

CLASS XII PROBABILITY CHAPTER 13

EX. 13.1 SOLUTIONS

Question 1:

Given that E and F are events such that $P(E) = 0.6$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$, find $P(E|F)$ and $P(F|E)$.

ANS :

It is given that $P(E) = 0.6$, $P(F) = 0.3$, and $P(E \cap F) = 0.2$

$$\Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

$$\Rightarrow P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3}$$

Question 2:

Compute $P(A|B)$, if $P(B) = 0.5$ and $P(A \cap B) = 0.32$

ANS :

It is given that $P(B) = 0.5$ and $P(A \cap B) = 0.32$

$$\Rightarrow P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{16}{25}$$

Question 3:

If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$, find

(i) $P(A \cap B)$ (ii) $P(A|B)$ (iii) $P(A \cup B)$

ANS :

It is given that $P(A) = 0.8$, $P(B) = 0.5$, and $P(B|A) = 0.4$

(i) $P(B|A) = 0.4$

$$\therefore \frac{P(A \cap B)}{P(A)} = 0.4$$

$$\Rightarrow \frac{P(A \cap B)}{0.8} = 0.4$$

$$\Rightarrow P(A \cap B) = 0.32$$

$$(ii) P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A|B) = \frac{0.32}{0.5} = 0.64$$

(iii)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = 0.5 + 0.5 - 0.32 = 0.98$$

Question 4:

Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$

ANS :

$$\text{It is given that, } 2P(A) = P(B) = \frac{5}{13}$$

$$\Rightarrow P(A) = \frac{5}{26} \text{ and } P(B) = \frac{5}{13}$$

$$P(A|B) = \frac{2}{5}$$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{2}{5}$$

$$\Rightarrow P(A \cap B) = \frac{2}{5} \times P(B) = \frac{2}{5} \times \frac{5}{13} = \frac{2}{13}$$

It is known that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow P(A \cup B) = \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$$

$$\Rightarrow P(A \cup B) = \frac{5+10-4}{26}$$

$$\Rightarrow P(A \cup B) = \frac{11}{26}$$

Question 5:

If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, find

(i) $P(A \cap B)$ (ii) $P(A|B)$ (iii) $P(B|A)$

ANS :

It is given that $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$, and $P(A \cup B) = \frac{7}{11}$

$$(i) P(A \cup B) = \frac{7}{11}$$

$$\therefore P(A) + P(B) - P(A \cap B) = \frac{7}{11}$$

$$\Rightarrow \frac{6}{11} + \frac{5}{11} - P(A \cap B) = \frac{7}{11}$$

$$\Rightarrow P(A \cap B) = \frac{11}{11} - \frac{7}{11} = \frac{4}{11}$$

(ii) It is known that, $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$\Rightarrow P(A|B) = \frac{\frac{4}{11}}{\frac{5}{11}} = \frac{4}{5}$$

(iii) It is known that, $P(B|A) = \frac{P(A \cap B)}{P(A)}$

$$\Rightarrow P(B|A) = \frac{\frac{4}{11}}{\frac{6}{11}} = \frac{4}{6} = \frac{2}{3}$$

Question 6:

A coin is tossed three times, where

(i) E: head on third toss, F: heads on first two tosses

(ii) E: at least two heads, F: at most two heads

(iii) E: at most two tails, F: at least one tail

ANS :

If a coin is tossed three times, then the sample space S is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

It can be seen that the sample space has 8 elements.

