac{c}{1 \times 3 - 2 \times 3} \\ \Rightarrow \frac{a}{-7} &= \frac{b}{8} = \frac{c}{-3} = k \text{ (say)} \\ \Rightarrow a &= -7k, b = 8k, c = -3k \end{aligned}$$

Substituting the values of  $a, b$ , and  $c$  in equation (1), we obtain

$$\begin{aligned} -7k(x + 1) + 8k(y - 3) - 3k(z - 2) &= 0 \\ \Rightarrow (-7x - 7) + (8y - 24) - 3z + 6 &= 0 \\ \Rightarrow -7x + 8y - 3z - 25 &= 0 \\ \Rightarrow 7x - 8y + 3z + 25 &= 0 \end{aligned}$$

#### Question 14:

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If the points  $(1, 1, p)$  and  $(-3, 0, 1)$  be equidistant from the plane  $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$ , then find the value of  $p$ .

#### ANS:

The position vector through the point  $(1, 1, p)$  is  $\vec{a}_1 = \hat{i} + \hat{j} + p\hat{k}$

Similarly, the position vector through the point  $(-3, 0, 1)$  is

$$\vec{a}_2 = -4\hat{i} + \hat{k}$$

The equation of the given plane is  $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$

It is known that the perpendicular distance between a point whose position vector is  $\vec{a}$  and the plane,  $\vec{r} \cdot \vec{N} = d$ , is given by,

$$D = \frac{|\vec{a} \cdot \vec{N} - d|}{|\vec{N}|}$$



Here,  $\vec{N} = 3\hat{i} + 4\hat{j} - 12\hat{k}$  and  $d = -13$

Therefore, the distance between the point  $(1, 1, p)$  and the given plane is

$$\begin{aligned}D_1 &= \frac{|(\hat{i} + \hat{j} + p\hat{k}) \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13|}{|3\hat{i} + 4\hat{j} - 12\hat{k}|} \\ \Rightarrow D_1 &= \frac{|3 + 4 - 12p + 13|}{\sqrt{3^2 + 4^2 + (-12)^2}} \\ \Rightarrow D_1 &= \frac{|20 - 12p|}{13} \quad \dots(1)\end{aligned}$$

Similarly, the distance between the point  $(-3, 0, 1)$  and the given plane is

$$\begin{aligned}D_2 &= \frac{|(-3\hat{i} + \hat{k}) \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13|}{|3\hat{i} + 4\hat{j} - 12\hat{k}|} \\ \Rightarrow D_2 &= \frac{|-9 - 12 + 13|}{\sqrt{3^2 + 4^2 + (-12)^2}} \\ \Rightarrow D_2 &= \frac{8}{13} \quad \dots(2)\end{aligned}$$

It is given that the distance between the required plane and the points,  $(1, 1, p)$  and  $(-3, 0, 1)$ , is equal.

$$\therefore D_1 = D_2$$

$$\Rightarrow \frac{|20 - 12p|}{13} = \frac{8}{13}$$

$$\Rightarrow 20 - 12p = 8 \text{ or } -(20 - 12p) = 8$$

$$\Rightarrow 12p = 12 \text{ or } 12p = 28$$

$$\Rightarrow p = 1 \text{ or } p = \frac{7}{3}$$

Question 15:

Find the equation of the plane passing through the line of intersection of the planes  $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$  and  $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$  and parallel to x-axis.

**ANS:**

The given planes are

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$$

$$\Rightarrow \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 = 0$$

$$\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$$

The equation of any plane passing through the line of intersection of these planes is

$$[\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1] + \lambda [\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4] = 0$$

$$\vec{r} \cdot [(2\lambda + 1)\hat{i} + (3\lambda + 1)\hat{j} + (1 - \lambda)\hat{k}] + (4\lambda + 1) = 0 \quad \dots(1)$$

Its direction ratios are  $(2\lambda + 1)$ ,  $(3\lambda + 1)$ , and  $(1 - \lambda)$ .

The required plane is parallel to  $x$ -axis. Therefore, its normal is perpendicular to  $x$ -axis.

The direction ratios of  $x$ -axis are 1, 0, and 0.

$$\therefore 1 \cdot (2\lambda + 1) + 0(3\lambda + 1) + 0(1 - \lambda) = 0$$

$$\Rightarrow 2\lambda + 1 = 0$$

$$\Rightarrow \lambda = -\frac{1}{2}$$

Substituting  $\lambda = -\frac{1}{2}$  in equation (1), we obtain

$$\Rightarrow \vec{r} \cdot \left[ -\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k} \right] + (-3) = 0$$

$$\Rightarrow \vec{r} \cdot (\hat{j} - 3\hat{k}) + 6 = 0$$

Therefore, its Cartesian equation is  $y - 3z + 6 = 0$

This is the equation of the required plane.

Question 16:

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If  $O$  be the origin and the coordinates of  $P$  be  $(1, 2, -3)$ , then find the equation of the plane passing through  $P$  and perpendicular to  $OP$ .

