

CLASS XII PROBABILITY CHAPTER 13

EX. 13.4 SOLUTIONS

Question 1:

State which of the following are **not** the probability distributions of a random variable. Give reasons for your answer.

(i)

X	0	1	2
P (X)	0.4	0.4	0.2

(ii)

X	0	1	2	3	4
P (X)	0.1	0.5	0.2	-0.1	0.3

(iii)

Y	-1	0	1
P (Y)	0.6	0.1	0.2

(iv)

Z	3	2	1	0	-1
P (Z)	0.3	0.2	0.4	0.1	0.05

ANS :

It is known that the sum of all the probabilities in a probability distribution is one.

(i) Sum of the probabilities = $0.4 + 0.4 + 0.2 = 1$

Therefore, the given table is a probability distribution of random variables.

(ii) It can be seen that for $X = 3$, $P (X) = -0.1$

It is known that probability of any observation is not negative. Therefore, the given table is not a probability distribution of random variables.

(iii) Sum of the probabilities = $0.6 + 0.1 + 0.2 = 0.9 \neq 1$

Therefore, the given table is not a probability distribution of random variables.

(iv) Sum of the probabilities = $0.3 + 0.2 + 0.4 + 0.1 + 0.05 = 1.05 \neq 1$

Therefore, the given table is not a probability distribution of random variables.

Question 2:

An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represents the number of black balls. What are the possible values of X ? Is X a random variable?

ANS :

The two balls selected can be represented as BB, BR, RB, RR, where B represents a black ball and R represents a red ball.

X represents the number of black balls.

$$\therefore X(\text{BB}) = 2$$

$$X(\text{BR}) = 1$$

$$X(\text{RB}) = 1$$

$$X(\text{RR}) = 0$$

Therefore, the possible values of X are 0, 1, and 2.

Yes, X is a random variable.

Question 3:

Let X represents the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X ?

ANS :

A coin is tossed six times and X represents the difference between the number of heads and the number of tails.

$$\therefore X(6 \text{ H}, 0 \text{ T}) = |6 - 0| = 6$$

$$X(5 \text{ H}, 1 \text{ T}) = |5 - 1| = 4$$

$$X(4 \text{ H}, 2 \text{ T}) = |4 - 2| = 2$$

$$X(3 \text{ H}, 3 \text{ T}) = |3 - 3| = 0$$

$$X(2 \text{ H}, 4 \text{ T}) = |2 - 4| = 2$$

$$X(1 \text{ H}, 5 \text{ T}) = |1 - 5| = 4$$

$$X(0 \text{ H}, 6 \text{ T}) = |0 - 6| = 6$$

Thus, the possible values of X are 6, 4, 2, and 0.

Question 4:

Find the probability distribution of

- (i) number of heads in two tosses of a coin
- (ii) number of tails in the simultaneous tosses of three coins
- (iii) number of heads in four tosses of a coin

ANS :

(i) When one coin is tossed twice, the sample space is

{HH, HT, TH, TT}

Let X represent the number of heads.

$\therefore X(\text{HH}) = 2, X(\text{HT}) = 1, X(\text{TH}) = 1, X(\text{TT}) = 0$

Therefore, X can take the value of 0, 1, or 2.

It is known that,

$$P(\text{HH}) = P(\text{HT}) = P(\text{TH}) = P(\text{TT}) = \frac{1}{4}$$

$$P(X = 0) = P(\text{TT}) = \frac{1}{4}$$

$$P(X = 1) = P(\text{HT}) + P(\text{TH}) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$P(X = 2) = P(\text{HH}) = \frac{1}{4}$$

Thus, the required probability distribution is as follows.

X	0	1	2
P (X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(ii) When three coins are tossed simultaneously, the sample space is {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Let X represent the number of tails.

It can be seen that X can take the value of 0, 1, 2, or 3.

$$P(X=0) = P(\text{HHH}) = \frac{1}{8}$$

$$P(X=1) = P(\text{HHT}) + P(\text{HTH}) + P(\text{THH}) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

$$P(X=2) = P(\text{HTT}) + P(\text{THT}) + P(\text{TTH}) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

$$P(X=3) = P(\text{TTT}) = \frac{1}{8}$$

Thus, the probability distribution is as follows.

