

# CLASS XII LINEAR PROGRAMMING CHAPTER 12

## EX. 12.1 SOLUTIONS

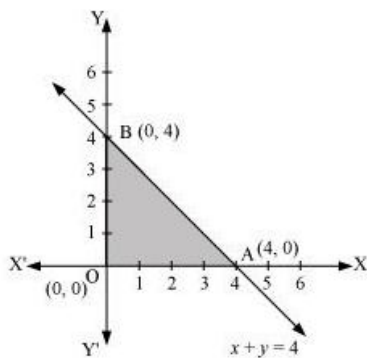
### Question 1:

Maximise  $Z = 3x + 4y$

Subject to the constraints:  $x + y \leq 4, x \geq 0, y \geq 0$ .

ANS :

The feasible region determined by the constraints,  $x + y \leq 4, x \geq 0, y \geq 0$ , is as follows.



The corner points of the feasible region are O (0, 0), A (4, 0), and B (0, 4). The values of Z at these points are as follows.

Corner point	$Z = 3x + 4y$
O(0, 0)	0
A(4, 0)	12
B(0, 4)	16

→ Maximum

Therefore, the maximum value of Z is 16 at the point B (0, 4).

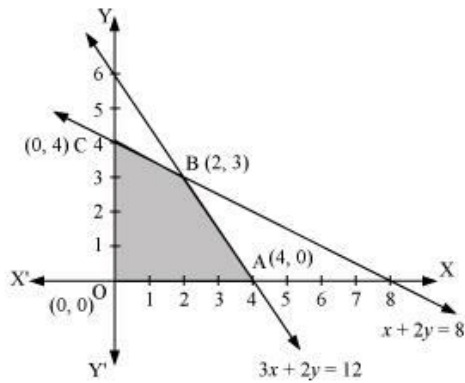
### Question 2:

Minimise  $Z = -3x + 4y$

subject to  $x + 2y \leq 8, 3x + 2y \leq 12, x \geq 0, y \geq 0$ .

ANS :

The feasible region determined by the system of constraints,  $x + 2y \leq 8$ ,  $3x + 2y \leq 12$ ,  $x \geq 0$ , and  $y \geq 0$ , is as follows.



The corner points of the feasible region are O (0, 0), A (4, 0), B (2, 3), and C (0, 4).

The values of Z at these corner points are as follows.

Corner point	$Z = -3x + 4y$	
O(0, 0)	0	→ Minimum
A(4, 0)	-12	
B(2, 3)	6	
C(0, 4)	16	

Therefore, the minimum value of Z is -12 at the point (4, 0).

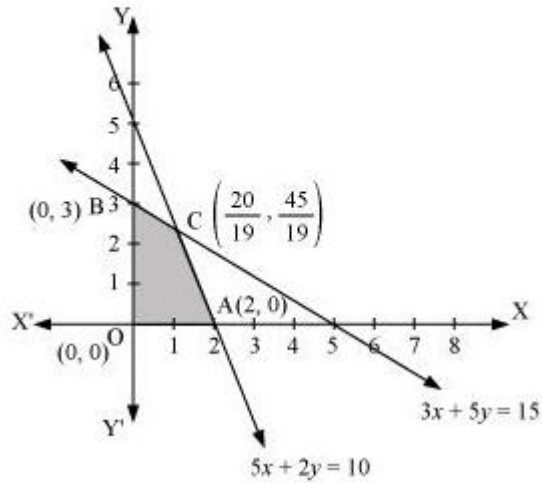
### Question 3:

Maximise  $Z = 5x + 3y$

subject to  $3x + 5y \leq 15$ ,  $5x + 2y \leq 10$ ,  $x \geq 0$ ,  $y \geq 0$ .

ANS :

The feasible region determined by the system of constraints,  $3x + 5y \leq 15$ ,  $5x + 2y \leq 10$ ,  $x \geq 0$ , and  $y \geq 0$ , are as follows.