(i) $E = \{HHH, HTH, THH, TTH\}$

$$F = \{HHH, HHT\}$$

$$\therefore E \cap F = \{HHH\}$$

$$P(F) = \frac{2}{8} = \frac{1}{4} \text{ and } P(E \cap F) = \frac{1}{8}$$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{4}{8} = \frac{1}{2}$$

(ii) $E = \{HHH, HHT, HTH, THH\}$

$$F = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$$\therefore E \cap F = \{HHT, HTH, THH\}$$

Clearly, $P(E \cap F) = \frac{3}{8}$ and $P(F) = \frac{7}{8}$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{3}{8}}{\frac{7}{8}} = \frac{3}{7}$$

$$(iii) E = \{HHH, HHT, HTT, HTH, THH, THT, TTH\}$$

$$F = \{HHT, HTT, HTH, THH, THT, TTH, TTT\}$$

$$\therefore E \cap F = \{HHT, HTT, HTH, THH, THT, TTH\}$$

$$P(F) = \frac{7}{8} \text{ and } P(E \cap F) = \frac{6}{8}$$

$$\text{Therefore, } P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{6}{8}}{\frac{7}{8}} = \frac{6}{7}$$

Question 7:

Two coins are tossed once, where

(i) E: tail appears on one coin, F: one coin shows head

(ii) E: not tail appears, F: no head appears

ANS :

If two coins are tossed once, then the sample space S is

$$S = \{HH, HT, TH, TT\}$$

$$(i) E = \{HT, TH\}$$

$$F = \{HT, TH\}$$

$$\therefore E \cap F = \{HT, TH\}$$

$$P(F) = \frac{2}{4} = \frac{1}{2}$$

$$P(E \cap F) = \frac{2}{4} = \frac{1}{2}$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

$$(ii) E = \{HH\}$$

$$F = \{TT\}$$

$$\therefore E \cap F = \emptyset$$

$$P(F) = \frac{1}{4} \text{ and } P(E \cap F) = 0$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{\frac{1}{4}} = 0$$

Question 8:

A die is thrown three times,

E: 4 appears on the third toss, F: 6 and 5 appears respectively on first two tosses

ANS :

If a die is thrown three times, then the number of elements in the sample space will be $6 \times 6 \times 6 = 216$

$$E = \left\{ \begin{array}{l} (1,1,4), (1,2,4), \dots, (1,6,4) \\ (2,1,4), (2,2,4), \dots, (2,6,4) \\ (3,1,4), (3,2,4), \dots, (3,6,4) \\ (4,1,4), (4,2,4), \dots, (4,6,4) \\ (5,1,4), (5,2,4), \dots, (5,6,4) \\ (6,1,4), (6,2,4), \dots, (6,6,4) \end{array} \right\}$$

$$F = \{(6,5,1), (6,5,2), (6,5,3), (6,5,4), (6,5,5), (6,5,6)\}$$

$$\therefore E \cap F = \{(6,5,4)\}$$

$$P(F) = \frac{6}{216} \text{ and } P(E \cap F) = \frac{1}{216}$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{216}}{\frac{6}{216}} = \frac{1}{6}$$

Question 9:

Mother, father and son line up at random for a family picture

E: son on one end, F: father in middle

ANS :

If mother (M), father (F), and son (S) line up for the family picture, then the sample space will be

$$S = \{MFS, MSF, FMS, FSM, SMF, SFM\}$$

$$\Rightarrow E = \{MFS, FMS, SMF, SFM\}$$

$$F = \{MFS, SFM\}$$

$$\therefore E \cap F = \{MFS, SFM\}$$

$$P(E \cap F) = \frac{2}{6} = \frac{1}{3}$$

$$P(F) = \frac{2}{6} = \frac{1}{3}$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$$

Question 10:

A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

ANS :

Let the first observation be from the black die and second from the red die.

When two dice (one black and another red) are rolled, the sample space S has $6 \times 6 = 36$ number of elements.

1. Let

A: Obtaining a sum greater than 9

$$= \{(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$$

B: Black die results in a 5.

$$= \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$$

$$\therefore A \cap B = \{(5, 5), (5, 6)\}$$

The conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5, is given by $P(A|B)$.

