CRPF PUBLIC SCHOOL, ROHINI

SECOND Intra School Mathematics Olympiad 2011

CLASS XII

Max. Marks: 50 **Max. Time: 1 hour 30 minutes**

General Instructions:

1. Q1-15 (Section A) each MCQ carries 2 mark. Each question has five choices (A, B, C, D or E). Select the correct answer to each question and darken the corresponding circle in the Answer Sheet provided to you. **THERE IS NO NEGATIVE MARKING.** Marking of more than one circle for an answer shall be awarded zero mark.
2. Q16-20 (Section B) each question carries 4 mark. You are to give the complete solution. Marking will be done stepwise.

SECTION - A

Q1. A fruit vendor makes a display with her peaches in a pyramid structure. The pyramid has a square base. She manages to make a display six layers high. The number of peaches she needs for this display is:

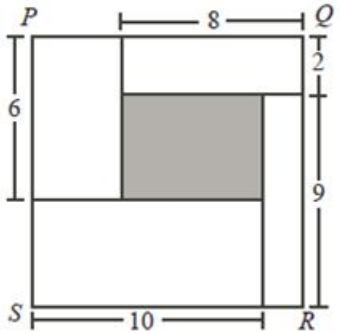
(A) 54 (B) 16 (C) 30 (D) 91 (E) None of these

Q2. The total number of zeroes at the end of the product $125^{20} \times 16^{12}$ when expanded in decimal system is:

(A) 45 (B) 46 (C) 48 (D) 50 (E)52

Q3. In the given diagram, $PQRS$ is a square. Square $PQRS$ is divided into five rectangles, as shown. The area of the shaded rectangle is:

(A) 49 (B) 28 (C) 22 (D) 57 (E) 16

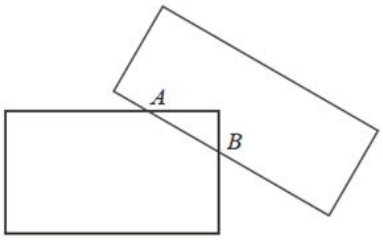


Q4. The total number of points where the graph of $y=\left(x^2-5x+9\right)\left(x^2-6x+9\right)\left(x^2-7x+9\right)$ intersect the x-axis are:

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Q5. In the given diagram, two rectangles intersect at exactly two points, A and B. The maximum possible finite number of point of intersection of any two rectangles is:

(A) 3 (B) 4 (C) 12 (D) 8 (E) 6



Q6. Three different numbers are chosen at random from the set $\{1,2,3,4,5\}$. The numbers are arranged in increasing order. The probability that the resulting sequence is an arithmetic sequence is:

(A) $\frac{2}{3}$ (B) $\frac{3}{5}$ (C) $\frac{1}{3}$ (D) $\frac{2}{5}$ (E) None of these

Q7. The value of $\sum_{i=1}^{100}\frac{1}{4i^2-1}$ when expressed as a simple fraction is given by:

(A) $\frac{100}{201}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{3998}{3999}$ (E) None of these

Q8. The real value of x satisfying the equation $\sqrt{x}=\sqrt{x-\frac{1}{x}}+\sqrt{1-\frac{1}{x}}$ is given by:

(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $\frac{5}{4}$ (E) No such real x exists

Q9. If $f(x,y)=xy+2x+y+1$, the value of $f\left(f\left(2,f\left(3,4\right)\right),5\right)$ is given by:

(A) 50 (B) 74 (C) 320 (D) 444 (E) 524

Q10. The value of x such that $2^{2^{3^{2^2}}} = 4^{4^x}$, is given by:

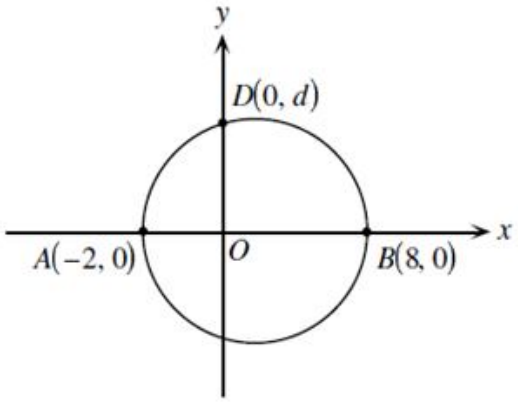
(A) 10 (B) 30 (C) 40 (D) 20 (E) 50

Q11. The number 2.5081081081... can be written as $\frac{a}{b}$ where a and b are natural numbers with no common factors. The value of $a + b$ is:

(A) 249 (B) 649 (C) 49 (D) 9 (E) None of these

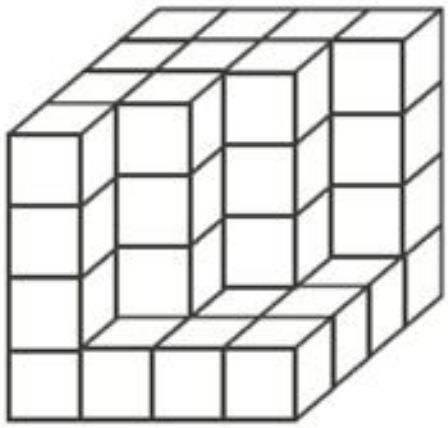
Q12. A circle, with diameter AB as shown, intersects the positive y-axis at a point $D(0, d)$. The value of d is:

(A) 2 (B) 3 (C) 4 (D) 4.5 (E) None of these



Q13. Sixty-four identical cubes are stacked in a $4 \times 4 \times 4$ arrangement and then some of the cubes are removed from the front as shown. No cube hidden from sight has been removed. The total number of cubes which remains in the arrangement is:

(A) 46 (B) 40 (C) 52 (D) 55 (E) 49



Q14. If $\frac{1}{1+2}+\frac{1}{1+2+3}+...+\frac{1}{1+2+...+20}=\frac{m}{n}$ where m and n are positive integers with no common divisor, the value of $m+n$ is:

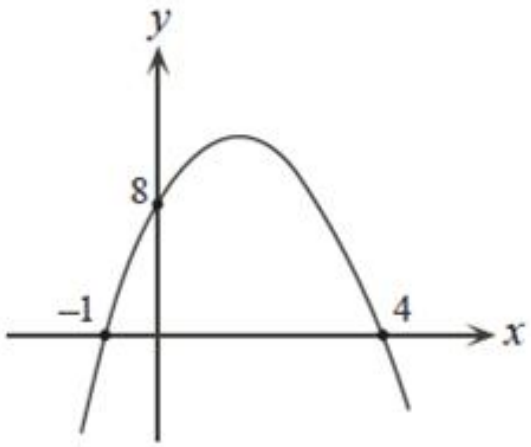
(A) 10 (B) 30 (C) 50 (D) 20 (E) 40

Q15. The sum of all the integral values of x such that $\left(x^2-3\right)\left(x^2+4\right)<0$, is given by:

(A) 0 (B) 1 (C) 2 (D) 3 (E) None of these

SECTION – B

Q16. In the given diagram, the parabola has x-intercepts -1 and 4, and y-intercept 8. If the parabola passes through the point $(3,a)$, what is the value of a?



Q17. Given that $3\sin x+4\cos x=5$, where $x\in\left(0,\frac{\pi}{2}\right)$. Find the value of $2\sin x+\cos x+4\tan x$.

Q18. Find the total number of addition problems in which a two-digit number is added to a second two-digit number to give a two-digit answer, such as in the given three examples:

$$\frac{23}{42}+\frac{65}{}, \quad \frac{36}{36}+\frac{72}{}, \quad \frac{42}{23}+\frac{65}{}.$$

Q19. Find the remainder when $f(x)=x^{200}-2x^{199}+x^{50}-2x^{49}+x^2+x+1$ is divided by $(x-1)(x-2)$.

Q20. If x and y are real numbers, determine all solutions (x,y) of the system of equations:

$$x^2-xy+8=0$$
$$x^2-8x+y=0$$

NOTE: The **Solution Key** of this paper will be available on School's blog www.crpfpsrohini.blogspot.com today after 6 pm. The **Result** will be declared on 22 December (Date of Birth of Great Indian Mathematician Ramanujan) and will be available on School's blog.