PRACTICE PAPER
MATHEMATICS Class XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.

SECTION – A

1. If $A=\{1,2,3\}$, then how many number of binary operations can be defined on A.

2. If $A=\begin{bmatrix} a & b & c & d \end{bmatrix}$, write the value of AA^T.

3. If $0<x<\pi$ and the matrix $\begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$ is singular, write the value(s) of x.

4. Is the Modulus function $f:R\to R$, given by $f(x)=|x|$ is bijective? Justify.

5. Write the value of $\cos\left[\tan^{-1}\left(\frac{3}{4}\right)\right]$.

6. Write the value of $\int \frac{3-\sin x}{\cos^2 x}dx$.

7. Write the integrating factor of the differential equation $\left(1+x^2\right)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$.

8. If $|\vec{a}|=\sqrt{26}$, $|\vec{b}|=7$ and $|\vec{a}\times\vec{b}|=35$, write the value of $\vec{a}.\vec{b}$.

9. Write the vector equation of the line through the point $(5, 2, -4)$ and which is parallel to the vector $3\hat{i}-4\hat{j}-\hat{k}$.

10. If $\vec{a}=\hat{i}+\hat{j}$, $\vec{b}=\hat{j}+k$, $\vec{c}=k+\hat{i}$, find a unit vector in the direction of $\vec{a}+\vec{b}+\vec{c}$.

SECTION – B

11. Show that the function $f:R\to R$ defined by $f(x)=\frac{2x-1}{3}$; $x\in R$ is one-one and onto function. Also find the inverse of the function f.

OR

Define a binary operation $\ast$ on the set $\{0, 1, 2, 3, 4, 5\}$ as

$$a\ast b=\begin{cases} a+b & \text{if } a+b<6 \\ a+b-6 & \text{if } a+b\geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element a of the set is invertible with $6-a$ being the inverse of a.

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12. Prove that: $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{63}{16}$.

13. Using properties of determinants, show that:

$$\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = -2\left(x^3+y^3\right).$$

14. Show that $y=\log\left(1+x\right)-\frac{2x}{2+x}$, $x>-1$, is an increasing function of x throughout its domain.

15. Find $\frac{dy}{dx}$, if $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$.

OR

If $x^a y^b=\left(x+y\right)^{(a+b)}$, prove that $\frac{dy}{dx}=\frac{y}{x}$.

16. Using differentials, find the approximate value of $\left(33.1\right)^{1/5}$

17. Evaluate the following: $\int\frac{4x-3}{x^2+3x-10}dx$ **OR** $\int\frac{1}{\left(x-2\right)\left(x^2+4\right)}dx$

18. Evaluate the following integral as limit of sums: $\int_0^2\left(x^2-3x\right)dx$

19. Solve the following differential equation:

$$x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)+x=y\sin\left(\frac{y}{x}\right),\ y(1)=\frac{\pi}{2}$$

OR

Solve the following differential equation:

$$\left(1+x^2\right)\frac{dy}{dx}+y=\tan^{-1}x.$$

20. Show that the points A, B and C with position vectors $2\hat{i}-\hat{j}+k$, $\hat{i}-3\hat{j}-5k$, $3\hat{i}-4\hat{j}-4k$ respectively, are the vertices of the right triangle. Also, find the remaining angles of the triangle.

21. Find the foot of perpendicular drawn from the point $P\left(1,6,3\right)$ on the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also, find its distance from P.

22. A company has two plants to manufacture bicycles. The first plant manufatures 60% of the bicycles and the second plant, 40%. Also, 80% of the bicycles are rated of standard quality at the first plant and 90% of standard quality at the second plant. A bicycle is picked at random and found to be of standard quality. Find the probability that it comes from the second plant.

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SECTION – C

23. If $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$, find AB and use this result to solve following equations:

$$2x-y+z=-1\,;\,-x+2y-z=4\,;\,x-y+2z=-3\,.$$

24. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

OR

A window is in the form of an rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10m. Find the dimensions of the window to admit maximum light through the whole opening.

25. Using the properties of definite integral, evaluate the following: $\int_0^{\pi/2}\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$.

26. Using integration, find area of ΔABC whose vertices have coordinates $A(2,0), B(4,5)$ and $C(6,3)$.

27. Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3$, $5x-3y+4z+9=0$, and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.

OR

Define skew lines and the line of the shortest distance. Also find the shortest distance between the following pair of lines:

$$\vec{r}=(1+\lambda)\hat{i}+(2-\lambda)\hat{j}+(\lambda+1)\hat{k} \text{ and } \vec{r}=\left(2\hat{i}-\hat{j}-\hat{k}\right)+\mu\left(2\hat{i}+\hat{j}+2\hat{k}\right)$$

28. The probability that a student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university:

(i) none will graduate (ii) only one will graduate (iii) all will graduate

29. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foos A and B are available at the cost of Rs 5 and Rs 4 per unit respectively. One unit of the food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories, while one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the foods A and B should be used to have least cost. Solve this LPP graphically.

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