

ENRICHMENT PROBLEMS

CLASS XII

INDEFINITE INTEGRAL

Question

Answer

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| 1. $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$ | $\tan x - \cot x - 3x + c$ |
| 2. $\int \frac{1}{\sin x + \sqrt{3} \sin x} dx$ | $\frac{1}{2} \log \left \tan \left(\frac{x}{2} + \frac{\pi}{6} \right) \right + c$ |
| 3. $\int \frac{x^3}{\sqrt{1+2x}} dx$ | $\frac{1}{8} \left\{ \frac{(2x+1)^{7/2}}{7} - \frac{3}{5} (2x+1)^{5/2} + (2x+1)^{3/2} - (2x+1)^{1/2} \right\} + c$ |
| 4. $\int \frac{x^2 + 1}{(x^2 - 1)^2} dx$ | $\frac{x}{1-x^2} + c$ |
| 5. $\int \frac{x^2 - 1}{(x^4 + 3x^2 + 1) \tan^{-1} \left(x + \frac{1}{x} \right)} dx$ | $\log \left \tan^{-1} \left(x + \frac{1}{x} \right) \right + c$ |
| 6. $\int \frac{\cos 5x + \cos 4x}{1 - 2 \cos 3x} dx$ | $-\left(\frac{\sin 2x}{2} + \sin x \right) + c$ |
| 7. $\int e^x \left(\frac{1-x}{1+x^2} \right)^2 dx$ | $\frac{e^x}{1+x^2} + c$ |
| 8. $\int \frac{x^2}{(x \sin x + \cos x)^2} dx$ | $\frac{-x \sec x}{(x \sin x + \cos x)} + \tan x + c$ |
| 9. $\int \frac{\sin(x+a)}{\sin(x+b)} dx$ | $(x+b) \cos(a-b) + \sin(a-b) \log \left \sin(x+b) \right + c$ |
| 10. $\int \sqrt{\frac{x}{a^3 - x^3}} dx$ | $\frac{2}{3} \sin^{-1} \left(\frac{x^{3/2}}{a^{3/2}} \right) + c$ |
| 11. $\int \sqrt{\frac{\sin(x-a)}{\sin(x+a)}} dx$ | $-\cos a \sin^{-1} \left(\frac{\cos x}{\cos a} \right) - \sin a \log \left \sin x - \sqrt{\sin^2 x - \sin^2 a} \right + c$ |
| 12. $\int \log(\sqrt{1-x} + \sqrt{1+x}) dx$ | $x \log(\sqrt{1-x} + \sqrt{1+x}) - \frac{1}{2}x + \frac{1}{2} \sin^{-1} x + c$ |
| 13. $\int (1 + \tan x \cdot \tan(x+a)) dx$ | $\cot a \cdot \{ \log \sec(x+a) - \log \sec x \} + c$ |
| 14. $\int \frac{\sqrt{\cos 2x}}{\sin x} dx$ | $-\log \left \cot x + \sqrt{\cot^2 x - 1} \right + \sqrt{2} \log \left \cos x + \sqrt{\cos^2 x - 1/2} \right + c$ |
| 15. $\int \frac{1}{\sec x + \cos ecx} dx$ | $\frac{1}{2} \left\{ (\sin x - \cos x) + \frac{1}{\sqrt{2}} \log \left \frac{\tan(x/2) - 1 - \sqrt{2}}{\tan x - 1 + \sqrt{2}} \right \right\} + c$ |
| 16. $\int \sin 4x e^{\tan^2 x} dx$ | $-2 \cos^4 x \cdot e^{\tan^2 x} + c$ |
| 17. $\int \frac{1}{2 \sin x + \sec x} dx$ | $\frac{1}{2\sqrt{2}} \log \left \cos ec \left(x + \frac{\pi}{4} \right) - \cot \left(x + \frac{\pi}{4} \right) \right - \frac{1}{2(\sin x + \cos x)} + c$ |
| 18. $\int \sqrt{x} (1+x^{1/3})^4 dx$ | $6 \left\{ x^{2/3} + \frac{4}{11} x^{11/6} + \frac{6}{13} x^{13/6} + \frac{4}{15} x^{5/2} + \frac{1}{17} x^{17/6} \right\} + c$ |
| 19. $\int \frac{1}{(x-1)^{3/4} (x+2)^{5/4}} dx$ | $\frac{4}{3} \left(\frac{x-1}{x+2} \right)^{1/4} + c$ |