The corner points of the feasible region are  $O (0, 0)$ ,  $A (2, 0)$ ,  $B (0, 3)$ , and  $C \left( \frac{20}{19}, \frac{45}{19} \right)$ .

The values of  $Z$  at these corner points are as follows.

Corner point	$Z = 5x + 3y$	
$O(0, 0)$	0	
$A(2, 0)$	10	
$B(0, 3)$	9	
$C \left( \frac{20}{19}, \frac{45}{19} \right)$	$\frac{235}{19}$	→ Maximum

Therefore, the maximum value of  $Z$  is  $\frac{235}{19}$  at the point  $\left( \frac{20}{19}, \frac{45}{19} \right)$ .

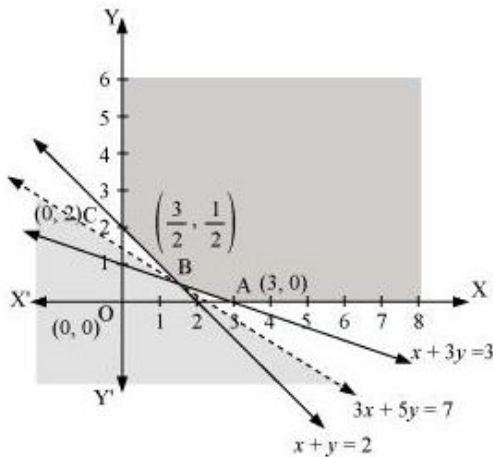
#### Question 4:

Minimise  $Z = 3x + 5y$

such that  $x + 3y \geq 3$ ,  $x + y \geq 2$ ,  $x, y \geq 0$ .

ANS :

The feasible region determined by the system of constraints,  $x + 3y \geq 3$ ,  $x + y \geq 2$ , and  $x, y \geq 0$ , is as follows.



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are A (3, 0), B  $\left(\frac{3}{2}, \frac{1}{2}\right)$ , and C (0, 2).

The values of Z at these corner points are as follows.

Corner point	$Z = 3x + 5y$	
A(3, 0)	9	
B $\left(\frac{3}{2}, \frac{1}{2}\right)$	7	→ Smallest
C(0, 2)	10	

As the feasible region is unbounded, therefore, 7 may or may not be the minimum value of Z.

For this, we draw the graph of the inequality,  $3x + 5y < 7$ , and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with  $3x + 5y < 7$  Therefore, the minimum value of Z is 7 at  $\left(\frac{3}{2}, \frac{1}{2}\right)$ .

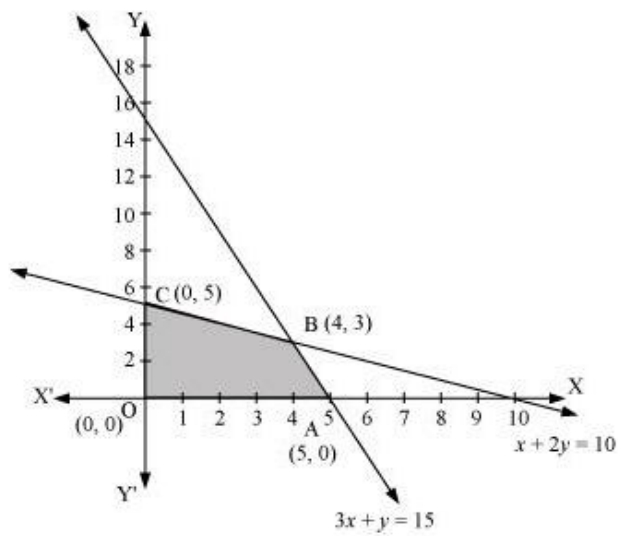
#### Question 5:

Maximise  $Z = 3x + 2y$

subject to  $x + 2y \leq 10$ ,  $3x + y \leq 15$ ,  $x, y \geq 0$ .