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii) When a coin is tossed four times, the sample space is

$$S = \left\{ \begin{array}{l} \text{HHHH, HHHT, HHTH, HHTT, HTHT, HTHH, HTTH, HTTT,} \\ \text{TTHH, THTH, THTT, TTHH, TTHT, TTTT} \end{array} \right\}$$

Let X be the random variable, which represents the number of heads.

It can be seen that X can take the value of 0, 1, 2, 3, or 4.

$$P(X=0) = P(\text{TTTT}) = \frac{1}{16}$$

$$P(X=1) = P(\text{TTHH}) + P(\text{TTHT}) + P(\text{THTT}) + P(\text{HTTT})$$

$$= \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{4}{16} = \frac{1}{4}$$

$$P(X=2) = P(\text{HHTT}) + P(\text{THHT}) + P(\text{TTHH}) + P(\text{HTTH}) + P(\text{HTHT})$$

$$+ P(\text{THTH})$$

$$= \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{6}{16} = \frac{3}{8}$$

$$P(X=3) = P(\text{HHHT}) + P(\text{HHTH}) + P(\text{HTHH}) + P(\text{THHH})$$

$$= \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{4}{16} = \frac{1}{4}$$

$$P(X=4) = P(\text{HHHH}) = \frac{1}{16}$$

Thus, the probability distribution is as follows.

X	0	1	2	3	4
P(X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Question 5:

Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

- (i) number greater than 4
- (ii) six appears on at least one die

ANS :

When a die is tossed two times, we obtain $(6 \times 6) = 36$ number of observations.

Let X be the random variable, which represents the number of successes.

- i. Here, success refers to the number greater than 4.

$$P(X = 0) = P(\text{number less than or equal to 4 on both the tosses}) = \frac{4}{6} \times \frac{4}{6} = \frac{4}{9}$$

$P(X = 1) = P(\text{number less than or equal to 4 on first toss and greater than 4 on second toss}) + P(\text{number greater than 4 on first toss and less than or equal to 4 on second toss})$

$$= \frac{4}{6} \times \frac{2}{6} + \frac{4}{6} \times \frac{2}{6} = \frac{4}{9}$$

$P(X = 2) = P(\text{number greater than 4 on both the tosses})$

$$= \frac{2}{6} \times \frac{2}{6} = \frac{1}{9}$$

Thus, the probability distribution is as follows.

X	1	1	2
P (X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

- (ii) Here, success means six appears on at least one die.

$$P(Y = 0) = P(\text{six does not appear on any of the dice}) = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$$

$$P(Y = 1) = P(\text{six appears on at least one of the dice}) = \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} = \frac{5}{36} + \frac{5}{36} = \frac{10}{36}$$

Thus, the required probability distribution is as follows.

Y	0	1
P (Y)	$\frac{25}{36}$	$\frac{10}{36}$

Question 6:

From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

ANS :

It is given that out of 30 bulbs, 6 are defective.

$$\Rightarrow \text{Number of non-defective bulbs} = 30 - 6 = 24$$

4 bulbs are drawn from the lot with replacement.

Let X be the random variable that denotes the number of defective bulbs in the selected bulbs.

$$\therefore P(X=0) = P(4 \text{ non-defective and } 0 \text{ defective}) = {}^4C_0 \cdot \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} = \frac{256}{625}$$

$$P(X=1) = P(3 \text{ non-defective and } 1 \text{ defective}) = {}^4C_1 \cdot \left(\frac{1}{5}\right) \cdot \left(\frac{4}{5}\right)^3 = \frac{256}{625}$$

$$P(X=2) = P(2 \text{ non-defective and } 2 \text{ defective}) = {}^4C_2 \cdot \left(\frac{1}{5}\right)^2 \cdot \left(\frac{4}{5}\right)^2 = \frac{96}{625}$$

$$P(X=3) = P(1 \text{ non-defective and } 3 \text{ defective}) = {}^4C_3 \cdot \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right) = \frac{16}{625}$$

$$P(X=4) = P(0 \text{ non-defective and } 4 \text{ defective}) = {}^4C_4 \cdot \left(\frac{1}{5}\right)^4 \cdot \left(\frac{4}{5}\right)^0 = \frac{1}{625}$$

Therefore, the required probability distribution is as follows.

