

CLASS XII DIFFERENTIAL EQUATIONS CHAPTER 9

EX. 9.3 SOLUTIONS

Question 1:

$$\frac{x}{a} + \frac{y}{b} = 1$$

ANS :

$$\frac{x}{a} + \frac{y}{b} = 1$$

Differentiating both sides of the given equation with respect to x , we get:

$$\begin{aligned}\frac{1}{a} + \frac{1}{b} \frac{dy}{dx} &= 0 \\ \Rightarrow \frac{1}{a} + \frac{1}{b} y' &= 0\end{aligned}$$

Again, differentiating both sides with respect to x , we get:

$$\begin{aligned}0 + \frac{1}{b} y'' &= 0 \\ \Rightarrow \frac{1}{b} y'' &= 0 \\ \Rightarrow y'' &= 0\end{aligned}$$

Hence, the required differential equation of the given curve is $y'' = 0$.

Question 2:

$$y^2 = a(b^2 - x^2)$$

ANS :

$$y^2 = a(b^2 - x^2)$$

Differentiating both sides with respect to x , we get:

$$\begin{aligned}2y \frac{dy}{dx} &= a(-2x) \\ \Rightarrow 2yy' &= -2ax \\ \Rightarrow yy' &= -ax\end{aligned}$$

...(1)

Again, differentiating both sides with respect to x , we get:

$$\begin{aligned}y' \cdot y' + yy'' &= -a \\ \Rightarrow (y')^2 + yy'' &= -a \quad \dots(2)\end{aligned}$$

Dividing equation (2) by equation (1), we get:

$$\begin{aligned}\frac{(y')^2 + yy''}{yy'} &= \frac{-a}{-ax} \\ \Rightarrow xyy'' + x(y')^2 - yy'' &= 0\end{aligned}$$

This is the required differential equation of the given curve.

Question 3:

$$y = ae^{3x} + be^{-2x}$$

ANS :

$$y = ae^{3x} + be^{-2x} \quad \dots(1)$$

Differentiating both sides with respect to x , we get:

$$y' = 3ae^{3x} - 2be^{-2x} \quad \dots(2)$$

Again, differentiating both sides with respect to x , we get:

$$y'' = 9ae^{3x} + 4be^{-2x} \quad \dots(3)$$

Multiplying equation (1) with (2) and then adding it to equation (2), we get:

$$\begin{aligned}(2ae^{3x} + 2be^{-2x}) + (3ae^{3x} - 2bc^{-2x}) &= 2y + y' \\ \Rightarrow 5ae^{3x} &= 2y + y' \\ \Rightarrow ae^{3x} &= \frac{2y + y'}{5}\end{aligned}$$

Now, multiplying equation (1) with equation (3) and subtracting equation (2) from it, we get:

$$\begin{aligned}(2ae^{3x} + 2be^{-2x}) + (3ae^{3x} - 2bc^{-2x}) &= 2y + y' \\ \Rightarrow 5ae^{3x} &= 2y + y' \\ \Rightarrow ae^{3x} &= \frac{2y + y'}{5}\end{aligned}$$

Now, multiplying equation (1) with equation (3) and subtracting equation (2) from it, we get:

$$\begin{aligned}(3ae^{3x} + 3be^{-2x}) - (3ae^{3x} - 2be^{-2x}) &= 3y - y' \\ \Rightarrow 5be^{-2x} &= 3y - y' \\ \Rightarrow be^{-2x} &= \frac{3y - y'}{5}\end{aligned}$$

Substituting the values of ae^{3x} and be^{-2x} in equation (3), we get:

$$\begin{aligned}y'' &= 9 \cdot \frac{(2y + y')}{5} + 4 \cdot \frac{(3y - y')}{5} \\ \Rightarrow y'' &= \frac{18y + 9y'}{5} + \frac{12y - 4y'}{5} \\ \Rightarrow y'' &= \frac{30y + 5y'}{5} \\ \Rightarrow y'' &= 6y + y' \\ \Rightarrow y'' - y' - 6y &= 0\end{aligned}$$

This is the required differential equation of the given curve.