ANS :

The feasible region determined by the constraints,  $x + 2y \leq 10$ ,  $3x + y \leq 15$ ,  $x \geq 0$ , and  $y \geq 0$ , is as follows.



The corner points of the feasible region are A (5, 0), B (4, 3), and C (0, 5).

The values of Z at these corner points are as follows.

Corner point	$Z = 3x + 2y$	
A(5, 0)	15	→ Maximum
B(4, 3)	18	
C(0, 5)	10	

Therefore, the maximum value of Z is 18 at the point (4, 3).

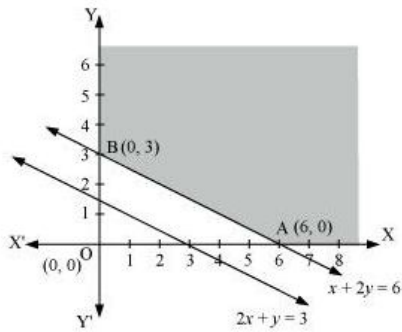
**Question 6:**

Minimise  $Z = x + 2y$

subject to  $2x + y \geq 3$ ,  $x + 2y \geq 6$ ,  $x, y \geq 0$ .

**ANS :**

The feasible region determined by the constraints,  $2x + y \geq 3$ ,  $x + 2y \geq 6$ ,  $x \geq 0$ , and  $y \geq 0$ , is as follows.



The corner points of the feasible region are A (6, 0) and B (0, 3).

The values of Z at these corner points are as follows.

Corner point	$Z = x + 2y$
A(6, 0)	6
B(0, 3)	6

It can be seen that the value of Z at points A and B is same. If we take any other point such as (2, 2) on line  $x + 2y = 6$ , then  $Z = 6$

Thus, the minimum value of Z occurs for more than 2 points.

Therefore, the value of Z is minimum at every point on the line,  $x + 2y = 6$

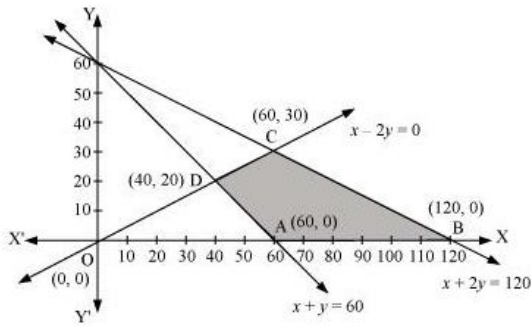
### Question 7:

Minimise and Maximise  $Z = 5x + 10y$

subject to  $x + 2y \leq 120$ ,  $x + y \geq 60$ ,  $x - 2y \geq 0$ ,  $x, y \geq 0$ .

ANS :

The feasible region determined by the constraints,  $x + 2y \leq 120$ ,  $x + y \geq 60$ ,  $x - 2y \geq 0$ ,  $x \geq 0$ , and  $y \geq 0$ , is as follows.



The corner points of the feasible region are A (60, 0), B (120, 0), C (60, 30), and D (40, 20).

The values of Z at these corner points are as follows.

Corner point	$Z = 5x + 10y$	
A(60, 0)	300	→ Minimum
B(120, 0)	600	→ Maximum
C(60, 30)	600	→ Maximum
D(40, 20)	400	

The minimum value of Z is 300 at (60, 0) and the maximum value of Z is 600 at all the points on the line segment joining (120, 0) and (60, 30).