X	0	1	2	3	4
P(X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Question 7:

A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

ANS :

Let the probability of getting a tail in the biased coin be x .

$$\therefore P(T) = x$$

$$\Rightarrow P(H) = 3x$$

For a biased coin, $P(T) + P(H) = 1$

$$\Rightarrow x + 3x = 1$$

$$\Rightarrow 4x = 1$$

$$\Rightarrow x = \frac{1}{4}$$

$$\therefore P(T) = \frac{1}{4} \text{ and } P(H) = \frac{3}{4}$$

When the coin is tossed twice, the sample space is $\{HH, TT, HT, TH\}$.

Let X be the random variable representing the number of tails.

$$\therefore P(X=0) = P(\text{no tail}) = P(H) \times P(H) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

$$P(X=1) = P(\text{one tail}) = P(HT) + P(TH)$$

$$\begin{aligned}
 &= \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{3}{4} \\
 &= \frac{3}{16} + \frac{3}{16} \\
 &= \frac{3}{8}
 \end{aligned}$$

$$P(X=2) = P(\text{two tails}) = P(TT) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

Therefore, the required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

Question 8:

A random variable X has the following probability distribution.

X	0	1	2	3	4	5	6	7
P(X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Determine

- (i) k
- (ii) $P(X < 3)$
- (iii) $P(X > 6)$
- (iv) $P(0 < X < 3)$

ANS :

(i) It is known that the sum of probabilities of a probability distribution of random variables is one.

$$\begin{aligned}
 \therefore 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + (7k^2 + k) &= 1 \\
 \Rightarrow 10k^2 + 9k - 1 &= 0 \\
 \Rightarrow (10k - 1)(k + 1) &= 0 \\
 \Rightarrow k = -1, \frac{1}{10}
 \end{aligned}$$

$k = -1$ is not possible as the probability of an event is never negative.

$$\therefore k = \frac{1}{10}$$

(ii) $P(X < 3) = P(X=0) + P(X=1) + P(X=2)$

$$\begin{aligned}
 &= 0 + k + 2k \\
 &= 3k \\
 &= 3 \times \frac{1}{10} \\
 &= \frac{3}{10}
 \end{aligned}$$

$$(iii) P(X > 6) = P(X = 7)$$

$$= 7k^2 + k$$

$$= 7 \times \left(\frac{1}{10}\right)^2 + \frac{1}{10}$$

$$= \frac{7}{100} + \frac{1}{10}$$

$$= \frac{17}{100}$$

$$(iv) P(0 < X < 3) = P(X = 1) + P(X = 2)$$

$$= k + 2k$$

$$= 3k$$

$$= 3 \times \frac{1}{10}$$

$$= \frac{3}{10}$$

Question 9:

The random variable X has probability distribution $P(X)$ of the following form, where k is some number:

$$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) Determine the value of k .

(b) Find $P(X < 2)$, $P(X \geq 2)$, $P(X \geq 2)$.

ANS :

(a) It is known that the sum of probabilities of a probability distribution of random variables is one.

$$\therefore k + 2k + 3k + 0 = 1$$

$$\Rightarrow 6k = 1$$

$$\Rightarrow k = \frac{1}{6}$$

$$(b) P(X < 2) = P(X = 0) + P(X = 1)$$

$$= k + 2k$$

$$= 3k$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

$$\begin{aligned}
P(X \leq 2) &= P(X = 0) + P(X = 1) + P(X = 2) \\
&= k + 2k + 3k \\
&= 6k \\
&= \frac{6}{6} \\
&= 1
\end{aligned}$$

$$\begin{aligned}
P(X \geq 2) &= P(X = 2) + P(X > 2) \\
&= 3k + 0 \\
&= 3k \\
&= \frac{3}{6} \\
&= \frac{1}{2}
\end{aligned}$$

Question 10:

Find the mean number of heads in three tosses of a fair coin.

ANS :

Let X denote the success of getting heads.

Therefore, the sample space is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

It can be seen that X can take the value of 0, 1, 2, or 3.

$$\begin{aligned}
\therefore P(X = 0) &= P(TTT) \\
&= P(T) \cdot P(T) \cdot P(T) \\
&= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\
&= \frac{1}{8}
\end{aligned}$$

$$\therefore P(X = 1) = P(HHT) + P(HTH) + P(THH)$$

$$\begin{aligned}
&= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\
&= \frac{3}{8}
\end{aligned}$$

$$\therefore P(X = 2) = P(HHT) + P(HTH) + P(THH)$$

$$\begin{aligned}
&= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\
&= \frac{3}{8}
\end{aligned}$$

$$\begin{aligned}\therefore P(X=3) &= P(\text{HHH}) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8}\end{aligned}$$

Therefore, the required probability distribution is as follows.

