ASSIGNMENT CLASS XII

(Minimum Learning Level) Based on Book - 1

RELATIONS AND FUNCTIONS

1. Check whether the relation R in **R** defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

2. Let $f : R-\{2\} \rightarrow R-\{1\}$ is a mapping defined by $f(x)=\frac{x-1}{x-2}$, show that f is bijective.

3. Let $f : N \rightarrow R$ be a function defined as $f(x)=4x^2+12x+15$. Show that $f : N \rightarrow R_f$ is invertible. Also find the inverse of f.

4 Show that $f : [-1, 1] \rightarrow \mathbf{R}$, given by $f(x) = \frac{x}{(x+2)}$ is one-one. Find the inverse of the function $f : [-1, 1] \rightarrow$ Range f.

5. Define a binary operation $*$ on the set $\{0, 1, 2, 3, 4, 5\}$ as

$$a * b = \begin{cases} a+b, & \text{if } a+b < 6 \\ a+b-6 & \text{if } a+b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element a of the set is invertible with $6 - a$ being the inverse of a.

6. Show that the operation $*$ on Z, defined by $a*b=a+b+1$, satisfies

(*i*) closure property (*ii*) associative property (*iii*) commutative property.

Also find the identity element and inverse of an element $a \in A$.

7. Find gof and fog when and $f : R \rightarrow R$ and $g : R \rightarrow R$ are defined by: $f(x) = 2x+3$ *and* $g(x) = x^2+5$.

8. Show that the function $f : R \rightarrow R$ defined by $f(x) = x^3+3$ is invertible. Also find the inverse of f.

9 Show that the relation R in **R** defined as $R = \{(a, b) : a \leq b\}$, is reflexive and transitive but not symmetric.

10. Check whether the relation R in **R** defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

11. Consider $f : \mathbf{R}_+ \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where $\mathbf{R}_+$ is the set of all non-negative real numbers.

INVERSE TRIGONOMETRY

1. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ \frac{-1}{\sqrt{2}}\le x\le 1$

2. Prove that: (i) $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{63}{16}$ (ii) $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\tan^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{4}=\frac{\pi}{4}$

3. Solve the following equations:

(i) $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{6}{17}$ (ii) $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

4. If $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, then find the value of x

5.

Solve $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$

6. Write the following in the simplest form: (a) $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ (b) $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

MATRICES AND DETERMINANTS

1. For the matrix $A=\begin{bmatrix}3&2\\1&1\end{bmatrix}$, find λ and μ such that $A^2+\lambda A+\mu I=0$. Hence find A^{-1}.

2. Express the given matrix as the sum of symmetric and a skew-symmetric matrix: $\begin{bmatrix}1&3&5\\-6&8&3\\-4&6&5\end{bmatrix}$

3. Find x, if $\begin{bmatrix}x&-5&-1\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix}=O$

4. Using properties of determinants prove the following:

(i) $\begin{vmatrix}x&y&z\\x^2&y^2&z^2\\x^3&y^3&z^3\end{vmatrix}=xyz(x-y)(y-z)(z-x)$ (ii) $\begin{vmatrix}1+a&1&1\\1&1+b&1\\1&1&1+c\end{vmatrix}=abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

(iii) $\begin{vmatrix}a-b-c&2a&2a\\2b&b-c-a&2b\\2c&2c&c-a-b\end{vmatrix}=(a+b+c)^3$ (iv) $\begin{vmatrix}b+c&a&b\\c+a&c&a\\a+b&b&c\end{vmatrix}=(a+b+c)(a-c)^2$

(v) $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$

(vi) $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$

(vii) $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc+bc+ca+ab$

5. Using properties of determinants, solve for x : $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

6. Using matrix method, solve the following system of linear equations:

(*i*) $2x+y+z=7$, $x-y-z=-4$, $3x+2y+z=10$

(*ii*) $x+y+z=6$, $x+2z=7$, $3x+y+z=12$

(*iii*) $x+2y+z=1$, $2x-y+z=5$, $3x+y-z=0$

7. If $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$, find AB and use this result to solve the following equations:

$2x-y+z=-1$; $-x+2y-z=4$; $x-y+2z=-3$. Ans: $x=1, y=2, z=-1$

DIFFERENTIATION

1. Show that the function $f(x)=\begin{cases} \dfrac{x-|x|}{x}, & x\neq 0 \\ 2 & x=0 \end{cases}$ is discontinuous at $x=0$.

2. Discuss the continuity and differentiability of $f(x)=|x-1|+|x-2|$.

2. 3. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

4. If $y=x^{\cos x}+\cos x^{\sin x}$, find $\frac{dy}{dx}$.

5. 5. If $x^a y^b=(x+y)^{(a+b)}$, prove that $\frac{dy}{dx}=\frac{y}{x}$.

6. If $y=\tan^{-1}x$, show that $(1+x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=0$.

7. Find $\frac{d^2y}{dx^2}$ if $x=a\left(\cos t+\log\tan\frac{t}{2}\right)$, $y=a\sin t$.

8. Find $\frac{dy}{dx}$, if $y=\sin^{-1}x+\sin^{-1}\sqrt{1-x^2}$, $-1\le x\le 1$

9. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ for $-1<x<1$ prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$

APPLICATIONS OF DERIVATIVES

1. Find the intervals on which the following functions are (i) increasing (ii) decreasing:

   (a) $f(x)=5+36x+3x^2-2x^3$ (b) $f(x)=(x-1)^3(x-2)^2$

2. Find the equation of the tangent to the curve $x^2+3y=3$, which is parallel to the line $y-4x+5=0$.
3. A right circular cylinder is inscribed in a given cone. Show that the curved surface area of the cylinder is maximum when diameter of cylinder is equal to the radius of the base of cone.
4. Show that a closed right circular cylinder of given surface area and maximum volume is such that its height is equal to the diameter of the base.
5. Show that the height of the right circular cylinder of maximum volume that can be inscribed in a given right circular cone of height h is $\frac{h}{3}$.
6. Using differentials, find the approximate value of $(15)^{1/4}$.
7. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cm^3 of gas per second. Find rate at which radius of the balloon increases when radius is 15 cm.

8. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

9. Prove that $y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$ is an increasing function of θ in $\left[0,\frac{\pi}{2}\right]$.

10. Find the points on the curve $x^2+y^2-2x-3=0$ at which the tangents are parallel to the x-axis.

11. Find both the maximum value and the minimum value of $3x^4-8x^3+12x^2-48x+25$ on the interval [0, 3].

12. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

13. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.