Question 4:

$$y = e^{2x}(a + bx)$$

ANS :

$$y = e^{2x}(a + bx) \quad \dots(1)$$

Differentiating both sides with respect to x , we get:

$$\begin{aligned}y' &= 2e^{2x}(a + bx) + e^{2x} \cdot b \\ \Rightarrow y' &= e^{2x}(2a + 2bx + b) \quad \dots(2)\end{aligned}$$

Multiplying equation (1) with equation (2) and then subtracting it from equation (2), we get:

$$\begin{aligned}y' - 2y &= e^{2x}(2a + 2bx + b) - e^{2x}(2a + 2bx) \\ \Rightarrow y' - 2y &= be^{2x} \quad \dots(3)\end{aligned}$$

Differentiating both sides with respect to x , we get:

$$y'' - 2y' = 2be^{2x} \quad \dots(4)$$

Dividing equation (4) by equation (3), we get:

$$\begin{aligned}\frac{y'' - 2y'}{y' - 2y} &= 2 \\ \Rightarrow y'' - 2y' &= 2y' - 4y \\ \Rightarrow y'' - 4y' + 4y &= 0\end{aligned}$$

This is the required differential equation of the given curve.

Question 5:

$$y = e^x (a \cos x + b \sin x)$$

ANS :

$$y = e^x (a \cos x + b \sin x) \quad \dots(1)$$

Differentiating both sides with respect to x , we get:

$$\begin{aligned} y' &= e^x (a \cos x + b \sin x) + e^x (-a \sin x + b \cos x) \\ \Rightarrow y' &= e^x [(a+b) \cos x - (a-b) \sin x] \quad \dots(2) \end{aligned}$$

Again, differentiating with respect to x , we get:

$$\begin{aligned} y'' &= e^x [(a+b) \cos x - (a-b) \sin x] + e^x [-(a+b) \sin x - (a-b) \cos x] \\ y'' &= e^x [2b \cos x - 2a \sin x] \\ y'' &= 2e^x (b \cos x - a \sin x) \\ \Rightarrow \frac{y''}{2} &= e^x (b \cos x - a \sin x) \quad \dots(3) \end{aligned}$$

Adding equations (1) and (3), we get:

$$\begin{aligned} y + \frac{y''}{2} &= e^x [(a+b) \cos x - (a-b) \sin x] \\ \Rightarrow y + \frac{y''}{2} &= y' \\ \Rightarrow 2y + y'' &= 2y' \\ \Rightarrow y'' - 2y' + 2y &= 0 \end{aligned}$$

This is the required differential equation of the given curve.

Question 6:

Form the differential equation of the family of circles touching the y -axis at the origin.

ANS :

The centre of the circle touching the y -axis at origin lies on the x -axis.

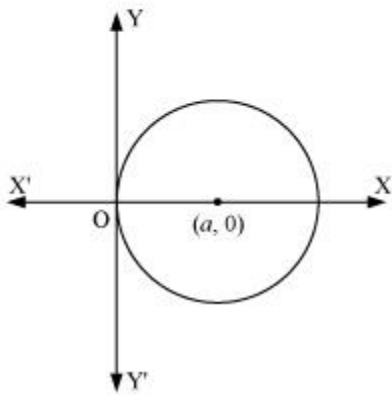
Let $(\alpha, 0)$ be the centre of the circle.

Since it touches the y -axis at origin, its radius is α .

Now, the equation of the circle with centre $(a, 0)$ and radius (a) is

$$(x-a)^2 + y^2 = a^2.$$

$$\Rightarrow x^2 + y^2 = 2ax \quad \dots(1)$$



Differentiating equation (1) with respect to x , we get:

$$2x + 2yy' = 2a$$

$$\Rightarrow x + yy' = a$$

Now, on substituting the value of a in equation (1), we get:

$$x^2 + y^2 = 2(x + yy')x$$

$$\Rightarrow x^2 + y^2 = 2x^2 + 2xyy'$$

$$\Rightarrow 2xyy' + x^2 = y^2$$

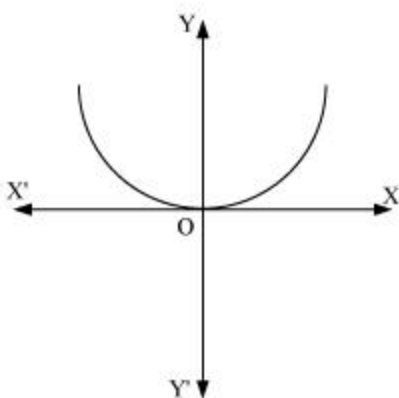
This is the required differential equation.