**ANS:**

The coordinates of the points, O and P, are (0, 0, 0) and (1, 2, -3) respectively.

Therefore, the direction ratios of OP are  $(1 - 0) = 1$ ,  $(2 - 0) = 2$ , and  $(-3 - 0) = -3$

It is known that the equation of the plane passing through the point  $(x_1, y_1, z_1)$  is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \text{ where, } a, b, \text{ and } c \text{ are the direction ratios of normal.}$$

Here, the direction ratios of normal are 1, 2, and -3 and the point P is (1, 2, -3).

Thus, the equation of the required plane is

$$\begin{aligned} 1(x - 1) + 2(y - 2) - 3(z + 3) &= 0 \\ \Rightarrow x + 2y - 3z - 14 &= 0 \end{aligned}$$

Question 17:

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Find the equation of the plane which contains the line of intersection of the planes  $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$ ,  $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$  and which is perpendicular to the plane  $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$ .

**ANS:**

The equations of the given planes are

$$\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0 \quad \dots(1)$$

$$\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0 \quad \dots(2)$$

The equation of the plane passing through the line intersection of the plane given in equation (1) and equation (2) is

$$\begin{aligned} [\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4] + \lambda [\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5] &= 0 \\ \vec{r} \cdot [(2\lambda + 1)\hat{i} + (\lambda + 2)\hat{j} + (3 - \lambda)\hat{k}] + (5\lambda - 4) &= 0 \quad \dots(3) \end{aligned}$$

The plane in equation (3) is perpendicular to the plane,  $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$

$$\therefore 5(2\lambda + 1) + 3(\lambda + 2) - 6(3 - \lambda) = 0$$

$$\Rightarrow 19\lambda - 7 = 0$$

$$\Rightarrow \lambda = \frac{7}{19}$$

Substituting  $\lambda = \frac{7}{19}$  in equation (3), we obtain

$$\Rightarrow \vec{r} \cdot \left[ \frac{33}{19}\hat{i} + \frac{45}{19}\hat{j} + \frac{50}{19}\hat{k} \right] - \frac{41}{19} = 0$$

$$\Rightarrow \vec{r} \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) - 41 = 0 \quad \dots(4)$$

This is the vector equation of the required plane.

The Cartesian equation of this plane can be obtained by substituting  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  in equation (3).

$$\begin{aligned}(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) - 41 &= 0 \\ \Rightarrow 33x + 45y + 50z - 41 &= 0\end{aligned}$$

**Question 18:**

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Find the distance of the point  $(-1, -5, -10)$  from the point of intersection of the line  $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$  and the plane  $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ .

**ANS:**

The equation of the given line is

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \quad \dots(1)$$

The equation of the given plane is

$$\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5 \quad \dots(2)$$

Substituting the value of  $\vec{r}$  from equation (1) in equation (2), we obtain

$$\begin{aligned}[2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})] \cdot (\hat{i} - \hat{j} + \hat{k}) &= 5 \\ \Rightarrow [(3\lambda + 2)\hat{i} + (4\lambda - 1)\hat{j} + (2\lambda + 2)\hat{k}] \cdot (\hat{i} - \hat{j} + \hat{k}) &= 5 \\ \Rightarrow (3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) &= 5 \\ \Rightarrow \lambda &= 0\end{aligned}$$

Substituting this value in equation (1), we obtain the equation of the line as

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$$

This means that the position vector of the point of intersection of the line and the plane is  $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$

This shows that the point of intersection of the given line and plane is given by the coordinates,  $(2, -1, 2)$ . The point is  $(-1, -5, -10)$ .

The distance  $d$  between the points,  $(2, -1, 2)$  and  $(-1, -5, -10)$ , is

$$d = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2} = \sqrt{9+16+144} = \sqrt{169} = 13$$

**Question 19:**

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Find the vector equation of the line passing through  $(1, 2, 3)$  and parallel to the planes  $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$  and  $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$ .

**ANS:**

Let the required line be parallel to vector  $\vec{b}$  given by,

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

The position vector of the point  $(1, 2, 3)$  is  $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

The equation of line passing through (1, 2, 3) and parallel to  $\vec{b}$  is given by,

$$\begin{aligned}\vec{r} &= \vec{a} + \lambda \vec{b} \\ \Rightarrow \vec{r}(\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) &\quad \dots(1)\end{aligned}$$

The equations of the given planes are

$$\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5 \quad \dots(2)$$

$$\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6 \quad \dots(3)$$

The line in equation (1) and plane in equation (2) are parallel. Therefore, the normal to the plane of equation (2) and the given line are perpendicular.

$$\begin{aligned}\Rightarrow (\hat{i} - \hat{j} + 2\hat{k}) \cdot \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) &= 0 \\ \Rightarrow \lambda(b_1 - b_2 + 2b_3) &= 0 \\ \Rightarrow b_1 - b_2 + 2b_3 &= 0 \quad \dots(4)\end{aligned}$$

$$\begin{aligned}\text{Similarly, } (3\hat{i} + \hat{j} + \hat{k}) \cdot \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) &= 0 \\ \Rightarrow \lambda(3b_1 + b_2 + b_3) &= 0 \\ \Rightarrow 3b_1 + b_2 + b_3 &= 0 \quad \dots(5)\end{aligned}$$

