

## CLASS XII INTEGRALS CHAPTER 7

### EX. 7.10 SOLUTIONS

Question 1: .....

$$\int_0^1 \frac{x}{x^2+1} dx$$

**ANS:**

$$\int_0^1 \frac{x}{x^2+1} dx$$

$$\text{Let } x^2+1=t \Rightarrow 2x dx = dt$$

When  $x=0$ ,  $t=1$  and when  $x=1$ ,  $t=2$

$$\begin{aligned} \therefore \int_0^1 \frac{x}{x^2+1} dx &= \frac{1}{2} \int_1^2 \frac{dt}{t} \\ &= \frac{1}{2} [\log|t|]_1^2 \\ &= \frac{1}{2} [\log 2 - \log 1] \\ &= \frac{1}{2} \log 2 \end{aligned}$$

Question 2: .....

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin \phi} \cos^5 \phi d\phi$$

**ANS:**

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^5\phi \, d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^4\phi \cos\phi \, d\phi$$

Also, let  $\sin\phi = t \Rightarrow \cos\phi \, d\phi = dt$

When  $\phi = 0$ ,  $t = 0$  and when  $\phi = \frac{\pi}{2}$ ,  $t = 1$

$$\begin{aligned} \therefore I &= \int_0^1 \sqrt{t} (1-t^2)^2 \, dt \\ &= \int_0^1 t^{\frac{1}{2}} (1+t^4 - 2t^2) \, dt \\ &= \int_0^1 \left[ t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}} \right] dt \\ &= \left[ \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}} - \frac{2t^{\frac{7}{2}}}{\frac{7}{2}} \right]_0^1 \\ &= \frac{2}{3} + \frac{2}{11} - \frac{4}{7} \\ &= \frac{154 + 42 - 132}{231} \\ &= \frac{64}{231} \end{aligned}$$

Question 3:

$$\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$$

**ANS:**

$$\text{Let } I = \int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$$

Also, let  $x = \tan\theta \Rightarrow dx = \sec^2\theta \, d\theta$

When  $x = 0$ ,  $\theta = 0$  and when  $x = 1$ ,  $\theta = \frac{\pi}{4}$

$$\begin{aligned} I &= \int_0^{\frac{\pi}{4}} \sin^{-1}\left(\frac{2 \tan\theta}{1 + \tan^2\theta}\right) \sec^2\theta \, d\theta \\ &= \int_0^{\frac{\pi}{4}} \sin^{-1}(\sin 2\theta) \sec^2\theta \, d\theta \\ &= \int_0^{\frac{\pi}{4}} 2\theta \cdot \sec^2\theta \, d\theta \\ &= 2 \int_0^{\frac{\pi}{4}} \theta \cdot \sec^2\theta \, d\theta \end{aligned}$$

Taking  $\theta$  as first function and  $\sec^2 \theta$  as second function and integrating by parts, we obtain

$$\begin{aligned}
 I &= 2 \left[ \theta \int \sec^2 \theta d\theta - \int \left( \frac{d}{dx} \theta \right) \int \sec^2 \theta d\theta \right]_0^{\frac{\pi}{4}} \\
 &= 2 \left[ \theta \tan \theta - \int \tan \theta d\theta \right]_0^{\frac{\pi}{4}} \\
 &= 2 \left[ \theta \tan \theta + \log |\cos \theta| \right]_0^{\frac{\pi}{4}} \\
 &= 2 \left[ \frac{\pi}{4} \tan \frac{\pi}{4} + \log \left| \cos \frac{\pi}{4} \right| - \log |\cos 0| \right] \\
 &= 2 \left[ \frac{\pi}{4} + \log \left( \frac{1}{\sqrt{2}} \right) - \log 1 \right] \\
 &= 2 \left[ \frac{\pi}{4} - \frac{1}{2} \log 2 \right] \\
 &= \frac{\pi}{2} - \log 2
 \end{aligned}$$

**Question 4:**

$$\int_0^2 x\sqrt{x+2} \quad (\text{Put } x+2 = t^2)$$

**ANS:**

$$\text{Let } x+2 = t^2 \Rightarrow dx = 2tdt$$

$$\text{When } x=0, t = \sqrt{2} \text{ and when } x=2, t=2$$

$$\begin{aligned}
 \therefore \int_0^2 x\sqrt{x+2} dx &= \int_{\sqrt{2}}^2 (t^2 - 2)\sqrt{t^2} 2tdt \\
 &= 2 \int_{\sqrt{2}}^2 (t^2 - 2)t^2 dt \\
 &= 2 \int_{\sqrt{2}}^2 (t^4 - 2t^2) dt \\
 &= 2 \left[ \frac{t^5}{5} - \frac{2t^3}{3} \right]_{\sqrt{2}}^2 \\
 &= 2 \left[ \frac{32}{5} - \frac{16}{3} - \frac{4\sqrt{2}}{5} + \frac{4\sqrt{2}}{3} \right] \\
 &= 2 \left[ \frac{96 - 80 - 12\sqrt{2} + 20\sqrt{2}}{15} \right] \\
 &= 2 \left[ \frac{16 + 8\sqrt{2}}{15} \right] \\
 &= \frac{16(2 + \sqrt{2})}{15} \\
 &= \frac{16\sqrt{2}(\sqrt{2} + 1)}{15}
 \end{aligned}$$

