ASSIGNMENT CLASS X TRIGONOMETRY

Q1. In ΔABC, right angled at B, if AB = 12 cm and BC = 5 cm, find (i) sin A and tan A (ii) sin C and cot C.

Q2. Given $\cot\theta = \frac{20}{21}$ find all other trigonometric ratios.

Q3. If $\cos A = \frac{12}{13}$ verify that: $\sin A(1 - \tan A) = \frac{35}{156}$.

Q4. (i) If $7\cot\theta = 24$, prove that $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1}{7}$ (ii) If $4\cot\theta = 5$, show that: $\frac{5\sin\theta + 3\cos\theta}{5\sin\theta - 2\cos\theta} = \frac{7}{2}$.

Q6. If 21cosec θ = 29, find the value of : (i) $\frac{\cos^2\theta - \sin^2\theta}{1 - 2\sin^2\theta}$ (ii) $\frac{2\cos^2\theta - 1}{\cos^2\theta - \sin^2\theta}$

Q7. If $\tan\theta + \frac{1}{\tan\theta} = 2$; show that: $\tan^2\theta + \frac{1}{\tan^2\theta} = 2$.

Q8. Evaluate each of the following :

(i) $2\cos^2 60 \cot 30° + 6\sin^2 30° \operatorname{cosec}^2 60°$ (ii) $\frac{5\sin^2 30° + \cos^2 45° - 4\tan^2 30°}{2\sin 30°\cos 30° + \tan 45°}$

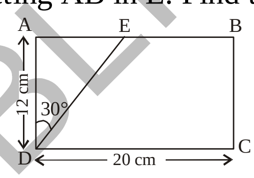
(iii) $2(\cos^2 45° + \tan^2 60°) - 6(\sin^2 45° - \tan^2 30°)$ (iv) $\frac{\tan^2 60° + 3\sec^2 30° + 4\cos^2 45° + 5\cos^2 90°}{\operatorname{cosec} 30° + \sec 60° - \cot^2 30°}$

Q9. If θ = 30°, verify that : (i) $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ (ii) $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ (iii) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Q10. Given that sin (A + B) = sin A cos B + cos A sin B, find the value of sin 75°.

Q11. If $\sin(A+2B) = \frac{\sqrt{3}}{2}$ and cos (A + 4B) = 0, find the values of angles A and B.

Q12. ABCD is a rectangle with AD =12 cm and DC = 20 cm as shown. The line segment DE is drawn making an angle of 30° with AD, intersecting AB in E. Find the lengths of DE and AE.



Q13. Evaluate each of the following:

(i) $\frac{\cos^2 20° + \cos^2 70°}{\sin^2 57° + \sin^2 33°}$ (ii) $\left(\frac{\sin 27°}{\cos 63°}\right)^2 + \left(\frac{\cos 63°}{\sin 27°}\right)^2$ (iii) cot 12° cot 38° cot 52° cot 60° cot 78°

Q14. Prove that:

(i) $\frac{\sin\theta.\cos(90°-\theta)\cos\theta}{\sin(90°-\theta)} + \frac{\cos\theta\sin(90°-\theta).\sin\theta}{\cos(90°-\theta)} = 1$ (ii) $\frac{\cos(90°-\theta).\sec(90°-\theta).\tan\theta}{\operatorname{cosec}(90°-\theta)\sin(90°-\theta).\cot(90°-\theta)} + \frac{\tan(90°-\theta)}{\cot\theta} = 2$

Q15. Without using trigonometric tables, find the value of each of the following:

(i) $\cos(40°+\theta) - \sin(50°-\theta) + \frac{\cos^2 40° + \cos^2 50°}{\sin^2 40° + \sin^2 50°}$

(ii) $\sec^2 10° - \cot^2 80° + \frac{\sin 15°\cos 75° + \cos 15°\sin 75°}{\cos\theta\sin(90°-\theta) + \sin\theta.\cos(90°-\theta)}$

(iii) $\frac{-\tan\theta\cot(90°-\theta) + \sec\theta\operatorname{cosec}(90°-\theta) + \sin^2 35° + \sin^2 55°}{\tan 10°\tan 20°\tan 45°\tan 70°\tan 80°}$

(iv) $\left(\frac{\tan 20°}{\operatorname{cosec}70°}\right)^2 + \left(\frac{\cot 20°}{\sec 70°}\right)^2 + 2\tan 15°\tan 37°\tan 53°\tan 60°\tan 75°$

(v) $\frac{\sec 39°}{\operatorname{cosec} 51°} + \frac{2}{\sqrt{3}}.\tan 17°\tan 38°\tan 60°\tan 52°\tan 73° - 3(\sin^2 31° + \sin^2 59°)$

Q16. If sec 5θ = cosec $(\theta - 36°)$, where 5θ is an acute angle, find the value of θ.

