SECOND TERMINAL EXAMINATION 2011-12
MATHEMATICS
Class XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. Let * be a binary operation on the set Q of non-zero rational numbers defined as $a*b=\frac{ab}{5}$.

   Write the identity element for * , if any.

2. Using the principal values, write the value of $\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{2}\right)$.

3. If $x\in R$, $0 \le x \le \frac{\pi}{2}$ and $\begin{vmatrix} 2\sin x & -1 \\ 1 & \sin x \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ -4 & \sin x \end{vmatrix}$, write the value(s) of x.

4. Construct a 3×2 matrix whose elements are given by $a_{ij}=\left|i-2j\right|$.

5. If $A=\begin{bmatrix} x+4 & 2 \\ 3 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} x & 2x-1 \\ 2 & 5 \end{bmatrix}$ and $\left|AB\right|=12$, find the value(s) of x.

6. Find $f(x)$ satisfying the following : $\int e^x\left(\sec^2 x+\tan x\right)dx=e^x f(x)+C$

7. Write value of the integral: $\int_{-\pi/4}^{\pi/4} \sin^3 x\, dx$

8. Find the position vector of the mid-point of the line-segment AB, where A is the point $(3,4,-2)$ and B is the point $(1,2,4)$.

9. Write the value of $\left|\vec{a}-\vec{b}\right|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $\left|\vec{a}\right|=2$, $\left|\vec{b}\right|=3$ and $\vec{a}.\vec{b}=4$.

10. Find the value of λ such that the line $\frac{x-2}{9}=\frac{y-1}{\lambda}=\frac{z-3}{-6}$ is perpendicular to the plane $3x-y-2z=7$.

SECTION – B

11. Show that $f:[-1\,,1]\rightarrow R$ given by $f(x)=\dfrac{x}{x+2}$ is one-one. Find the inverse of the function $f:[-1\,,1]\rightarrow$ Range of f.

OR

Show that the relation R on Z defined by $(a\,,b)\in R \Leftrightarrow a-b$ is divisible by 4 is an equivalence relation.

12. Prove that:

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}\quad ;x\in\left(0,\frac{\pi}{4}\right)$$

13. Using properties of determinants, prove that:

$$\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}=-2\left(x^3+y^3\right)$$

OR

Without expanding the determinants, show that:

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}=2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

14. The surface area of a spherical bubble is increasing at the rate of $2\,cm^2/\sec$. Find the rate at which the volume of the bubble is increasing at the instant its radius is 6 cm.

OR

Discuss the applicability of Lagrange's mean value theorem for the function $f(x)=x^3-2x^2-x+3$ on $[0\,,1]$

15. If $x^y=e^{x-y}$, then prove that $\dfrac{dy}{dx}=\dfrac{\log x}{(1+\log x)^2}$

16. Find the intervals in which the function $f(x)=2x^3-15x^2+36+1$ is strictly increasing or decreasing. Also, find the points on which the tangents are parallel to the x-axis.

17. Evaluate the following: $\int x \tan^{-1} x \, dx$

OR

Evaluate the following: $\int \frac{3x+1}{(x-2)^2 (x+2)} dx$

18. Solve the following differential equation:

$$\left(1+x^2\right)\frac{dy}{dx} + y = \tan^{-1} x$$

19. Solve the following differential equation:

$$2xy\,dx + \left(x^2 + 2y^2\right) dy = 0$$

20. Using vectors, find area of triangle whose vertices are given by: $A(2,3,5)$, $B(3,5,8)$ and $C(2,7,8)$.

21. Find the shortest distance between the following pair of lines:

$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$.

22. In a bolt factory machines, A, B and C manufacture respectively 25%, 35% and 40% of the total bolts. Of their output 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from producton and is found to be defective. What is the probability that it manufactured by the machine B.

SECTION – C

23. If $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix}$, find A^{-1}. Hence solve the following system of equations:

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

24. Using properties of definite integrals, evaluate the following:

$$\int_0^{\pi/2} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$$

25. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

OR

Given the sum of the perimeter of a square and a circle, show that the sum of their areas is minimum when the side of the square is equal to diameter of the circle.

26. Find the area bounded by the lines : $x+2y=2$; $y-x=1$; $2x+y=7$

OR

Sketch the region common to the circle $x^2+y^2=16$ and the parabola $x^2=6y$. Also, find the area of the region using integration.

27. Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3$, $5x-3y+4z+9=0$, and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.

28. Let X denote the number of colleges where you will apply after your results and $P(X=x)$ denotes your probability of getting admission in x number of colleges. It is given that:

$$P(X=x) = \begin{cases} kx & \text{if } x=0 \text{ or } 1 \\ 2kx & \text{if } x=2 \\ k(5-x) & \text{if } x=3 \text{ or } 4 \end{cases} \quad ; \ k \text{ is a positive constant}$$

(a) Find the value of k.

(b) What is the probability that you will get admission in exactly two colleges?

(c) Find the mean and variance of the probability distribution.

29. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foos A and B are available at the cost of Rs 5 and Rs 4 per unit respectively. One unit of the food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories, while one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the foods A and B should be used to have least cost.