SAMPLE QUESTIONS FOR SECOND INTRA SCHOOL MATHEMATICS OLYMPIAD 2011

CLASS XI

QUESTIONS

1.

The sum of k consecutive positive integers is 1000, where $k \geq 2$. What is the smallest possible value of k?

2.

The edges of a rectangular box are 2, 3, and 4. What is the length of the shortest path from one vertex to the opposite vertex on the surface of the box?

3.

Solve the system of equations

$$\begin{aligned} |x| + y &= 12 \\ x + |y| &= 6. \end{aligned}$$

What is the product of the values of x and y in the solution (x, y) ?

4.

What is the smallest positive integer n such that $\sqrt{n} - \sqrt{n-1} < 0.01$?

5.

If $f(x) = \dfrac{x^2}{x^2 - 1}$, what is the value of $51 \cdot f(50) \cdot f(49) \cdot \cdots \cdot f(3) \cdot f(2)$?

SOLUTIONS

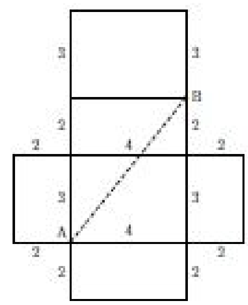
1.

k has to be at least 2, but 2 consecutive integers always add up to an odd number, since exactly one of them is odd. Can we have $k = 3$? No, the sum of three consecutive numbers would always be divisible by 3, since $x + (x+1) + (x+2) = 3x + 3$. Can $k = 4$ give a solution? The sum of four consecutive numbers x, $x+1$, $x+2$, and $x+3$ would be $4x + 6$, which cannot be divisible by 4. But 1000 is divisible by 4, therefore $k = 4$ will not provide a solution. Trying further with $k = 5$, the numbers should average to 200, one can easily find the solution of $198 + 199 + 200 + 201 + 202 = 1000$.

So, answer is 5.

2.

The best way to visualize the problem is to unfold the box and draw its net in two dimensions (see picture). The straight line connecting points A and B is the shortest path connecting opposite vertices. Using the Pythagorean theorem $\sqrt{4^2 + 5^2} = \sqrt{41}$, is the length of the shortest path. Notice that one may need to unfold the net three different ways to find the actual shortest path represented by a straight line segment.



3.

Let's first consider the situation when x is positive or zero, that is when $|x| = x$. Subtracting the first equation from the second, we will get $|y| - y = -6$, which is impossible since this difference is always positive or 0.

We found that x must be negative, so we can replace $|x|$ by $-x$. Now we add the two equations and get $y + |y| = 18$. This can only happen if $y = 9$ and then $x = -3$. The product of x and y is -27.

4.

We seek the smallest integer n such that $\sqrt{n} - \sqrt{n-1} < \frac{1}{100}$.

Since, for positive a, $a < c \Leftrightarrow \frac{1}{a} > \frac{1}{c}$, we must find the smallest integer such that

$\frac{1}{\sqrt{n}-\sqrt{n-1}} = \sqrt{n} + \sqrt{n-1} > 100.$

Since $\sqrt{2500} + \sqrt{2499} < 100$ and $\sqrt{2501} + \sqrt{2500} > 100$, the least such integer is 2501.

5.

Remember that $x^2 - 1 = (x-1)(x+1)$

Check $f(50) = \frac{50^2}{50^2 - 1} = \frac{50^2}{(50-1)(50+1)} = \frac{50^2}{(49)(51)}$

This pattern repeats with $f(49) = \frac{49^2}{(48)(50)}$ and $f(48) = \frac{48^2}{(47)(49)}$. Thus:

$$
\begin{aligned}
51 \cdot f(50) \cdot f(49) \cdot \cdots \cdot f(3) \cdot f(2) &= 51 \cdot \frac{50^2}{(49)(51)} \cdot \frac{49^2}{(48)(50)} \cdot \frac{48^2}{(47)(49)} \cdot \cdots \cdot \frac{3^2}{(2)(4)} \cdot \frac{2^2}{(1)(2)} \\
&= 50 \cdot 2 \\
&= 100
\end{aligned}
$$