ASSIGNMENT CLASS XII

(Minimum Learning Level)

RELATIONS AND FUNCTIONS

1. Check whether the relation R in **R** defined by $R = \{(a, b) : a \le b^3\}$ is reflexive, symmetric or transitive.

2. Let $f : R - \{2\} \to R - \{1\}$ is a mapping defined by $f(x) = \frac{x-1}{x-2}$, show that f is bijective.

3. Let $f : N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to R_f$ is invertible. Also find the inverse of f.

4. Show that $f : [-1, 1] \to \mathbf{R}$, given by $f(x) = \frac{x}{(x+2)}$ is one-one. Find the inverse of the function $f : [-1, 1] \to$ Range f.

5. Define a binary operation $*$ on the set $\{0, 1, 2, 3, 4, 5\}$ as

$$a * b = \begin{cases} a+b, & \text{if } a+b < 6 \\ a+b-6 & \text{if } a+b \ge 6 \end{cases}$$

Show that zero is the identity for this operation and each element a of the set is invertible with $6 - a$ being the inverse of a.

6. Show that the operation $*$ on Z, defined by $a * b = a + b + 1$, satisfies

(*i*) closure property (*ii*) associative property (*iii*) commutative property.

Also find the identity element and inverse of an element $a \in A$.

INVERSE TRIGONOMETRY

1. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x, \ \frac{-1}{\sqrt{2}} \le x \le 1$

2. Prove that: (i) $\sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5} = \tan^{-1}\frac{63}{16}$ (ii) $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \tan^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{4} = \frac{\pi}{4}$

3. Solve the following equations:

(i) $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{6}{17}$ (ii) $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$

4. If $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$, then find the value of x

MATRICES AND DETERMINANTS

1. For the matrix $A=\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}$, find λ and μ such that $A^2+\lambda A+\mu I=0$. Hence find A^{-1}.

2. Express the given matrix as the sum of symmetric and a skew-symmetric matrix: $\begin{bmatrix}1 & 3 & 5\\ -6 & 8 & 3\\ -4 & 6 & 5\end{bmatrix}$

3. Find x, if $\begin{bmatrix}x & -5 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\ 4\\ 1\end{bmatrix}=O$

4. Using properties of determinants prove the following:

(i) $\begin{vmatrix}x & y & z\\ x^2 & y^2 & z^2\\ x^3 & y^3 & z^3\end{vmatrix}=xyz(x-y)(y-z)(z-x)$

(ii) $\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}=abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

(iii) $\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}=(a+b+c)^3$

(iv) $\begin{vmatrix}b+c & a & b\\ c+a & c & a\\ a+b & b & c\end{vmatrix}=(a+b+c)(a-c)^2$

(v) $\begin{vmatrix}1+a^2-b^2 & 2ab & -2b\\ 2ab & 1-a^2+b^2 & 2a\\ 2b & -2a & 1-a^2-b^2\end{vmatrix}=(1+a^2+b^2)^3$

5. Using matrix method, solve the following system of linear equations:

(i) $2x+y+z=7$
$x-y-z=-4$
$3x+2y+z=10$

(ii) $x+y+z=6$
$x+2z=7$
$3x+y+z=12$

(iii) $x+2y+z=1$
$2x\,y+z=5$
$3x+y-z=0$

DIFFERENTIATION

1. Show that the function $f(x)=\begin{cases}\dfrac{x-|x|}{x}, & x\neq 0\\ 2 & x=0\end{cases}$ is discontinuous at $x=0$.
2. Discuss the continuity and differentiability of $f(x)=|x-1|+|x-2|$.
3. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.
4. If $y=x^{\cos x}+\cos x^{\sin x}$, find $\frac{dy}{dx}$.
5. 5. If $x^a y^b=(x+y)^{(a+b)}$, prove that $\frac{dy}{dx}=\frac{y}{x}$.
6. If $y=\tan^{-1}x$, show that $(1+x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=0$.
7. Find $\frac{d^2y}{dx^2}$ if $x=a\left(\cos t+\log\tan\frac{t}{2}\right)$, $y=a\sin t$.