PRACTICE PAPER
MATHEMATICS
Class XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, Internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. Write the all possible orders of a matrix having 15 elements.

2. Give an example of two non-zero matrices A and B such that $AB=0$.

3. If $A=\begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 1\\ 3 & 6\end{bmatrix}$, write the value of $|AB|$.

4. Find all the points of discontinuity(if any) of f defined by $f(x)=|x-1|+|x+2|$.

5. The radius of a circle is increasing at the rate of $0.7\,cm/\sec$. What is the rate of increase of its circumference in terms of π.

6. Evaluate $\int \frac{e^{5\log x}-e^{3\log x}}{x^2-1}dx$

7. Write order and degree of the following differential equation:

$$\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2=x^2\log\left(\frac{d^2y}{dx^2}\right)$$

8. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|=3$, $|\vec{b}|=\frac{\sqrt{2}}{3}$ and $\vec{a}\times\vec{b}$ is a unit vector. What is the angle between $\vec{a}$ and $\vec{b}$?

9. If $P(A)=\frac{6}{11}, P(B)=\frac{5}{11}$ and $P(A\cup B)=\frac{7}{11}$, write the value of $P(B|A)$.

10. Amit and Sanjeev appear for an interview for two vacancies in a company. The probabilities of their selection are respectively $\frac{1}{5}$ and $\frac{1}{6}$. what is the probability that none of them is selected?

SECTION – B

11. Prove that:

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2} \quad ; x\in\left(0,\frac{\pi}{4}\right)$$

12. Using properties of determinants, show that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$$

OR

If $A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, prove that $(aI+bA)^n = a^n I + na^{n-1}bA$, where I is a unit matrix of order 2 and n is a positive integer.

13. If $x^a y^b = (x+y)^{(a+b)}$, prove that $\frac{dy}{dx}=\frac{y}{x}$.

OR

If $y\sqrt{x^2+1}=\log\left(\sqrt{x^2+1}-x\right)$, show that $\left(x^2+1\right)\frac{dy}{dx}+xy+1=0$.

14. If $x=a\sin 2t(1+\cos 2t), y=b\cos 2t(1-\cos 2t)$, find $\left(\frac{dy}{dx}\right)_{at\ t=\frac{\pi}{4}}$.

15. Show that $y=\log(1+x)-\frac{2x}{2+x}$, $x>-1$, is an increasing function of x throughout its domain.

16. Evaluate the following: $\int \frac{3x+1}{(x-2)^2(x+2)}dx$

17. Evaluate the following: $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx$

18. Evaluate the following: $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$

OR

Evaluate the following: $\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}dx$

19. Solve the following differential equation:

$$2xy\,dx+\left(x^2+2y^2\right)dy=0$$

OR

Solve the following differential equation:

$$\left(1+x^2\right)\frac{dy}{dx}-2xy=\left(x^2+2\right)\left(x^2+1\right)$$

20. If $\vec{a},\vec{b}$ and $\vec{c}$ are vectors such that $\left|\vec{a}\right|=5,\ \left|\vec{b}\right|=4,\ \left|\vec{c}\right|=3$ and each is perpendicular to the sum of the other two, find the value of $\left|\vec{a}+\vec{b}+\vec{c}\right|$.

21. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10},\frac{1}{5},\frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4},\frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?

22. If a fair coin is tossed 8 times, find the probability of

(i) exactly three heads (ii) at least three heads (iii) at most three heads

SECTION – C

23. Show that the operation $*$ on Z, defined by $a*b=a+b+1$, satisfies

(i) associative property (ii) commutative property.

Also find the identity element and inverse of an element $a\in Z$.

24. $A=\begin{bmatrix}1 & 2 & 5\\ 1 & -1 & -1\\ 2 & 3 & -1\end{bmatrix}$ find A^{-1}. Hence solve the following:

$$x+2y+5z=10$$
$$x-y-z=-2$$
$$2x+3y-z=-11$$

25. Show that the volume of the largest cylinder which can be inscribed in a cone of height h and semi-vertical angle 30^0 is $\frac{4}{81}\pi h^3$.

OR

A window is in the form of an rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10m. Find the dimensions of the window to admit maximum light through the whole opening.

26. Find the area of the region enclosed between the circles $x^2+y^2=16$ and $(x-4)^2+y^2=16$.

27. Define skew lines and the line of the shortest distance. Also find the shortest distance between the following pair of lines:

$$\frac{x-1}{2}=\frac{y-1}{-1}=\frac{z}{1} \text{ and } \frac{x-2}{4}=\frac{y-1}{-2}=\frac{z+1}{2}.$$

28. Find the equation of the plane passing through the intersection of the planes $2x+3y-z+1=0$ and $x+y-2z+3=0$, and perpendicular to plane $3x-y-2z-4=0$. Also find the inclination of this plane with xy - plane.

OR

Find the foot of the perpendicular from $P(1,2,3)$ on the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Also, obtain the equation of the plane containing the line and the point $(1,2,3)$.

29. A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair is Rs. 30 while by selling one table the profit is Rs. 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit? Formulate the problem as L.P.P. and solve it graphically.