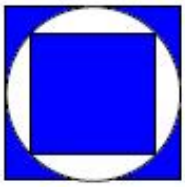
SAMPLE QUESTIONS FOR SECOND INTRA SCHOOL MATHEMATICS OLYMPIAD 2011

CLASS IX

QUESTIONS

1. In the diagram, a circle is inscribed in a square with sides of 6 metres and then a square is inscribed in the circle. What is the area of the shaded region?



2. Mr. Rahul was recording marks of his class. He calculated the class average to be 84, however, he had accidentally recorded a mark of 59 as 95. What was the actual class average if there are 20 students in his class?

3. If all the integers from 1 to 100 are written, how many times will the digit 5 appear?

4. If $x = 20^{49}$, $y = 25^{100}$ and $z = 36^{101}$, how many zeroes are there at the end of product xyz?

5. If a, b, c and d are positive integers such that

$$a + \frac{1}{b + \frac{d}{c}} = \frac{2005}{101}.$$

What is the value of $a+b+c+d$?

SOLUTIONS

1.

The area of the large square is 36 m^2.
The diameter of the circle is equal to the length of one side of the large square, so the radius of the circle is 3 m. Thus, the area of the circle is 9π m^2.
The diagonal of the small square is equal to the diameter of the circle. The diagonal divides the small square into two congruent right-angled isosceles triangles. Using the Pythagorean theorem, we can determine the length of each side of the small square. If each side of the smaller square is of length a, then

$$a^2 + a^2 = 6^2$$

$$2a^2 = 36$$

$$a^2 = 18$$

Thus, the area of the small square is 18 m^2.
The area of the shaded region is $36 - 9\pi + 18 = 54 - 9\pi \doteq 25.73$ m^2.

2. If there are 20 students in his class, then he recorded the sum of their marks as 20 x 84 =1680 . If he accidentally recorded a mark of 59 as 95, then he added 36 additional marks into the total class mark. Thus the correct sum is 1644 giving us an average of 82.2.

3.

Looking at the one's digit, a 5 will appear in each set of 10 numbers, so in the first one hundred numbers, there will be ten 5's in the one's digit. A 5 will appear in the ten's digit from 50-59. Thus, there will be ten 5's in the ten's digit. In total, the digit 5 will appear 20 times.

4.

A zero occurs when a number is multiplied by 10. Since the only prime factors of 10 are 5 and 2, we can see how many times a number will be multiplied by 10 by finding how many pairs of 5 and 2 there are in the prime factors.

$$x = 20^{99} = (5)^{99}(2^2)^{99} = 5^{99} \times 2^{198}$$

$$y = (5^2)^{100} = 5^{200}$$

$$z = (3^2)^{101}(2^2)^{101} = 3^{202} \times 2^{202}$$

The product xyz equals $(5^{299})(2^{400})(3^{202})$ or $(5^{299})(2^{299})(2^{101})(3^{202})$ which equals $(10^{299})(2^{101})(3^{202})$. Therefore, there are 299 factors of 10, so there will be 299 zeroes at the end of the product.

5.

$$\frac{2005}{101} = 19 + \frac{86}{101} = 19 + \frac{1}{\frac{101}{86}} = 19 + \frac{1}{1 + \frac{15}{86}}.$$

Hence $a+b+c+d$=19+1+86+15=121.