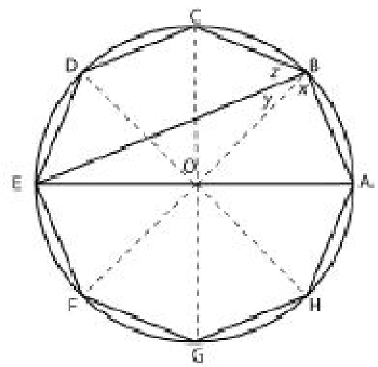
SAMPLE QUESTIONS FOR SECOND INTRA SCHOOL MATHEMATICS OLYMPIAD 2011

CLASS X

QUESTIONS

1. How may integer values of x satisfy $\frac{x-2}{5} < \frac{8}{3} < \frac{x+6}{7}$?

2. In the diagram, what are the measures of the angles x, y and z in the following regular octagon.



3. Shallu was given the task of removing all multiples of 2 and 3 from the set of numbers from 1 to 100. Find the number of numbers that remained.
4. Vicky was writing the page numbers of a book he wrote. If his book has 112 pages, how many digits did Jim write in all?

5. What is the sum of the solutions of the equation $|2-|1-x||=1$?

SOLUTIONS

1. Multiplying the inequation by the product of $3\times5\times7=105$ results in

$$\frac{x-2}{5} < \frac{8}{3} < \frac{x+6}{7}$$

$$\frac{(105)(x-2)}{5} < \frac{(105)(8)}{3} < \frac{(105)(x+6)}{7}$$

$$21(x-2) < 280 < 15(x+6)$$

In order to solve the inequality then,

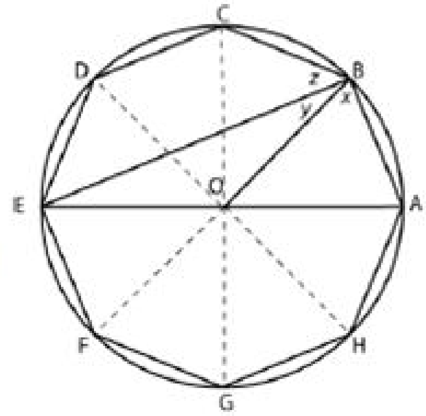
$21x-42<280$ and $15x+90>280$ and $21x-42<15x+90$

$21x<322$ and $15x>190$ and $6x<132$

$x<15\frac{1}{3}$ and $x>12\frac{2}{3}$ and $x<22$

Therefore, the only integers that satisfy all three conditions are 13, 14, and 15.

2.



Since the shape is a regular octagon then $\angle AOB = \frac{360°}{8} = 45°$ and ΔAOB is an isosceles triangle so $\angle OBA = \angle OAB$. Thus angle $x = \frac{180° - 45°}{2} = 67.5°$. Also using the property that all inscribed angles subtended by a diameter are right angles, we determine that $y = 90° - x = 90° - 67.5° = 22.5°$. Another way to find y is to notice that $\angle EOB = 180° - 45° = 135°$ and ΔEOB is isosceles. Thus $y = \frac{180° - 135°}{2} = 22.5°$.

Since ΔCOB is congruent to ΔBOA then $\angle OBC = \angle OBA$. Thus $x = y + z$ and since $x = 67.5°$ and $y = 22.5°$ then $z = 67.5° - 22.5° = 45°$.

Alternatively, it can be determined that $\angle CBA = 135°$. Since $x + y = 90°$ then $z = 45°$.

3. There are 50 multiples of 2 from 1 to 100, both numbers included, namely 2, 4, 6, etc. There are 33 multiples of 3 from 1 to 100, both numbers included, namely 3, 6, 9, etc. When added up, 83 numbers were counted, of which some were counted twice, namely all the numbers which are both multiples of 2 and 3, e.g. 6, 12, 18, etc. The numbers that are both multiples of 2 and 3, at the same time, are also multiples of 6, therefore if the number of multiples of 6 are removed from the above number once, it will cover the doubles. There are 16 multiples of 6 from 1 to 100, both numbers included, namely 6, 12, 18, etc. Therefore 83 – 16 = 67 numbers have been removed and 33 remained.
4. No. from 1, 2, …, 9 : sum of digits = 1 + 2+ …+ 9 = 45

No. from 10, 11, …, 19 sum of digits = 1+0 + 1+1 + … + 1+9= 1*10 + 45

Similarly, 90 + 91 +… 99 sum of digits = 9*10 + 45

So, sum till here = 45*10 + (10 + 20+… + 90) = 450 + 450 = 900

Now, 100, 101, … 112 sum of digits =

(1+0+ 0) + (1 + 0 + 1) +… + (1 + 1 + 2) =

(1 + 2 + … 10) + (1+ 1) + (1 + 1+ 1) + (1+ 1+ 2) = 55 + 9 = 64.

So, total sum = 900 + 64 = 964.

5.

To satisfy the equation, $2 - |1-x| = 1$ or -1. Correspondingly, $|1-x| = 1$ or $|1-x| = 3$. The first equation is satisfied by $x = 0$ and $x = 2$, while the second by $x = -2$ and $x = 4$. The sum of these four solutions is 4.