Q17. Simplify the following expressions:

(i) $(1 + \cos\theta)(\text{cosec}\,\theta - \cot\theta)$ (ii) $\text{cosec}\,\theta\,(1+ \cos\theta)(\text{cosec}\,\theta - \cot\theta)$ (iii) $\frac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}$ (iv) $\frac{\sin^4 A-\cos^4 A}{\sin^2 A-\cos^2 A}$

Q18. Prove that following identities:

(i) $\text{cosec}^2\theta + \sec^2\theta = \text{cosec}^2\theta \cdot \sec^2\theta$ (ii) $2\sec^2\theta - \sec^4\theta - 2\,\text{cosec}^2\theta + \text{cosec}^4\theta = \cot^4\theta - \tan^4\theta$

(iii) $\frac{1+\tan^2\theta}{1+\cot^2\theta} = \left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2$ (iv) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1+\tan\theta+\cot\theta = 1+\sec\theta\,\text{cosec}\,\theta$

(v) $\sqrt{\frac{1-\sin A}{1+\sin A}} = \frac{\cos A}{1+\sin A}$ (vi) $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = 2\,\text{cosec}\,\theta$

(vii) $\frac{\sin\theta}{\cot\theta+\text{cosec}\,\theta} = 2 + \frac{\sin\theta}{\cot\theta-\text{cosec}\,\theta}$ (viii) $\left(1+\frac{1}{\tan^2 A}\right)\left(1+\frac{1}{\cot^2 A}\right) = \frac{1}{\sin^2 A-\sin^4 A}$

(ix) $\frac{\sin^2\theta}{1-\cot\theta} + \frac{\cos^3\theta}{\cos\theta-\sin\theta} = 1+\sin\theta\cos\theta$ (x) $\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta-\cos\theta} = 1+\sin\theta\cos\theta$

(xi) $\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \cot\theta$ (xii) $\frac{1}{\text{cosec}\,A-\cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\text{cosec}\,A+\cot A}$

(xiii) $\tan^2 A - \tan^2 B = \frac{\cos^2 B-\cos^2 A}{\cos^2 B.\cos^2 A} = \frac{\sin^2 A-\sin^2 B}{\cos^2 A\cos^2 B}$ (xiv) $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

(xv) $\frac{\sin\theta-\sin\alpha}{\cos\theta+\cos\alpha} + \frac{\cos\theta-\cos\alpha}{\sin\theta+\sin\alpha} = 0$ (xvi) $\frac{\cot A+\text{cosec}\,A-1}{\cot A-\text{cosec}\,A+1} = \frac{1+\cos A}{\sin A}$

Q19. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

Q20. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$, show that $q(p^2-1) = 2p$.

Q21. If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$, prove that $x^2 - y^2 = a^2 - b^2$.

Q22.(i) If $\sec\theta = x+\frac{1}{4x}$, prove that $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$. (ii) If $\sec\theta+\tan\theta = p$, prove that $\frac{p^2-1}{p^2+1} = \sin\theta$.

Q23. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, prove that $m^2 - n^2 = 4\sqrt{mn}$.

Q24. If $\sin\theta + \sin^2\theta = 1$, prove that $\cos^2\theta + \cos^4\theta = 1$

Q25. If $3\sin\theta+5\cos\theta = 5$, prove that $5\sin\theta-3\cos\theta = \pm 3$.

Q26. If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$, prove that $x^2 - y^2 = a^2 - b^2$.

Q27. If $a\cos\theta = x$ and $b\cot\theta = y$, show that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.

Q28. If $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta = m$ and $\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta = n$, prove that $\frac{x^2}{a^2}+\frac{y^2}{b^2} = m^2+n^2$.

ANSWERS

1. (i) $\frac{5}{13}, \frac{5}{12}$ (ii) $\frac{12}{13}, \frac{5}{12}$ 2. $\sin\theta = \frac{21}{29}, \cos\theta = \frac{20}{29}, \tan\theta = \frac{21}{20}, \text{cosec}\,\theta = \frac{29}{21}, \sec\theta = \frac{29}{20}$

6. (i) 1 (ii) 1 8. (i) $\frac{\sqrt{3}+4}{2}$ (ii) $\frac{5}{6}(2-\sqrt{3})$ (iii) 6 (iv) 9 10. $\frac{\sqrt{3}+1}{2\sqrt{2}}$

11. A = 30°, B = 15° 12. DE = $8\sqrt{3}$ cm, AE = $4\sqrt{3}$ cm

13. (i) 1 (ii) 2 (iii) $\frac{1}{\sqrt{3}}$ 15. (i) 1 (ii) 2 (iii) 2 (iv) $1+2\sqrt{3}$ (v) 0

16. 21° 17. (i) $\sin\theta$ (ii) 1 (iii) $1 - \sin\theta\cos\theta$ (iv) 1