KENDRIYA VIDYALAYA SANGATHAN

CLASS - XII

MATHEMATICS

HOTS

2009 - 10

2009 - 10

CHAPTER – 1

RELATIONS AND FUNCTIONS

1 . Let f: R → R be defined by $f(x) = x|x|$
State whether the function f(x) is onto.

2. Let∗ be the binary operations on Z given by $a * b = a + b + 1 \; \forall\, a, b \in Z$. Find the identify element for ∗ on Z, if any.

3. State with reason whether the function f: X→Y have inverse, where $f(x)= \frac{1}{x} \;\; \forall x \in X$. and X=Q-{o}, Y=Q .

4. Let $Y = \{n^2: n\in N\}$ be a subset of N and let "f" be a function f : N →Y defined as $f(x) = x^2$.
Show that "f" is invertible and find inverse of "f".

5. Show that the function f : N →N given by $f(x) = x - (-1)^x$ is bijective.

6. If f be the greatest integer function and g be the absolute value function; find the value of (fog)(-3/2) + (gof)(4/3).

7.Consider the mapping f :[0,2] → [0,2] defined by $f(x)=\sqrt{4-x^2}$. Show that f is invertible and hence find f^{-1}.

8. Give examples of two functions f N→N and g :Z→Z such that gof is injective but g is not injective

9..Give examples of two functions f:N→N and g: N→N such that gof is onto but f is not onto.

10..Let f: R- {-3/5}→ R be a function defined as $f(x) = \frac{2x}{5x+3}$, find the inverse of f.

11. Show that the relation R defined by (a, b) R (c, d)⇒ a + d=b + c on the set $N \times N$ is an equivalence relation.

12.Let Q^+ be the set of all positive rational numbers.

- Show that the operation ∗ on Q^+ defined by $a*b = \frac{1}{2}(a+b)$ is a binary operation.
- Show that ∗ is commutative.
- Show that ∗ is not associative.

13. Let A= N × N. Let ∗ be a binary operation on A defined by $(a,b) * (c,d) = (ad + bc, bd)$ $\forall$ a,b,c,d ∈N. Show that (i) ∗ is commutative (ii) ∗ is associative

(iii) identity element w.r.t. * does not exist.

14. Draw the graph of the function $f(x) = x^2$ on R and show that it is not invertible. Restrict its domain suitably so that f^{-1} may exist, find f^{-1} and draw its graph.

15. Show that the relation " congruence modulo 2" on the set Z is an equivalence relation.Also find the equivalence class of 1

CHAPTER – 2
INVERSE TRIGONOMETRIC FUNCTIONS

1) Prove that $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$

2) Solve $\tan^{-1}\frac{x-1}{x+1} + \tan^{-1}\frac{2x-1}{2x+1} = \tan^{-1}\frac{23}{56}$

3) Write $\tan^{-1}\left(x+\sqrt{1+x^2}\right)$, $x \in R$, in the simplest form.

4) Solve that $\tan^{-1}(x+1)+\tan^{-1}x+\tan^{-1}(x-1)=\tan^{-1}3$

5) Prove that $\frac{\alpha^2}{2}\,cosec^2\left(\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}\right)+\frac{\beta^2}{2}\,sec^2\left(\frac{1}{2}\tan^{-1}\frac{\beta}{\alpha}\right) = (\alpha+\beta)\ (\alpha^{\uparrow}2+\beta^{\uparrow}2\ )$

6) Solve for x : $\sin^{-1}\frac{2\alpha}{1+\alpha^2} + \sin^{-1}\frac{2\beta}{\beta^2+1} = 2\tan^{-1}x$

7) If $\tan^{-1}a + \tan^{\uparrow}(-1)\ b + \tan^{\uparrow}(-1)\ c = \pi$, prove that a + b + c = abc

8) Prove that $\cos(\tan^{\uparrow}(-1)(\sin[(\cot^{\uparrow}(-1)x]))) = \sqrt{\frac{x^2+1}{x^2+2}}$

9) What is the principal value of $\cos^{-1}\cos\left(\frac{8\pi}{7}\right)$?

10) If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z = \frac{\pi}{2}$, prove that xy + yz + zx = 1

11) Show that 4 $\tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{120}{119}\right)$

12) If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z = \frac{3\pi}{2}$, *then find the value of* $x^{100}+y^{100}+z^{100}-\frac{1}{x^{101}+y^{101}+z^{101}}$

13) If $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\beta$, *prove that* $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\beta+\frac{y^2}{b^2}=\sin^2\beta$

14) If $x + \frac{1}{x} = 2$, find the value of $\sin^{-1} x$

15) Find the value of $\sin(2\sin^{-1} 0.8)$.