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{2}{6} = \frac{1}{3}$$

(b) E: Sum of the observations is 8.

$$= \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$$

F: Red die resulted in a number less than 4.

$$= \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), \\ (3,1), (3,2), (3,3), (4,1), (4,2), (4,3), \\ (5,1), (5,2), (5,3), (6,1), (6,2), (6,3) \end{array} \right\} \therefore E \cap F = \{(5,3), (6,2)\}$$
$$P(F) = \frac{18}{36} \text{ and } P(E \cap F) = \frac{2}{36}$$

The conditional probability of obtaining the sum equal to 8, given that the red die resulted in a number less than 4, is given by $P(E|F)$

$$\text{Therefore, } P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \frac{1}{9}$$

Question 11:

A fair die is rolled. Consider events $E = \{1, 3, 5\}$, $F = \{2, 3\}$ and $G = \{2, 3, 4, 5\}$

Find

(i) $P(E|F)$ and $P(F|E)$ (ii) $P(E|G)$ and $P(G|E)$

(ii) $P((E \cup F)|G)$ and $P((E \cap G)|G)$

ANS :

When a fair die is rolled, the sample space S will be

$$S = \{1, 2, 3, 4, 5, 6\}$$

It is given that $E = \{1, 3, 5\}$, $F = \{2, 3\}$, and $G = \{2, 3, 4, 5\}$

$$\therefore P(E) = \frac{3}{6} = \frac{1}{2}$$

$$P(F) = \frac{2}{6} = \frac{1}{3}$$

$$P(G) = \frac{4}{6} = \frac{2}{3}$$

(i) $E \cap F = \{3\}$

$$\therefore P(E \cap F) = \frac{1}{6}$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2}$$

$$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

(ii) $E \cap G = \{3, 5\}$

$$\therefore P(E \cap G) = \frac{2}{6} = \frac{1}{3}$$

$$\therefore P(E|G) = \frac{P(E \cap G)}{P(G)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$P(G|E) = \frac{P(E \cap G)}{P(E)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

$$(iii) E \cup F = \{1, 2, 3, 5\}$$

$$(E \cup F) \cap G = \{1, 2, 3, 5\} \cap \{2, 3, 4, 5\} = \{2, 3, 5\}$$

$$E \cap F = \{3\}$$

$$(E \cap F) \cap G = \{3\} \cap \{2, 3, 4, 5\} = \{3\}$$

$$\therefore P(E \cup G) = \frac{4}{6} = \frac{2}{3}$$

$$P((E \cup F) \cap G) = \frac{3}{6} = \frac{1}{2}$$

$$P(E \cap F) = \frac{1}{6}$$

$$P((E \cap F) \cap G) = \frac{1}{6}$$

$$\therefore P((E \cup F) | G) = \frac{P((E \cup F) \cap G)}{P(G)}$$

$$= \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$$

$$P((E \cap F) | G) = \frac{P((E \cap F) \cap G)}{P(G)}$$

$$= \frac{\frac{1}{6}}{\frac{3}{2}} = \frac{1}{6} \times \frac{2}{3} = \frac{1}{9}$$

Question 12:

Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl, (ii) at least one is a girl?

ANS :

Let b and g represent the boy and the girl child respectively. If a family has two children, the sample space will be

$$S = \{(b, b), (b, g), (g, b), (g, g)\}$$

Let A be the event that both children are girls.

$$\therefore A = \{(g, g)\}$$

(i) Let B be the event that the youngest child is a girl.

$$\therefore B = [(b, g), (g, g)]$$

$$\Rightarrow A \cap B = \{(g, g)\}$$

$$\therefore P(B) = \frac{2}{4} = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{4}$$

The conditional probability that both are girls, given that the youngest child is a girl, is given by $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Therefore, the required probability is $\frac{1}{2}$.