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean of X $E(X)$, $\mu = \sum X_i P(X_i)$

$$\begin{aligned}&= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} \\ &= \frac{3}{8} + \frac{3}{4} + \frac{3}{8} \\ &= \frac{3}{2} \\ &= 1.5\end{aligned}$$

Question 11:

Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

ANS :

Here, X represents the number of sixes obtained when two dice are thrown simultaneously. Therefore, X can take the value of 0, 1, or 2.

$$\therefore P(X=0) = P(\text{not getting six on any of the dice}) = \frac{25}{36}$$

$P(X=1) = P(\text{six on first die and no six on second die}) + P(\text{no six on first die and six on second die})$

$$= 2 \left(\frac{1}{6} \times \frac{5}{6} \right) = \frac{10}{36}$$

$$P(X=2) = P(\text{six on both the dice}) = \frac{1}{36}$$

Therefore, the required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Then, expectation of X $= E(X) = \sum X_i P(X_i)$

$$\begin{aligned}&= 0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36} \\ &= \frac{1}{3}\end{aligned}$$

Question 12:

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denotes the larger of the two numbers obtained. Find $E(X)$.

ANS :

The two positive integers can be selected from the first six positive integers without replacement in $6 \times 5 = 30$ ways

X represents the larger of the two numbers obtained. Therefore, X can take the value of 2, 3, 4, 5, or 6.

For $X = 2$, the possible observations are (1, 2) and (2, 1).

$$\therefore P(X = 2) = \frac{2}{30} = \frac{1}{15}$$

For $X = 3$, the possible observations are (1, 3), (2, 3), (3, 1), and (3, 2).

$$\therefore P(X = 3) = \frac{4}{30} = \frac{2}{15}$$

For $X = 4$, the possible observations are (1, 4), (2, 4), (3, 4), (4, 3), (4, 2), and (4, 1).

$$\therefore P(X = 4) = \frac{6}{30} = \frac{1}{5}$$

For $X = 5$, the possible observations are (1, 5), (2, 5), (3, 5), (4, 5), (5, 4), (5, 3), (5, 2), and (5, 1).

$$\therefore P(X = 5) = \frac{8}{30} = \frac{4}{15}$$

For $X = 6$, the possible observations are (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 4), (6, 3), (6, 2), and (6, 1).

$$\therefore P(X = 6) = \frac{10}{30} = \frac{1}{3}$$

Therefore, the required probability distribution is as follows.

X	2	3	4	5	6
$P(X)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$

Then, $E(X) = \sum X_i P(X_i)$

$$\begin{aligned} &= 2 \cdot \frac{1}{15} + 3 \cdot \frac{2}{15} + 4 \cdot \frac{1}{5} + 5 \cdot \frac{4}{15} + 6 \cdot \frac{1}{3} \\ &= \frac{2}{15} + \frac{2}{5} + \frac{4}{5} + \frac{4}{3} + 2 \\ &= \frac{70}{15} \\ &= \frac{14}{3} \end{aligned}$$

Question 13:

Let X denotes the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X .

ANS :

When two fair dice are rolled, $6 \times 6 = 36$ observations are obtained.

$$P(X=2) = P(1, 1) = \frac{1}{36}$$

$$P(X=3) = P(1, 2) + P(2, 1) = \frac{2}{36} = \frac{1}{18}$$

$$P(X=4) = P(1, 3) + P(2, 2) + P(3, 1) = \frac{3}{36} = \frac{1}{12}$$

$$P(X=5) = P(1, 4) + P(2, 3) + P(3, 2) + P(4, 1) = \frac{4}{36} = \frac{1}{9}$$

$$P(X=6) = P(1, 5) + P(2, 4) + P(3, 3) + P(4, 2) + P(5, 1) = \frac{5}{36}$$

$$P(X=7) = P(1, 6) + P(2, 5) + P(3, 4) + P(4, 3) + P(5, 2) + P(6, 1) = \frac{6}{36} = \frac{1}{6}$$

$$P(X=8) = P(2, 6) + P(3, 5) + P(4, 4) + P(5, 3) + P(6, 2) = \frac{5}{36}$$

$$P(X=9) = P(3, 6) + P(4, 5) + P(5, 4) + P(6, 3) = \frac{4}{36} = \frac{1}{9}$$

$$P(X=10) = P(4, 6) + P(5, 5) + P(6, 4) = \frac{3}{36} = \frac{1}{12}$$

$$P(X=11) = P(5, 6) + P(6, 5) = \frac{2}{36} = \frac{1}{18}$$

$$P(X=12) = P(6, 6) = \frac{1}{36}$$

Therefore, the required probability distribution is as follows.