CHAPTER - 3

MATRICES

1. If $\begin{bmatrix} 0 & 6-5x \\ x^2 & x-3 \end{bmatrix}$ is symmetric, find x.

2. If A= $\begin{bmatrix} x & y \\ z & -x \end{bmatrix}$ is such that A^2=I, then find the value of $2 - x^2 - yz$

3. If A = $\begin{pmatrix} -4 & 1 \\ 3 & 2 \end{pmatrix}$ then find f (A) when f(x) = $x^2 - 2x + 3$.

4. If A=$\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ find A^{4n} ,n∈N

5. Give an example of a square matrix which is both symmetric as well as skew symmetric.

6. If A and B are symmetric matrices, then show that AB + BA is also a symmetric matrix but AB-BA is skew symmetric matrix.

7. Show that all the positive integral powers of Symmetric matrix are Symmetric.

8. Find the matrix A satisfying the matrix equation $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} A \begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

9.If A=$\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, $a \neq 1$, Prove by induction that $A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$ for all positive integer n.

10.Find x if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

11. By using elementary row transformation , find A^{-1} where A = $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

12. If A = $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ prove that $(aI + bA)^3 = a^3 I + 3a^2 bA$.

13. If A and B are two matrices such that AB=B and BA=A find $A^2 + B^2$

14. If $A = [a_{ij}]_{m \times n}$ is a skew-symmetric matrix, what is the value of a_{ii} for every i ?

15. $A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$, find the matrix B such that AB = I

CHAPTER - 4

DETERMINANTS

1. If a,b,c are non-zero real numbers, then find the inverse of matrix A= $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.

2. If A= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ then what is the $|\text{adj}(\text{adj}A)|$?

3. If A is a sqaure matrix of order 3 such that |Adj A|=64, then find |A|

4. Find the value(s) of θ, if the matrix $\begin{bmatrix} 2\cos\theta & 1 \\ 3 & 2\cos\theta \end{bmatrix}$ is singular,

   where $0 < \theta < \pi$.

5. Evaluate the determinant $\begin{vmatrix} \log_a b & 1 \\ 1 & \log_b a \end{vmatrix}$

6. If $\begin{vmatrix} \lambda^2 + 3\lambda & \lambda - 1 & \lambda + 3 \\ \lambda + 1 & 2 - \lambda & \lambda - 3 \\ \lambda - 3 & \lambda + 4 & 3\lambda \end{vmatrix} = A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E$, then find the value of E

7. The value of a third order determinant is 12. Find the value of the square of the determinant formed by the cofactor

8. Let A be a skew symmetric matrix of odd order, then what will be $|A|$

9. If f(x) = $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$,then show that $\{f(x)\}^{-1} = f(-x)$

10. Prove the following by using the properties of determinants

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

11. Using the properties of determinants, solve for x. $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

12. If l, m, n are in A.P. then, find value of $\begin{vmatrix} 2x+4 & 5x+7 & 8x+l \\ 3x+4 & 6x+8 & 9x+m \\ 4x+6 & 7x+9 & 10x+n \end{vmatrix}$

13. If $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$ and α is not a root of the equation $ax^2+bx+c=0$,then show that a,b,c are in G.P.

14. Let $\begin{vmatrix} x^k & x^{k+2} & x^{k+3} \\ y^k & y^{k+2} & y^{k+3} \\ z^k & z^{k+2} & z^{k+3} \end{vmatrix}$=(x-y) (y-z) (z-x) ($\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$), then find k

15. Let $=\begin{bmatrix} 2 & 3 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}$, find A^{-1} . Hence solve the following system of equations

2x – y – z =7
3x + y – z = 7
x + y – z = 3

16. Given that A= $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B= $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ Find AB and use it to solve the system of equations x – y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1.

17. Prove that $\begin{vmatrix} (a+1)(a+2) & (a+2) & 1 \\ (a+2)(a+3) & (a+3) & 1 \\ (a+3)(a+4) & (a+4) & 1 \end{vmatrix} = -2$.

18.Using the properties of the determinants, prove that $\begin{vmatrix} mc_1 & mc_2 & mc_3 \\ nc_1 & nc_2 & nc_{3_1} \\ pc_1 & pc_2 & pc_3 \end{vmatrix} =$

$$\frac{\text{mpn (m - n) (n - p) (p - m)}}{12}.$$

19. Evaluate $\begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix}$

CHAPTER – 5
CONTINUITY AND DIFFERENIABILITY

1. Show that the function f(x) = $|Sinx + Cosx|$ is continuous at x= π
2. Show that the logarithmic function is continuous.
3. Let f(x) = $(x-a)Cos\frac{1}{x-a}$ for x $\neq$ a and let f(a)= 0 . Show that f is continuous at x=a but not derivable there at.
4. Let f(x)= x.$|x|$ for all x$\in$R. Discuss the continuity and defferentiability of f(x) at x=0.
5. Examine for continuity and differentiability of the following functions:-

$$f(x) = \begin{cases} |x| \text{ Sin } 1/x & x>0 \\ \mathbf{0} & \text{, if } x \leq 0 \quad \text{at } x=0 \end{cases}$$

6. Given that $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & , if\ x < 0 \\ a \quad , & if\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} , & if\ x > o \end{cases}$

   If f(x) is continuous at x=0, find the values of a.

