ASSIGNMENT CLASS XII

(Minimum Learning Level) Based on Book - 2

INTEGRATION

1. Integrate :

(a) $\frac{1}{1+\cot x}$ (b) $\frac{\sin x}{\sin(x-a)}$ (c) $\tan^3 x$ (d) $\frac{e^x}{e^{2x}+6e^x+5}$ (e) $\frac{3x+1}{\sqrt{5-2x-x^2}}$

(f) $x^2\tan^{-1}x$ (g) $\frac{3x+1}{(x-2)^2(x+2)}$ (h) $\frac{1}{(x+2)(x^2+4)}$ (i) $\frac{2+\sin 2x}{1+\cos 2x}e^x$

(j) $\frac{1}{\cos(x-a)\cos(x-b)}$ (k) $\frac{x}{(x+1)(x+2)}$ (l) $\frac{x\,e^x}{(1+x)^2}$

2. Evaluate: (a) $\int_0^{\pi/2}\frac{\sin^2 x}{\sin x+\cos x}dx$ (b) $\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$ (c) $\int_0^{\pi/4}\log(1+\tan x)dx$ (d) $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$

3. Evaluate $\int_0^2 (x^2-x+2)dx$ as limit of sum.

AREA UNDER CURVE

1. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.
2. Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.
3. Find the area bounded by curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.
4. Find area of region bounded by triangle whose vertices are (– 1, 0), (1, 3) and (3, 2).
5. Find the area of the region $\{(x, y): x^2+y^2 \le 1 \le x+y\}$.
6. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and the line $\frac{x}{4}+\frac{y}{3}=1$.

DIFFERENTIAL EQUATIONS

Solve:

1. $y(1-x^2)\frac{dy}{dx}=x(1+y^2)$
2. $e^x\tan y\,dx + (1-e^x)\sec^2 y\,dy = 0$
3. $(x^2-y^2)dx+2xy\,dy = 0$
4. $x^2\frac{dy}{dx}=x^2-2y^2+xy$
5. $(x^2-1)\frac{dy}{dx}+2xy=\frac{2}{x^2-1}$
6. $(1+x^2)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$
7. $(1+x^2)\frac{dy}{dx}-2xy=(x^2+2)(x^2+1)$
8. $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$
9. $(1+x^2)\,dy + 2xy\,dx = \cot x\,dx$

VECTOR ALGEBRA

1. If the position vectors of the vertices of a triangle ABC are $\hat{i}+2\hat{j}+3k$, $2\hat{i}+3\hat{j}+k$, $3\hat{i}+\hat{j}+2k$, prove that ΔABC is an equilateral triangle.
2. The dot products of a vector with the vectors $\hat{i}+\hat{j}-3k$, $\hat{i}+3\hat{j}-2k$ and $2\hat{i}+\hat{j}+4k$ are 0, 5, 8 respectively. Find the vector.
3. Find the area of the parallelogram whose diagonals are determined by the vectors $\vec{a}=2\hat{i}+3\hat{j}-6k$ and $\vec{b}=3\hat{i}-4\hat{j}-\hat{k}$.
4. If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.
5. Find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).
6. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=3, |\vec{b}|=4, |\vec{c}|=5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a}+\vec{b}+\vec{c}|$.
7. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

THREE DIMENSIONAL GEOMETRY

1. Find the image of the point $(5,9,3)$ in the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$.
2. Find the shortest distance between the following pair of lines:

   (a) $\vec{r}=(\lambda-1)\hat{i}+(\lambda+1)\hat{j}-(\lambda+1)k$ and $\vec{r}=(1-\mu)\hat{i}+2(2\mu-1)\hat{j}+(\mu+2)k$ Ans: $\frac{5\sqrt{2}}{2}$ units

   (b) $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$
3. Find vector equation of the plane passing through the intersection of the planes $\vec{r}.(2\hat{i}-7\hat{j}+4k)=3$ and $\vec{r}.(3\hat{i}-5\hat{j}+4k)+11=0$, and passing through point $(-2, 1, 3)$.
4. Find the distance between the point P (6, 5, 9) and plane determined by points A (3,– 1, 2), B (5, 2, 4) and C(– 1, – 1, 6).
5. Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3$ and $5x-3y+4z+9=0$, and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.
6. Find the equation of the plane passing through the point $(1,1,1)$ and perpendicular to each of the planes $x+2y+3z=7$ and $2x-3y+4z=0$.
7. Prove that the image of the point $(3,-2,1)$ in the plane $3x-y+4z=2$ lies on the plane $x+y+z+4=0$.
8. Find the distance between the point P (6, 5, 9) and the plane determined by the points A (3, – 1, 2), B (5, 2, 4) and C (– 1, – 1, 6).
9. Find the distance of the point (– 1, – 5, – 10) from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=5$.

LINEAR PROGRAMMING

1. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foos A and B are available at the cost of Rs 5 and Rs 4 per unit respectively. One unit of the food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories, while one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the foods A and B should be used to have least cost.
2. A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair is Rs. 30 while by selling one table the profit is Rs. 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit? Formulate the problem as L.P.P. and solve it graphically.
3. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 5760 to invest and has space for at most 20 items. A fan costs him Rs 360 and a sewing machine costs him Rs 240. He expects to sell a fan a profit of Rs 22 and a sewing machine at a profit of Rs 18. Assuming that he can sell all the items that he buys, how should he invest his money to maximize the profit? What is the maximum profit?
4. A dealer deals in two items A and B. He has Rs. 15,000 to invest and a space to store at the most 80 pieces. Item A costs him Rs. 300 and item B costs him Rs. 150. He can sell items A and B at profits of Rs. 40 and Rs. 25 respectively. Assuming that he can sell all that he buys, formulate the above as a linear programming problem for maximum profit and solve it graphically.
5. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit?

PROBABILITY

1. Sumit and Nishu appear for an interview for two vacancies in a company. The probabilities of their selection are respectively $\frac{1}{5}$ and $\frac{1}{6}$. What is the probability that:

   (i) both of them are selected (ii) only one of them is selected (iii) none of them is selected?

2. A candidate has to reach the examination centre in time. Probability of him going by bus or scooter or by other means of transport is $\frac{3}{10}, \frac{1}{10}, \frac{3}{5}$ respectively. The probability that he will be late is $\frac{1}{4}$ and $\frac{1}{3}$ respectively, if he travels by bus or scooter. But he reaches in time if he uses any other mode of transport. He reached late at centre. Find probability that he travelled by bus.
3. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of accidents are 0.01, 0.03 and 0.15 respectively. One of insured persons meets with accident. What is probability that he is a scooter driver?
4. In a bolt factory machines, A, B and C manufacture respectively 25%, 35% and 40% of the total bolts. Of their output 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from product.

   (i) What is the probability that the bolt drawn is defective?

   (ii) If the bolt is found to be defective find the probability that it is a product of machine B.

5. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.
6. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.
7. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.
8. A and B throw a die alternatively till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts first.
9. Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?
10. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.