X	2	3	4	5	6	7	8	9	10	11	12
$P(X)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

$$\text{Then, } E(X) = \sum X_i \cdot P(X_i)$$

$$\begin{aligned} &= 2 \times \frac{1}{36} + 3 \times \frac{1}{18} + 4 \times \frac{1}{12} + 5 \times \frac{1}{9} + 6 \times \frac{5}{36} + 7 \times \frac{1}{6} \\ &\quad + 8 \times \frac{5}{36} + 9 \times \frac{1}{9} + 10 \times \frac{1}{12} + 11 \times \frac{1}{18} + 12 \times \frac{1}{36} \\ &= \frac{1}{18} + \frac{1}{6} + \frac{1}{3} + \frac{5}{9} + \frac{5}{6} + \frac{7}{6} + \frac{10}{9} + 1 + \frac{5}{6} + \frac{11}{18} + \frac{1}{3} \\ &= 7 \end{aligned}$$

$$\begin{aligned}
 E(X^2) &= \sum X_i^2 \cdot P(X_i) \\
 &= 4 \times \frac{1}{36} + 9 \times \frac{1}{18} + 16 \times \frac{1}{12} + 25 \times \frac{1}{9} + 36 \times \frac{5}{36} + 49 \times \frac{1}{6} \\
 &\quad + 64 \times \frac{5}{36} + 81 \times \frac{1}{9} + 100 \times \frac{1}{12} + 121 \times \frac{1}{18} + 144 \times \frac{1}{36} \\
 &= \frac{1}{9} + \frac{1}{2} + \frac{4}{3} + \frac{25}{9} + 5 + \frac{49}{6} + \frac{80}{9} + 9 + \frac{25}{3} + \frac{121}{18} + 4 \\
 &= \frac{987}{18} = \frac{329}{6} = 54.833
 \end{aligned}$$

$$\begin{aligned}
 \text{Then, Var}(X) &= E(X^2) - [E(X)]^2 \\
 &= 54.833 - (7)^2 \\
 &= 54.833 - 49 \\
 &= 5.833
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Standard deviation} &= \sqrt{\text{Var}(X)} \\
 &= \sqrt{5.833} \\
 &= 2.415
 \end{aligned}$$

Question 14:

A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X ? Find mean, variance and standard deviation of X .

ANS :

There are 15 students in the class. Each student has the same chance to be chosen. Therefore, the probability of each student to be selected is $\frac{1}{15}$.

The given information can be compiled in the frequency table as follows.

X	14	15	16	17	18	19	20	21
f	2	1	2	3	1	2	3	1

$$P(X=14) = \frac{2}{15}, P(X=15) = \frac{1}{15}, P(X=16) = \frac{2}{15}, P(X=17) = \frac{3}{15},$$

$$P(X=18) = \frac{1}{15}, P(X=19) = \frac{2}{15}, P(X=20) = \frac{3}{15}, P(X=21) = \frac{1}{15}$$

Therefore, the probability distribution of random variable X is as follows.

X	14	15	16	17	18	19	20	21
f	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Then, mean of $X = E(X)$

$$\begin{aligned} &= \sum X_i P(X_i) \\ &= 14 \times \frac{2}{15} + 15 \times \frac{1}{15} + 16 \times \frac{2}{15} + 17 \times \frac{3}{15} + 18 \times \frac{1}{15} + 19 \times \frac{2}{15} + 20 \times \frac{3}{15} + 21 \times \frac{1}{15} \\ &= \frac{1}{15} (28 + 15 + 32 + 51 + 18 + 38 + 60 + 21) \\ &= \frac{263}{15} \\ &= 17.53 \end{aligned}$$