7. $\begin{cases} 3ax+b, & \text{if } x>1 \\ & \end{cases}$

If the function f(x) = $\begin{cases} 11 & \text{if } x=1 \\ 5ax - 2b & \text{if } x<1 \end{cases}$

is continuous at x=1, find the values of a and b

8. Discuss for continuity of the function at x=0

$$f(x)=\begin{cases} \dfrac{\sin 3x}{\tan 2x}, & \text{if } x<0 \\ 3/2, & \text{if } x=0 \\ \dfrac{\log(1+3x)}{e^{2x}-1} & \text{if } x>0 \end{cases}$$

9. Find all points of discontinuity of f where

$$f(x) = \begin{cases} \dfrac{\sin x}{x}, & \text{if } x<0 \\ x+1, & \text{if } x \geq 0 \end{cases}$$

10. Show that the function f(x) = $\begin{cases} 1+x, & \text{if } x \leq 2 \\ 5-x, & \text{if } x>2 \end{cases}$

is not differentiable at x=2

11. Is the function

$$f(x) = \begin{cases} \dfrac{[x]-1}{x-1} & x \neq 1 \\ -1, & x=1 \end{cases}$$

continuous at x=1 ?

12. Show that the function f is continuous at x=0 for all values of a. Also find the value of a for which f is derivable at x=0 when

$$f(x) = \begin{cases} x^2, & x \geq 0 \\ ax, & x<0 \end{cases}$$

13. Examine the continuity of the function $f(x) = \tan^{-1}(3x^3-2x+1)$

14. If $f(x)= \begin{cases} \text{Sin } 2x, & 0<x\leq \pi/6 \\ ax+b & \pi/6<x<1 \end{cases}$

is continuous and differentiable. Find a & b

15. Find whether the function $f(x) = \begin{cases} \dfrac{e^{1/x}-1}{e^{1/x}+1} & x\neq 0, \\ 0 & x=0 \end{cases}$ is continuous.

16. Find whether $f(x) = \begin{cases} \dfrac{x^4-5x^2+4}{|(x-1)(x-2)|}, & x\neq 1,2 \\ 6, & x=1 \\ 12, & x=2 \end{cases}$

is Continuous?

17). Find the value of derivative at x=2 of the function

$f(x)= |x-1| + |x-3|$

18). Find the derivative of the following w.r.t.x.

1) $y=\log\left(\frac{1}{1+x}\right)$.

2) $y=\sin(x^x)$.

3) $y=x\sin y$.

4) $x^y=e^{x-y}$

5) $y=e^{x^2}$.

6) $y=(\sin^{-1}x)^2$.

7) $y=\sin^{-1}\frac{2^{x+1}}{1+4^x}$

8) $y=\sin^{-1}\left(\frac{a+b\cos x}{b+a\cos x}\right)$

9) $y=b\tan^{-1}\left[\frac{x}{a}+\tan y/x\right]$

10) $y=\tan^{-1} x/(1+\sqrt{1-x^2})$

11) $y=\sin^{-1}[x^2\sqrt{(1-x^2)}+x\sqrt{(1-x^4)}]$

12) $y= \text{Cos}^x(x^x)$

13) $y = e^{-ax^2 \log\sin x}$

19). $x=\sin^3x/\sqrt{\cos 2t}$, $y=\cos^3x/\sqrt{\cos 2t}$

20). If $x^p y^q=(x+y)^{pq}$ then show that $\frac{dy}{dx} = \frac{y}{x}$

21). Differentiate $(\sin x)^x$ w.r.t.$x^{\sin x}$

23). If $x=a\sin 2t(1+\cos 2t)$ & $y=b\cos 2t(1-\cos 2t)$ Show that $\frac{dy}{dx}=\frac{b}{a}$ at $t=\frac{\pi}{4}$

24). Differentiate $\cos^{-1}[\frac{3\cos x-4\sin x}{5}]$ w.r.t.x

25). Differentiate $\sin^2x$ w.r.t $e^{\cos x}$.

26). Show that $y=c_1e^x +c_2e^{-x}$ is the general solution of $\frac{d^2y}{dx^2}-y=0$

27). Prove that the solution of $y = x\frac{dy}{dx}+a\frac{dx}{dy}$ is $y=cx+a/c$.