20. $\int \frac{1}{x+\sqrt{a^2-x^2}} dx$ $\frac{1}{2} \sin^{-1}\left(\frac{x}{a}\right) + \frac{1}{2} \log|x+\sqrt{a^2-x^2}| + c$
21. $\int \frac{1}{\sin^6 x + \cos^6 x} dx$ $\tan^{-1}(-2 \cot 2x) + c$
22. $\int \frac{1}{(x^2+a^2)^2} dx$ $\frac{1}{2a^2} \left\{ \frac{x}{x^2+a^2} + \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) \right\} + c$
23. $\int \frac{1}{(5+4 \cos x)^2} dx$ $\frac{10}{27} \tan^{-1}\left(\frac{\tan(x/2)}{3}\right) - \frac{4}{9} \left(\frac{\sin x}{5+4 \cos x}\right) + c$
24. $\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right) dx$ $(x+1) \tan^{-1}\left(\frac{2}{3}(x+1)\right) - \frac{3}{4} \log|4x^2+8x+13| + c$
25. $\int \frac{1}{\cos x + \cos ecx} dx$ $\frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + \sin x - \cos x}{\sqrt{3} - \sin x + \cos x} \right| - \tan^{-1}(\sin x + \cos x) + c$
26. $\int (x + \sqrt{1+x^2})^3 dx$ $\frac{1}{8} (x + \sqrt{1+x^2})^4 + \frac{1}{4} (x + \sqrt{1+x^2})^2 + c$
27. $\int \cot^{-1}(x^2+x+1) dx$ $x \{ \tan^{-1}(x+1) - \tan^{-1}x \} + \frac{1}{2} \log|1+x^2| - \frac{1}{2} \log|1+(x+1)^2| + \tan^{-1}(1+x) + c$
28. $\int \frac{\tan^{-1} x}{x^4} dx$ $-\frac{\tan^{-1} x}{3x^3} - \frac{1}{6} \log \left| \frac{x^2+1}{x^2} \right| - \frac{1}{6x^2} + c$
29. $\int \frac{1}{\tan x + \cot x + \sec x + \cos ecx} dx$ $\frac{1}{2} (\sin x - \cos x - x) + c$
30. $\int x^2 \log(1-x^2) dx$ $-\left\{ \frac{x^5}{1.5} + \frac{x^7}{2.7} + \frac{x^9}{3.9} + \dots \infty \right\}$
31. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$ $2 \cos^{-1} \sqrt{x} - 2 \log \left| \frac{1+\sqrt{1-x}}{\sqrt{x}} \right| + c$
32. $\int \sec^{-1}\left(\frac{1+x}{1-x}\right) dx$
33. $\int e^x \cdot \frac{x^3-x+2}{(x^2+1)^2} dx$
34. $\int \frac{\sqrt{x^2+2x-3}}{x+2} dx$ $\sqrt{t^2+2t-3} - \log|(t+1)+\sqrt{t^2+2t-3}| - \sqrt{3} \sin^{-1}\left(\frac{t+5}{t+2}\right) + c$ where $t=x+1/x$
35. $\int \frac{\sin x + \cos x}{\sin^4 x + \sin^2 x} dx$ $\tan^{-1}(\sin x - \cos x) + \frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + \sin x - \cos x}{\sqrt{3} - \sin x + \cos x} \right| + c$
36. $\int \frac{1}{\sqrt[4]{1+x^4}} dx$ $-\frac{1}{4} \log \left| \frac{(1+1/x^4)^{1/4}}{(1+1/x^4)^{1/4} + 1} \right| - \frac{1}{2} \tan^{-1}(1+1/x^4)^{1/4} - 1 + c$
37. $\int \frac{e^{\tan^{-1} x}}{(1+x^2)^2} dx$ $\frac{e^{\tan^{-1} x}}{5} \left(\frac{2x^2+2x+3}{1+x^2} \right) + c$
38. $\int \frac{\sqrt{5-x}}{x} dx$ $2\sqrt{5} \left[\log \left| \sqrt{\frac{5}{x}} - \sqrt{\frac{5-x}{x}} \right| + \sqrt{\frac{5-x}{x}} \right] + c$
39. $\int \frac{x e^x}{\sqrt{1+e^x}} dx$ $2(x-2)\sqrt{1-e^x} - 2 \log \left| \frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1} \right| + c$
40. $\int \frac{x^2-1}{x\sqrt{x^4+3x^2+1}} dx$ $\log \left| \frac{x^2+1+\sqrt{x^4+3x^2+1}}{x} \right| + c$