Question 5:

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos^2 x} dx$$

**ANS:**

$$\text{Let } \cos x = t \Rightarrow -\sin x \, dx = dt$$

$$\text{When } x = 0, t = 1 \text{ and when } x = \frac{\pi}{2}, t = 0$$

$$\begin{aligned} \Rightarrow \int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos^2 x} dx &= - \int_1^0 \frac{dt}{1+t^2} \\ &= - \left[ \tan^{-1} t \right]_1^0 \\ &= - \left[ \tan^{-1} 0 - \tan^{-1} 1 \right] \\ &= - \left[ -\frac{\pi}{4} \right] \\ &= \frac{\pi}{4} \end{aligned}$$

Question 6:

$$\int_0^2 \frac{dx}{x+4-x^2}$$

**ANS:**

$$\begin{aligned} \int_0^2 \frac{dx}{x+4-x^2} &= \int_0^2 \frac{dx}{-(x^2-x-4)} \\ &= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)} \\ &= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]} \\ &= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2} \end{aligned}$$

$$\text{Let } x - \frac{1}{2} = t \Rightarrow dx = dt$$

$$\text{When } x = 0, t = -\frac{1}{2} \text{ and when } x = 2, t = \frac{3}{2}$$

$$\begin{aligned} \therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2} &= \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2 - t^2} \\ &= \left[ \frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2} + t}{\frac{\sqrt{17}}{2} - t} \right]_{-\frac{1}{2}}^{\frac{3}{2}} \\ &= \frac{1}{\sqrt{17}} \left[ \log \frac{\frac{\sqrt{17}}{2} + \frac{3}{2}}{\frac{\sqrt{17}}{2} - \frac{3}{2}} - \log \frac{\frac{\sqrt{17}}{2} - \frac{1}{2}}{\frac{\sqrt{17}}{2} + \frac{1}{2}} \right] \\ &= \frac{1}{\sqrt{17}} \left[ \log \frac{\sqrt{17} + 3}{\sqrt{17} - 3} - \log \frac{\sqrt{17} - 1}{\sqrt{17} + 1} \right] \\ &= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17} + 3}{\sqrt{17} - 3} \times \frac{\sqrt{17} + 1}{\sqrt{17} - 1} \\ &= \frac{1}{\sqrt{17}} \log \left[ \frac{17 + 3 + 4\sqrt{17}}{17 + 3 - 4\sqrt{17}} \right] \\ &= \frac{1}{\sqrt{17}} \log \left[ \frac{20 + 4\sqrt{17}}{20 - 4\sqrt{17}} \right] \\ &= \frac{1}{\sqrt{17}} \log \left( \frac{5 + \sqrt{17}}{5 - \sqrt{17}} \right) \end{aligned}$$

$$= \frac{1}{\sqrt{17}} \log \left[ \frac{(5 + \sqrt{17})(5 + \sqrt{17})}{25 - 17} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left[ \frac{25 + 17 + 10\sqrt{17}}{8} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left( \frac{42 + 10\sqrt{17}}{8} \right)$$

$$= \frac{1}{\sqrt{17}} \log \left( \frac{21 + 5\sqrt{17}}{4} \right)$$

Question 7:

$$\int_{-1}^1 \frac{dx}{x^2 + 2x + 5}$$

**ANS:**

$$\int_{-1}^1 \frac{dx}{x^2 + 2x + 5} = \int_{-1}^1 \frac{dx}{(x^2 + 2x + 1) + 4} = \int_{-1}^1 \frac{dx}{(x+1)^2 + (2)^2}$$

$$\text{Let } x + 1 = t \Rightarrow dx = dt$$

When  $x = -1$ ,  $t = 0$  and when  $x = 1$ ,  $t = 2$

$$\begin{aligned} \therefore \int_{-1}^1 \frac{dx}{(x+1)^2 + (2)^2} &= \int_0^2 \frac{dt}{t^2 + 2^2} \\ &= \left[ \frac{1}{2} \tan^{-1} \frac{t}{2} \right]_0^2 \\ &= \frac{1}{2} \tan^{-1} 1 - \frac{1}{2} \tan^{-1} 0 \\ &= \frac{1}{2} \left( \frac{\pi}{4} \right) = \frac{\pi}{8} \end{aligned}$$

