PRE BOARD EXAMINATION 2011-12
MATHEMATICS
SET - A
Class XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. Let * be a binary operation on set of integers I, defined by $a*b = 2a+b-3$, find value of $3*4$.

2. For what value of x is the matrix $\begin{bmatrix} 6-x & 4 \\ 3-x & 1 \end{bmatrix}$ is singular?

3. Using principal values, write the value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

4. What is the maximum and minimum value of the function $f(x) = 3-|x+2|$?

5. Write the point of discontinuity and non-differentiability(if any) of the function f given by:

$$f(x) = |x+1| + |x-3|.$$

6. Evaluate $\int \frac{dx}{x+x\log x}$

7. Evaluate: $\int_0^1 \frac{2x}{1+x^2}dx$

8. Find a unit vector in the direction of $\vec{a} = 2\hat{i}-3\hat{j}+6\hat{k}$.

9. For what value of λ are the vectors $\vec{a} = \hat{i}+2\lambda\hat{j}+\hat{k}$ and $\vec{b} = 2\hat{i}+\hat{j}-3\hat{k}$ perpendicular?

10. The equation of a line is $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$. Find the direction cosines of a line parallel to the given line.

SECTION – B

11. If $f:R\to R$ be the function defined by $f(x)=4x^3+7$, show that f is a bijection. Also find f^{-1}.

12. Prove that: $2\tan^{-1}\left(\frac{3}{4}\right)-\tan^{-1}\left(\frac{17}{31}\right)=\frac{\pi}{4}$

13. If $A=\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}$, show that $A^2-5A-14I=0$. Hence find A^{-1}.

14. Using properties of determinants, prove that:

$$\begin{vmatrix}-a^2 & ab & ac\\ ba & -b^2 & bc\\ ac & bc & -c^2\end{vmatrix}=4a^2b^2c^2$$

15. Find the values of x for which $f(x)=\left(x(x-2)\right)^2$ is an:

(a) increasing function (b) decreasing function

OR

Find the equation of tangent and normal to the curve $y=x^4-6x^3+13x^2-10x+5$ at point $(1,3)$.

16. If $y=\left(\cot^{-1}x\right)^2$, then show that $\left(x^2+1\right)^2\frac{d^2y}{dx^2}+2x\left(x^2+1\right)\frac{dy}{dx}=2$

OR

Find the values of a and b such that the function defined as follows is continuous.

$$f(x)=\begin{cases}x+2, & x\le 2\\ ax+b, & 2<x<5\\ 3x-2, & x\ge 5\end{cases}$$

17. Solve the following differential equation:

$$\left[x\sin^2\left(\frac{y}{x}\right)-y\right]dx+x\,dy=0$$

OR

$$x\sqrt{1-y^2}\,dx+y\sqrt{1-x^2}\,dy=0$$

18. Solve the following differential equation:

$$\left(1+x^2\right)dy+2xy\,dx=\cot x\,dx\ ;\ x\neq 0$$

19. Evaluate: $\int\frac{x+2}{2x^2+6x+5}dx$ **OR** Evaluate: $\int\frac{1+\sin x}{1+\cos x}e^x\,dx$

20. If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $|\vec{a}|=3$, $|\vec{b}|=4$ and $|\vec{c}|=5$ and each one of them is perpendicular to the sum of the other two, then find $|\vec{a}+\vec{b}+\vec{c}|$.

21. Find the shortest distance between the following pair of lines:

$$\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+4\hat{k}\right) \text{ and } \vec{r}=\left(2\hat{i}+4\hat{j}+5\hat{k}\right)+\mu\left(3\hat{i}+4\hat{j}+5\hat{k}\right)$$

22. Daniel and Nishi appear for an interview for two vacancies in a company. The probabilities of their selection are respectively $\frac{1}{5}$ and $\frac{1}{6}$. what is the probability that:

(i) only one of them is selected (ii) none of them is selected?

SECTION – C

23. Solve the following system of equations:

$$3x-4y+2z=-1;\ 2x+3y+5z=7;\ x+z=2$$

24. Show that the right circular cylinder of given volume, open at the top, has minimum total surface area if its height is equal to the radius of the base.

25. Evaluate $\int_0^2 \left(3x^2-2\right)dx$ as limit of sums.

OR

Evaluate: $\int_0^{\pi/2} 2\sin x\cos x\tan^{-1}\left(\sin x\right)dx$

26. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$.

27. Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?

28. Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3$ and $5x-3y+4z+9=0$, and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.

OR

Find the coordinates of the point where the line through $(3,-4,-5)$ and $(2,-3,1)$ crosses the plane determined by the points $A(1,2,3)$, $B(2,2,1)$ and $C(-1,3,6)$.

29. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time. If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the number of tennis rackets and cricket bats that the factory must manufacture to earn the maximum profit. Make it as an L.P.P. and solve it graphically.