Question 7:

Form the differential equation of the family of parabolas having vertex at origin and axis along positive y -axis.

The equation of the parabola having the vertex at origin and the axis along the positive y -axis is:

$$x^2 = 4ay \quad \dots(1)$$



Differentiating equation (1) with respect to x , we get:

$$2x = 4ay' \quad \dots(2)$$

Dividing equation (2) by equation (1), we get:

$$\begin{aligned} \frac{2x}{x^2} &= \frac{4ay'}{4ay} \\ \Rightarrow \frac{2}{x} &= \frac{y'}{y} \\ \Rightarrow xy' &= 2y \\ \Rightarrow xy' - 2y &= 0 \end{aligned}$$

This is the required differential equation.

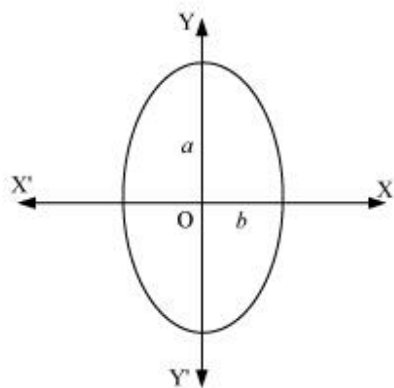
Question 8:

Form the differential equation of the family of ellipses having foci on y -axis and centre at origin.

ANS :

The equation of the family of ellipses having foci on the y -axis and the centre at origin is as follows:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \dots(1)$$



Differentiating equation (1) with respect to x , we get:

$$\begin{aligned} \frac{2x}{b^2} + \frac{2yy'}{a^2} &= 0 \\ \Rightarrow \frac{x}{b^2} + \frac{yy'}{a^2} &= 0 \quad \dots(2) \end{aligned}$$

Again, differentiating with respect to x , we get:

$$\begin{aligned}\frac{1}{b^2} + \frac{y' \cdot y' + y \cdot y''}{a^2} &= 0 \\ \Rightarrow \frac{1}{b^2} + \frac{1}{a^2}(y'^2 + yy'') &= 0 \\ \Rightarrow \frac{1}{b^2} &= -\frac{1}{a^2}(y'^2 + yy'')\end{aligned}$$

Substituting this value in equation (2), we get:

$$\begin{aligned}x \left[-\frac{1}{a^2}((y')^2 + yy'') \right] + \frac{yy'}{a^2} &= 0 \\ \Rightarrow -x(y')^2 - xyy'' + yy' &= 0 \\ \Rightarrow xyy'' + x(y')^2 - yy' &= 0\end{aligned}$$

This is the required differential equation.

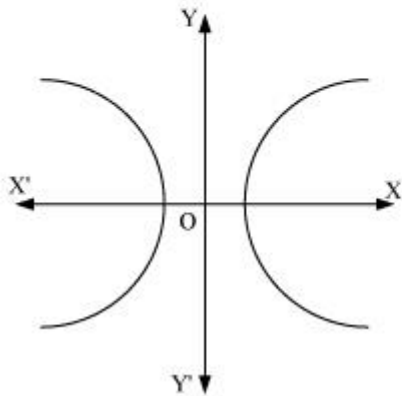
Question 9:

Form the differential equation of the family of hyperbolas having foci on x -axis and centre at origin.

ANS :

The equation of the family of hyperbolas with the centre at origin and foci along the x-axis is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \dots(1)$$



Differentiating both sides of equation (1) with respect to x , we get:

$$\begin{aligned} \frac{2x}{a^2} - \frac{2yy'}{b^2} &= 0 \\ \Rightarrow \frac{x}{a^2} - \frac{yy'}{b^2} &= 0 \quad \dots(2) \end{aligned}$$

Again, differentiating both sides with respect to x , we get:

$$\begin{aligned} \frac{1}{a^2} - \frac{y' \cdot y' + yy''}{b^2} &= 0 \\ \Rightarrow \frac{1}{a^2} &= \frac{1}{b^2} \left((y')^2 + yy'' \right) \end{aligned}$$

Substituting the value of $\frac{1}{a^2}$ in equation (2), we get:

$$\begin{aligned} \frac{x}{b^2} \left((y')^2 + yy'' \right) - \frac{yy'}{b^2} &= 0 \\ \Rightarrow x(y')^2 + xyy'' - yy' &= 0 \\ \Rightarrow xyy'' + x(y')^2 - yy' &= 0 \end{aligned}$$

This is the required differential equation.