From equations (4) and (5), we obtain

$$\begin{aligned}\frac{b_1}{(-1) \times 1 - 1 \times 2} &= \frac{b_2}{2 \times 3 - 1 \times 1} = \frac{b_3}{1 \times 1 - 3(-1)} \\ \Rightarrow \frac{b_1}{-3} &= \frac{b_2}{5} = \frac{b_3}{4}\end{aligned}$$

Therefore, the direction ratios of  $\vec{b}$  are -3, 5, and 4.

$$\therefore \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = -3\hat{i} + 5\hat{j} + 4\hat{k}$$

Substituting the value of  $\vec{b}$  in equation (1), we obtain

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$$

This is the equation of the required line.

#### Question 20:

Find the vector equation of the line passing through the point (1, 2, -4) and perpendicular to the two lines:

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

**ANS:**

Let the required line be parallel to the vector  $\vec{b}$  given by,  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

The position vector of the point  $(1, 2, -4)$  is  $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$

The equation of the line passing through  $(1, 2, -4)$  and parallel to vector  $\vec{b}$  is

$$\begin{aligned}\vec{r} &= \vec{a} + \lambda\vec{b} \\ \Rightarrow \vec{r} &= (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) \quad \dots(1)\end{aligned}$$

The equations of the lines are

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \quad \dots(2)$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5} \quad \dots(3)$$

Line (1) and line (2) are perpendicular to each other.

$$\therefore 3b_1 - 16b_2 + 7b_3 = 0 \quad \dots(4)$$

Also, line (1) and line (3) are perpendicular to each other.

$$\therefore 3b_1 + 8b_2 - 5b_3 = 0 \quad \dots(5)$$

From equations (4) and (5), we obtain

$$\begin{aligned}\frac{b_1}{(-16)(-5) - 8 \times 7} &= \frac{b_2}{7 \times 3 - 3(-5)} = \frac{b_3}{3 \times 8 - 3(-16)} \\ \Rightarrow \frac{b_1}{24} &= \frac{b_2}{36} = \frac{b_3}{72} \\ \Rightarrow \frac{b_1}{2} &= \frac{b_2}{3} = \frac{b_3}{6}\end{aligned}$$

$\therefore$  Direction ratios of  $\vec{b}$  are 2, 3, and 6.

$$\therefore \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

Substituting  $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$  in equation (1), we obtain

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

This is the equation of the required line.

**Question 21:**

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Prove that if a plane has the intercepts  $a, b, c$  and is at a distance of  $P$  units from the origin, then  $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$

**ANS:**

The equation of a plane having intercepts  $a, b, c$  with  $x, y,$  and  $z$  axes respectively is given by,

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \dots(1)$$

The distance ( $p$ ) of the plane from the origin is given by,

$$p = \left| \frac{\frac{0}{a} + \frac{0}{b} + \frac{0}{c} - 1}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}} \right|$$

$$\Rightarrow p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

$$\Rightarrow p^2 = \frac{1}{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

**Question 22:**

---

Distance between the two planes:  $2x + 3y + 4z = 4$  and  $4x + 6y + 8z = 12$  is

(A) 2 units (B) 4 units (C) 8 units

(D)  $\frac{2}{\sqrt{29}}$  units

**ANS:**

The equations of the planes are

$$2x + 3y + 4z = 4 \quad \dots(1)$$

$$4x + 6y + 8z = 12$$

$$\Rightarrow 2x + 3y + 4z = 6 \quad \dots(2)$$

It can be seen that the given planes are parallel.

It is known that the distance between two parallel planes,  $ax + by + cz = d_1$  and  $ax + by + cz = d_2$ , is given by,

$$D = \left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$

$$\Rightarrow D = \left| \frac{6 - 4}{\sqrt{(2)^2 + (3)^2 + (4)^2}} \right|$$

$$D = \frac{2}{\sqrt{29}}$$

Thus, the distance between the lines is  $\frac{2}{\sqrt{29}}$  units.

Hence, the correct answer is D.

**Question 23:**

The planes:  $2x - y + 4z = 5$  and  $5x - 2.5y + 10z = 6$  are

(A) Perpendicular (B) Parallel (C) intersect  $y$ -axis

(C) passes through  $\left(0, 0, \frac{5}{4}\right)$

**ANS:**

It can be seen that,

$$\frac{a_1}{a_2} = \frac{2}{5}$$

$$\frac{b_1}{b_2} = \frac{-1}{-2.5} = \frac{2}{5}$$

$$\frac{c_1}{c_2} = \frac{4}{10} = \frac{2}{5}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the given planes are parallel.

Hence, the correct answer is B.

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