CRPF PUBLIC SCHOOL, ROHINI

SECOND Intra School Mathematics Olympiad 2011

CLASS XI

Max. Marks: 50 **Max. Time: 1 hour 30 minutes**

General Instructions:

1. Q1-15 (Section A) each MCQ carries 2 mark. Each question has five choices (A, B, C, D or E). Select the correct answer to each question and darken the corresponding circle in the Answer Sheet provided to you. **THERE IS NO NEGATIVE MARKING.** Marking of more than one circle for an answer shall be awarded zero mark.
2. Q16-20 (Section B) each question carries 4 mark. You are to give the complete solution. Marking will be done stepwise.

SECTION - A

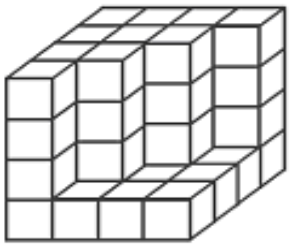
Q1.

If m and n are consecutive positive integers and $n^2 - m^2 > 20$, then the minimum possible value of $n^2 + m^2$ is

(A) 29 **(B)** 181 **(C)** 265 **(D)** 23 **(E)** 221

Q2.

Sixty-four identical cubes are stacked in a $4 \times 4 \times 4$ arrangement and then some of the cubes are removed from the front as shown. No cube hidden from sight has been removed. How many cubes remain in the arrangement?



(A) 46 **(B)** 40 **(C)** 52
(D) 55 **(E)** 49

Q3.

If $n > 0$ and $\sqrt{n^2 + n^2 + n^2 + n^2} = 64$, then n equals

(A) $\sqrt{8}$ **(B)** 16 **(C)** 4 **(D)** 32 **(E)** $\sqrt{2}$

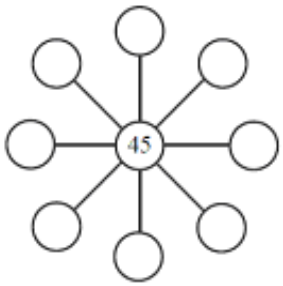
Q4.

The number of integers x for which the value of $\dfrac{-6}{x+1}$ is an integer is

(A) 8 **(B)** 9 **(C)** 2 **(D)** 6 **(E)** 7

Q5.

Different positive integers can be written in the eight empty circles so that the product of any three integers in a straight line is 3240. What is the largest possible sum of the eight numbers surrounding 45?



(A) 139 **(B)** 211 **(C)** 156
(D) 159 **(E)** 160

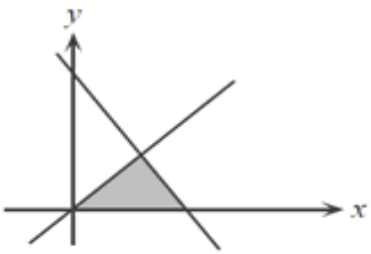
Q6.

The difference between the squares of two consecutive integers is 199. The sum of the squares of these two consecutive integers is

(A) 19 801 **(B)** 39 601 **(C)** 19 602 **(D)** 20 201 **(E)** 19 405

Q7.

In the diagram, the shaded region is bounded by the x-axis and the lines $y = x$, and $y = -2x + 3$. The area of the shaded region is

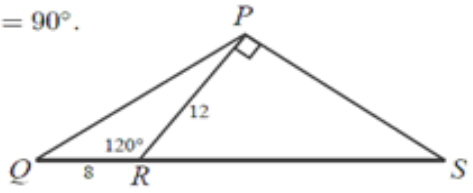


(A) $\frac{3}{4}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{9}{4}$

(D) 1 **(E)** $\frac{\sqrt{10}}{4}$

Q8.

In the diagram, R is on QS and $QR = 8$.
Also, $PR = 12$, $\angle PRQ = 120°$, and $\angle RPS = 90°$.
What is the area of $\triangle QPS$?



(A) $72\sqrt{3}$ **(B)** 72 **(C)** 36

(D) $60\sqrt{3}$ **(E)** $96\sqrt{3}$

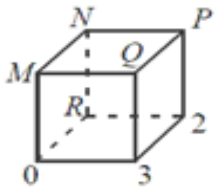
Q9.

Ivan trained for a cross-country meet.
On Monday, he ran a certain distance.
On Tuesday, he ran twice as far as he ran on Monday.
On Wednesday, he ran half as far as he ran on Tuesday.
On Thursday, he ran half as far as he ran on Wednesday.
On Friday, he ran twice as far as he ran on Thursday.
If the shortest distance that he ran on any of the five days is 5 km, how far did he run in total?

(A) 55 km **(B)** 25 km **(C)** 27.5 km **(D)** 17.5 km **(E)** 50 km

Q10.