$$E(X^2) = \sum X_i^2 P(X_i)$$

$$\begin{aligned} &= (14)^2 \cdot \frac{2}{15} + (15)^2 \cdot \frac{1}{15} + (16)^2 \cdot \frac{2}{15} + (17)^2 \cdot \frac{3}{15} + \\ &\quad (18)^2 \cdot \frac{1}{15} + (19)^2 \cdot \frac{2}{15} + (20)^2 \cdot \frac{3}{15} + (21)^2 \cdot \frac{1}{15} \\ &= \frac{1}{15} \cdot (392 + 225 + 512 + 867 + 324 + 722 + 1200 + 441) \\ &= \frac{4683}{15} \\ &= 312.2 \end{aligned}$$

$$\begin{aligned} \therefore \text{Variance}(X) &= E(X^2) - [E(X)]^2 \\ &= 312.2 - \left(\frac{263}{15}\right)^2 \\ &= 312.2 - 307.4177 \\ &= 4.7823 \\ &\approx 4.78 \end{aligned}$$

$$\begin{aligned} \text{Standard deviation} &= \sqrt{\text{Variance}(X)} \\ &= \sqrt{4.78} \\ &= 2.186 \approx 2.19 \end{aligned}$$

Question 15:

In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take $X = 0$ if he opposed, and $X = 1$ if he is in favour. Find $E(X)$ and $\text{Var}(X)$.

ANS :

It is given that $P(X = 0) = 30\% = \frac{30}{100} = 0.3$

$$P(X = 1) = 70\% = \frac{70}{100} = 0.7$$

Therefore, the probability distribution is as follows.

X	0	1
P(X)	0.3	0.7

$$\begin{aligned} \text{Then, } E(X) &= \sum X_i P(X_i) \\ &= 0 \times 0.3 + 1 \times 0.7 \\ &= 0.7 \end{aligned}$$

$$\begin{aligned} E(X^2) &= \sum X_i^2 P(X_i) \\ &= 0^2 \times 0.3 + (1)^2 \times 0.7 \\ &= 0.7 \end{aligned}$$

It is known that, $\text{Var}(X) = E(X^2) - [E(X)]^2$

$$= 0.7 - (0.7)^2$$

$$= 0.7 - 0.49$$

$$= 0.21$$

Question 16:

The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

(A) 1 (B) 2 (C) 5 (D) $\frac{8}{3}$

ANS :

Let X be the random variable representing a number on the die.

The total number of observations is six.

$$\therefore P(X = 1) = \frac{3}{6} = \frac{1}{2}$$

$$P(X = 2) = \frac{2}{6} = \frac{1}{3}$$

$$P(X = 5) = \frac{1}{6}$$

Therefore, the probability distribution is as follows.

X	1	2	5
P(X)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$$\text{Mean} = E(X) = \sum p_i x_i$$

$$= \frac{1}{2} \times 1 + \frac{1}{3} \times 2 + \frac{1}{6} \times 5$$

$$= \frac{1}{2} + \frac{2}{3} + \frac{5}{6}$$

$$= \frac{3+4+5}{6}$$

$$= \frac{12}{6}$$

$$= 2$$

The correct answer is B.

Question 17:

Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is

- (A) $\frac{37}{221}$ (B) $\frac{5}{13}$ (C) $\frac{1}{13}$ (D) $\frac{2}{13}$

ANS :

Let X denote the number of aces obtained. Therefore, X can take any of the values of 0, 1, or 2.

In a deck of 52 cards, 4 cards are aces. Therefore, there are 48 non-ace cards.

$$\therefore P(X=0) = P(0 \text{ ace and } 2 \text{ non-ace cards}) = \frac{{}^4C_0 \times {}^{48}C_2}{{}^{52}C_2} = \frac{1128}{1326}$$

$$P(X=1) = P(1 \text{ ace and } 1 \text{ non-ace cards}) = \frac{{}^4C_1 \times {}^{48}C_1}{{}^{52}C_2} = \frac{192}{1326}$$

$$P(X=2) = P(2 \text{ ace and } 0 \text{ non- ace cards}) = \frac{{}^4C_2 \times {}^{48}C_0}{{}^{52}C_2} = \frac{6}{1326}$$

Thus, the probability distribution is as follows.

X	0	1	2
P(X)	$\frac{1128}{1326}$	$\frac{192}{1326}$	$\frac{6}{1326}$

Then, $E(X) = \sum p_i x_i$

$$\begin{aligned} &= 0 \times \frac{1128}{1326} + 1 \times \frac{192}{1326} + 2 \times \frac{6}{1326} \\ &= \frac{204}{1326} \\ &= \frac{2}{13} \end{aligned}$$

Therefore, the correct answer is D.