(ii) Let C be the event that at least one child is a girl.

$$\therefore C = \{(b, g), (g, b), (g, g)\}$$

$$\Rightarrow A \cap C = \{(g, g)\}$$

$$\Rightarrow P(C) = \frac{3}{4}$$

$$P(A \cap C) = \frac{1}{4}$$

The conditional probability that both are girls, given that at least one child is a girl, is given by $P(A|C)$.

$$\text{Therefore, } P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Question 13:

An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

ANS :

The given data can be tabulated as

	True/False	Multiple choice	Total
Easy	300	500	800
Difficult	200	400	600
Total	500	900	1400

Let us denote E = easy questions, M = multiple choice questions, D = difficult questions, and T = True/False questions

Total number of questions = 1400

Total number of multiple choice questions = 900

Therefore, probability of selecting an easy multiple choice question is

$$P(E \cap M) = \frac{500}{1400} = \frac{5}{14}$$

Probability of selecting a multiple choice question, P(M), is

$$\frac{900}{1400} = \frac{9}{14}$$

P(E|M) represents the probability that a randomly selected question will be an easy question, given that it is a multiple choice question.

$$\therefore P(E|M) = \frac{P(E \cap M)}{P(M)} = \frac{\frac{5}{14}}{\frac{9}{14}} = \frac{5}{9}$$

Therefore, the required probability is $\frac{5}{9}$.

Question 14:

Given that the two numbers appearing on throwing the two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

ANS :

When dice is thrown, number of observations in the sample space = $6 \times 6 = 36$

Let A be the event that the sum of the numbers on the dice is 4 and B be the event that the two numbers appearing on throwing the two dice are different.

$$\therefore A = \{(1, 3), (2, 2), (3, 1)\}$$

$$B = \left\{ \begin{array}{l} (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

$$A \cap B = \{(1,3), (3,1)\}$$

$$\therefore P(B) = \frac{30}{36} = \frac{5}{6} \text{ and } P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$

Let P(A|B) represent the probability that the sum of the numbers on the dice is 4, given that the two numbers appearing on throwing the two dice are different.

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{18}}{\frac{5}{6}} = \frac{1}{15}$$

Therefore, the required probability is $\frac{1}{15}$.

Question 15:

Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

ANS :

The outcomes of the given experiment can be represented by the following tree diagram.

The sample space of the experiment is,

$$S = \left\{ \begin{array}{l} (1, H), (1, T), (2, H), (2, T), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), \\ (4, H), (4, T), (5, H), (5, T), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \end{array} \right\}$$

Let A be the event that the coin shows a tail and B be the event that at least one die shows 3.

$$\therefore A = \{(1, T), (2, T), (4, T), (5, T)\}$$

$$B = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 3)\}$$

$$\Rightarrow A \cap B = \phi$$

$$\therefore P(A \cap B) = 0$$

$$\text{Then, } P(B) = P(\{3, 1\}) + P(\{3, 2\}) + P(\{3, 3\}) + P(\{3, 4\}) + P(\{3, 5\}) + P(\{3, 6\}) + P(\{6, 3\})$$

$$\begin{aligned} &= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} \\ &= \frac{7}{36} \end{aligned}$$

Probability of the event that the coin shows a tail, given that at least one die shows 3, is given by $P(A|B)$.

Therefore,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{\frac{7}{36}} = 0$$

Question 16:

If $P(A) = \frac{1}{2}$, $P(B) = 0$, then $P(A|B)$ is

(A) 0 (B) $\frac{1}{2}$

(C) not defined (D) 1

ANS :

It is given that $P(A) = \frac{1}{2}$ and $P(B) = 0$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{0}$$

Therefore, $P(A|B)$ is not defined.

Thus, the correct answer is C.

Question 17:

If A and B are events such that $P(A|B) = P(B|A)$, then

(A) $A \subset B$ but $A \neq B$ (B) $A = B$

(C) $A \cap B = \Phi$ (D) $P(A) = P(B)$

ANS :

It is given that, $P(A|B) = P(B|A)$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)}$$

$$\Rightarrow P(A) = P(B)$$

Thus, the correct answer is D.