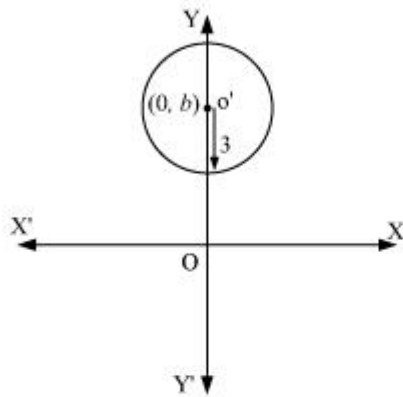
Question 10:

Form the differential equation of the family of circles having centre on y -axis and radius 3 units.

ANS :

The differential equation of the family of circles with centre at $(0, b)$ and radius 3 is as follows:

$$\begin{aligned}x^2 + (y - b)^2 &= 3^2 \\ \Rightarrow x^2 + (y - b)^2 &= 9 \quad \dots(1)\end{aligned}$$



Differentiating equation (1) with respect to x , we get:

$$\begin{aligned}2x + 2(y - b) \cdot y' &= 0 \\ \Rightarrow (y - b) \cdot y' &= -x \\ \Rightarrow y - b &= \frac{-x}{y'}\end{aligned}$$

Substituting the value of $(y - b)$ in equation (1), we get:

$$\begin{aligned}x^2 + \left(\frac{-x}{y'}\right)^2 &= 9 \\ \Rightarrow x^2 \left[1 + \frac{1}{(y')^2}\right] &= 9 \\ \Rightarrow x^2 \left((y')^2 + 1\right) &= 9(y')^2 \\ \Rightarrow (x^2 - 9)(y')^2 + x^2 &= 0\end{aligned}$$

This is the required differential equation.

Question 11:

Which of the following differential equations has $y = c_1e^x + c_2e^{-x}$ as the general solution?

A. $\frac{d^2y}{dx^2} + y = 0$

B.

C. $\frac{d^2y}{dx^2} + 1 = 0$

D. $\frac{d^2y}{dx^2} - 1 = 0$

ANS :

The given equation is:

$$y = c_1e^x + c_2e^{-x} \quad \dots(1)$$

Differentiating with respect to x , we get:

$$\frac{dy}{dx} = c_1e^x - c_2e^{-x}$$

Again, differentiating with respect to x , we get:

$$\frac{d^2y}{dx^2} = c_1e^x + c_2e^{-x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = y$$

$$\Rightarrow \frac{d^2y}{dx^2} - y = 0$$

This is the required differential equation of the given equation of curve.

Hence, the correct answer is B.

Question 12:

Which of the following differential equation has $y = x$ as one of its particular solution?

A. $\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = x$

B. $\frac{d^2y}{dx^2} + x \frac{dy}{dx} + xy = x$

C. $\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = 0$

D. $\frac{d^2y}{dx^2} + x \frac{dy}{dx} + xy = 0$

ANS :

The given equation of curve is $y = x$.

Differentiating with respect to x , we get:

$$\frac{dy}{dx} = 1 \quad \dots(1)$$

Again, differentiating with respect to x , we get:

$$\frac{d^2y}{dx^2} = 0 \quad \dots(2)$$

Now, on substituting the values of y , $\frac{d^2y}{dx^2}$, and $\frac{dy}{dx}$ from equation (1) and (2) in each of the given alternatives, we find that only the differential equation given in alternative C is correct.

$$\begin{aligned} \frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy &= 0 - x^2 \cdot 1 + x \cdot x \\ &= -x^2 + x^2 \\ &= 0 \end{aligned}$$

Hence, the correct answer is C.