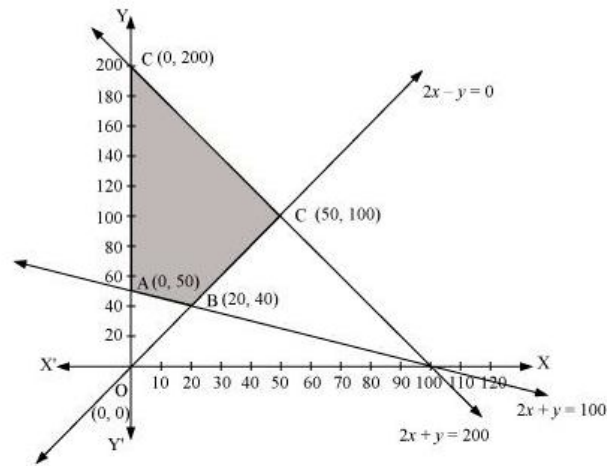
### Question 8:

Minimise and Maximise  $Z = x + 2y$

subject to  $x + 2y \geq 100$ ,  $2x - y \leq 0$ ,  $2x + y \leq 200$ ;  $x, y \geq 0$ .

ANS :

The feasible region determined by the constraints,  $x + 2y \geq 100$ ,  $2x - y \leq 0$ ,  $2x + y \leq 200$ ,  $x \geq 0$ , and  $y \geq 0$ , is as follows.



The corner points of the feasible region are A(0, 50), B(20, 40), C(50, 100), and D(0, 200).

The values of  $Z$  at these corner points are as follows.

Corner point	$Z = x + 2y$	
A(0, 50)	100	→ Minimum
B(20, 40)	100	→ Minimum
C(50, 100)	250	
D(0, 200)	400	→ Maximum

The maximum value of  $Z$  is 400 at (0, 200) and the minimum value of  $Z$  is 100 at all the points on the line segment joining the points (0, 50) and (20, 40).

### Question 9:

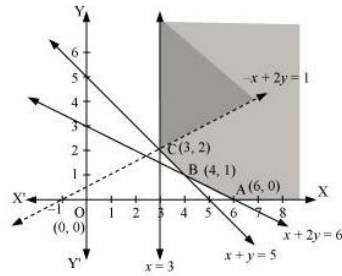
Maximise  $Z = -x + 2y$ , subject to the constraints:

$$x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0.$$

ANS :



The feasible region determined by the constraints,  $x \geq 3$ ,  $x + y \geq 5$ ,  $x + 2y \geq 6$ , and  $y \geq 0$ , is as follows.



It can be seen that the feasible region is unbounded.

The values of  $Z$  at corner points  $A(6, 0)$ ,  $B(4, 1)$ , and  $C(3, 2)$  are as follows.

Corner point	$Z = -x + 2y$
$A(6, 0)$	$Z = -6$
$B(4, 1)$	$Z = -2$
$C(3, 2)$	$Z = 1$

As the feasible region is unbounded, therefore,  $Z = 1$  may or may not be the maximum value.

For this, we graph the inequality,  $-x + 2y > 1$ , and check whether the resulting half plane has points in common with the feasible region or not.

The resulting feasible region has points in common with the feasible region.

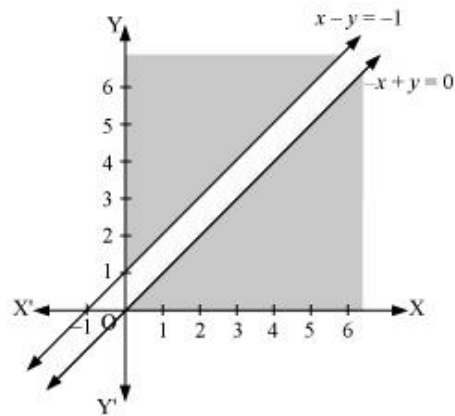
Therefore,  $Z = 1$  is not the maximum value.  $Z$  has no maximum value.

**Question 10:**

Maximise  $Z = x + y$ , subject to  $x - y \leq -1$ ,  $-x + y \leq 0$ ,  $x, y \geq 0$ .

**ANS :**

The region determined by the constraints,  $x - y \leq -1$ ,  $-x + y \leq 0$ ,  $x, y \geq 0$ , is as follows.



There is no feasible region and thus,  $Z$  has no maximum value.

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