Question 8:

$$\int_1^2 \left( \frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx$$

**ANS:**

$$\int_1^2 \left( \frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx$$

$$\text{Let } 2x = t \Rightarrow 2dx = dt$$

When  $x = 1$ ,  $t = 2$  and when  $x = 2$ ,  $t = 4$

$$\begin{aligned}\therefore \int_2^4 \left( \frac{1}{x} - \frac{1}{2x^2} \right) e^{2x} dx &= \frac{1}{2} \int_2^4 \left( \frac{2}{t} - \frac{2}{t^2} \right) e^t dt \\ &= \int_2^4 \left( \frac{1}{t} - \frac{1}{t^2} \right) e^t dt\end{aligned}$$

Let  $\frac{1}{t} = f(t)$

Then,  $f'(t) = -\frac{1}{t^2}$

$$\begin{aligned}\Rightarrow \int_2^4 \left( \frac{1}{t} - \frac{1}{t^2} \right) e^t dt &= \int_2^4 e^t [f(t) + f'(t)] dt \\ &= [e^t f(t)]_2^4 \\ &= \left[ e^t \cdot \frac{2}{t} \right]_2^4 \\ &= \left[ \frac{e^t}{t} \right]_2^4 \\ &= \frac{e^4}{4} - \frac{e^2}{2} \\ &= \frac{e^2(e^2 - 2)}{4}\end{aligned}$$

**Question 9:**

The value of the integral  $\int_{\frac{1}{3}}^1 \frac{(x-x^3)^{\frac{1}{3}}}{x^4} dx$  is

- A. 6
- B. 0
- C. 3
- D. 4

**ANS:**

Let  $I = \int_{\frac{1}{3}}^1 \frac{(x-x^3)^{\frac{1}{3}}}{x^4} dx$

Also, let  $x = \sin \theta \Rightarrow dx = \cos \theta d\theta$

When  $x = \frac{1}{3}$ ,  $\theta = \sin^{-1}\left(\frac{1}{3}\right)$  and when  $x = 1$ ,  $\theta = \frac{\pi}{2}$

$$\begin{aligned}
\Rightarrow I &= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin \theta - \sin^3 \theta)^{\frac{1}{3}}}{\sin^4 \theta} \cos \theta d\theta \\
&= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin \theta)^{\frac{1}{3}} (1 - \sin^2 \theta)^{\frac{1}{3}}}{\sin^4 \theta} \cos \theta d\theta \\
&= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin \theta)^{\frac{1}{3}} (\cos \theta)^{\frac{2}{3}}}{\sin^4 \theta} \cos \theta d\theta \\
&= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin \theta)^{\frac{1}{3}} (\cos \theta)^{\frac{2}{3}}}{\sin^2 \theta \sin^2 \theta} \cos \theta d\theta \\
&= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\cos \theta)^{\frac{5}{3}}}{(\sin \theta)^{\frac{5}{3}}} \operatorname{cosec}^2 \theta d\theta \\
&= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} (\cot \theta)^{\frac{5}{3}} \operatorname{cosec}^2 \theta d\theta
\end{aligned}$$

Let  $\cot \theta = t \Rightarrow -\operatorname{cosec}^2 \theta d\theta = dt$

When  $\theta = \sin^{-1}\left(\frac{1}{3}\right)$ ,  $t = 2\sqrt{2}$  and when  $\theta = \frac{\pi}{2}$ ,  $t = 0$

$$\begin{aligned}
\therefore I &= - \int_{2\sqrt{2}}^0 (t)^{\frac{5}{3}} dt \\
&= - \left[ \frac{3}{8} (t)^{\frac{8}{3}} \right]_{2\sqrt{2}}^0 \\
&= - \frac{3}{8} \left[ (t)^{\frac{8}{3}} \right]_{2\sqrt{2}}^0 \\
&= - \frac{3}{8} \left[ -(2\sqrt{2})^{\frac{8}{3}} \right] \\
&= \frac{3}{8} \left[ (\sqrt{8})^{\frac{8}{3}} \right] \\
&= \frac{3}{8} \left[ (8)^{\frac{4}{3}} \right] \\
&= \frac{3}{8} [16] \\
&= 3 \times 2 \\
&= 6
\end{aligned}$$

Hence, the correct answer is A.



Question 10:

If  $f(x) = \int_0^x t \sin t \, dt$ , then  $f'(x)$  is

A.  $\cos x + x \sin x$

B.  $x \sin x$

C.  $x \cos x$

D.  $\sin x + x \cos x$

**ANS:**

$$f(x) = \int_0^x t \sin t \, dt$$

Integrating by parts, we obtain

$$\begin{aligned} f(x) &= t \int_0^x \sin t \, dt - \int_0^x \left\{ \left( \frac{d}{dt} t \right) \int \sin t \, dt \right\} dt \\ &= [t(-\cos t)]_0^x - \int_0^x (-\cos t) \, dt \\ &= [-t \cos t + \sin t]_0^x \\ &= -x \cos x + \sin x \end{aligned}$$

$$\begin{aligned} \Rightarrow f'(x) &= -[x(-\sin x)] + \cos x \\ &= x \sin x - \cos x + \cos x \\ &= x \sin x \end{aligned}$$

Hence, the correct answer is B.

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