In the diagram, each of the numbers 0, 1, 2, 3, 4, 5, 6, and 7 is to be used to label a vertex of the cube. The numbers 0, 2 and 3 are placed as shown. The sum of the numbers at the ends of each edge must be a prime number. (Note: 1 is not a prime number.) The value of $M + N + P + Q$ must be

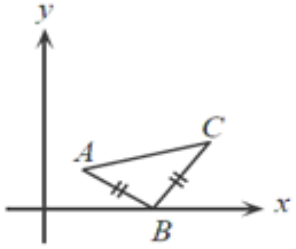


(A) 16 **(B)** 17 **(C)** 18

(D) 19 **(E)** 22

Q11.

In the diagram, $A(2, 2)$ and $C(8, 4)$ are two of the vertices of an isosceles right-angled triangle ABC. If the vertex B is located on the x-axis and $\angle ABC = 90°$, the x-coordinate of B is



(A) 3 **(B)** 4 **(C)** 5

(D) 6 **(E)** 7

Q12.

In seven term sequence, 5, p, q, 13, r, 40, x, each term after the third term is the sum of the preceding three terms. The value of x is

(A) 21 **(B)** 61 **(C)** 67 **(D)** 74 **(E)** 80

Q13.

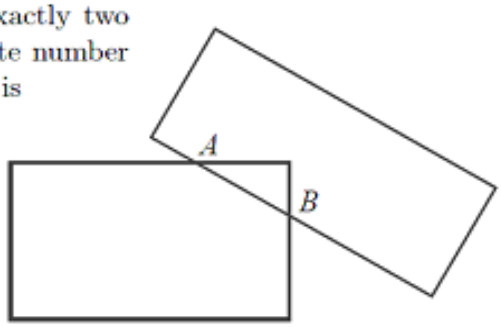
How many four-digit integers between 5000 and 6000 are there for which the thousands digit equals the sum of the other three digits? (The thousands digit of 5124 is 5.)

(A) 5 **(B)** 15 **(C)** 21 **(D)** 30 **(E)** 12

Q14.

In the diagram, two rectangles intersect at exactly two points, A and B. The maximum possible finite number of points of intersection of *any* two rectangles is

(A) 3 **(B)** 4 **(C)** 12 **(D)** 8 **(E)** 6



Q15.

The points $(5,3)$ and $(1,-1)$ are plotted on a sheet of graph paper. The sheet of graph paper is folded along a line so that the point $(5,3)$ lands on top of the point $(1,-1)$. The equation of the line that represents the fold is

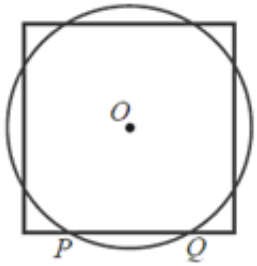
(A) $y=-x+1$ **(B)** $y=-x+2$ **(C)** $y=-x+3$
(D) $y=-x+4$ **(E)** $y=-x+5$

SECTION – B

Q16. Find the total number of pairs of positive integers (p,q), with $p+q \le 100$, that satisfy the equation $\dfrac{p+q^{-1}}{p^{-1}+q}=17$.

Q17.

In the diagram, the circle and the square have the same centre O and equal areas. The circle has radius 1 and intersects one side of the square at P and Q. What is the length of PQ?



Q18.

Suppose that a, b, c, and d are positive integers that satisfy the equations

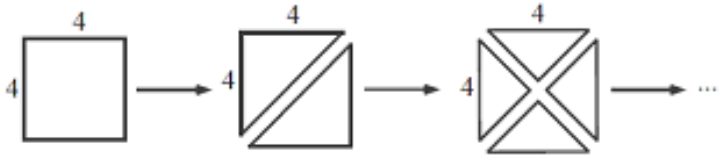
$$\begin{aligned} ab + cd &= 38 \\ ac + bd &= 34 \\ ad + bc &= 43 \end{aligned}$$

What is the value of $a + b + c + d$?

Q19. An arithmetic sequence is a sequence in which each term after the first term is obtained by adding a constant to the previous term.
If the first four terms of arithmetic sequence are $a, 2a, b$ and $a-6-b$ for some numbers a and b. Find the value of 100^{th} term.

Q20.

A 4×4 square piece of paper is cut into two identical pieces along its diagonal. The resulting triangular pieces of paper are each cut into two identical pieces.



Each of the four resulting pieces is cut into two identical pieces. Each of the eight new resulting pieces is finally cut into two identical pieces.

Find the length of the longest edge of one of these final sixteen pieces of paper.

NOTE: The **Solution Key** of this paper will be available on School's blog www.crpfpsrohini.blogspot.com today after 6 pm. The **Result** will be declared on 22 December (Date of Birth of Great Indian Mathematician Ramanujan) and will be available on School's blog.