28). If $y=x\log(\frac{x}{a+bx})$, Prove that $\frac{d^2y}{dx^2}=\frac{1}{x}(\frac{a}{a+bx})^2$

29). Differentiate $y=\log_7(\log x)$ w.r.t.x

30). Differentiate $y=\sin(\sqrt{\cos x})$ w.r.t.x.

31). Differentiate $y=\sqrt{a+\sqrt{a+x}}$ w.r.t.x.

32). Differentiate $y=\tan^{-1}(\frac{\sqrt{x}+\sqrt{a}}{1-\sqrt{ax}})$, w.r.t.x.

33). Differentiate $y=\log\{\tan(\Pi/4+x/2)\}$ w.r.t.x

34). Verify Rolle's Theorem for $f(x) = \log(x^2+2) - \log 3$ on $[-1, 1]$

35).Verify Rolle's Theorem for $f(x) = \text{Sin}^4x + \text{Cos}^4x$ in $[0, \frac{\Pi}{2}]$

36). Verify Rolle's Theorem for $f(x) = e^{-x}\,\text{Sin}x$ in $[0, \Pi]$

37). Verify LMV theorem for f(x) = Sinx – Sin 2x on [0 , Π]

38).Find a point on the Parabola $y = (x - 3)^2$ where the tangent is parallel to the chord joining (3 , 0)and (4 , 1)

CHAPTER – 6
APPLICATIONS OF DERIVATIVES

1) Show that the rate of change of the perimeter of a square is 4 times the rate of change the length of its sides.

2) Using differentials ,find the approximate value of $\log_e 4.01$, given that $\log_e 4$ = 1.3863

3) The pressure p and the volume v of a gas are connected by the relation pv =1.4 = constant. . Find the percentage error in p corresponding to a decrease of 1/2 % in v.

4) If there is an error of 2 % in measuring the length of a simple pendulum ,then find the percentage error in its time period.

5) While measuring the side of an equilateral triangle , an error of 5% is made. Find the percentage error in its area .

6) For what value of x is the rate of increase of x^3-5x^2+8 is twice the rate of increase of x ?

7). If the rate of change of area of a circle is equal to the rate of change of its diameter ,find the radius.

8). The side of an equilateral triangle is increasing at the rate of 1/3 cm/sec. Find the rate of increase of its perimeter.

9). Find 'a" for which f(x)=a(x + Sinx)+a is increasing

10). Let g(x) =f(x) +f(2a-x) and f ''(x)>0 for all $x \in [0,2a]$ then g(x) increasing or decreasing on [0,a]?

11). Let f(x)= $\tan^{-1}$ g(x), where g(x) is monotonically increasing for $0<x<\pi/2$, then find f(x) is increasing or decreasing on (0, $\pi/2$).

12). Find whether the function $f(x) = \tan^{-1}(\text{Sin } x + \text{Cos } x)$ on $[0, \pi/4]$ is either strictly increasing or strictly decreasing..

13). For what value of 'λ' for which the function $f(x)= \cos x - 2\lambda x$ is monotonic decreasing.

14).Find the value of 'a' for which function $f(x) = \log_a x$ is increasing on R,

15). If the slope of tangent to curve $y=x^3+ax+b$ at (1,-6) is -1. Find a & b .

16). If $x+y=k$ is normal to the curve $y^2=12x$, then find the value of K.

17). Find the point at which the curves $x^2=y$ and $y^2=x$ cut orthogonally.

18).Find whether the function $f(x)= \frac{x}{1+|x|}$ is increasing or decreasing

19).Is the function $f(x)=2^x$ is strictly increasing on R?

20. Find the angle of intersection of the curves $xy=a^2$ and $x^2-y^2=2a^2$

21. Find the condition for which the curve $y=ae^x$ and $y=be^{-x}$ cut orthogonally.

22. Find the slope of tangent of curve $y=3x^2+4x$ at the point whose abscissa is -2?

23. What is the slope of Normal to curve $y=2x^2+3\text{Sin } x$ at $x=0$?

24. If the function $f(x) =x^2-kx+5$ is increasing on [2,4] then find the value of k.

25).Find the interval for which the function $f(x) =x^x$ is decreasing

26). A man 2 meters high, walks at a uniform speed 6 meters per minute away from a lamp-post ,5 meters high. Find the rate at which the length of its shadow increases .

27) A kite is 120 m high and 130 m string is out. If the kite is moving away horizontally at the rate of 52m/sec find the rate at which the string is being paid out .

28) An inverted cone has a depth of 10 cm and a base of radius 5 cm. Water is poured into it at the rate of 3/2 cc per minute. Find the rate at which the level of water in the cone is rising when the depth is 4 cm.

29) The time T of complete oscillation of a simple pendulum of length l is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$,where g is a constant.What is the percentage error in T when l is increased by 1% ?

30) Find the approximate value of $\tan 46^\circ$ if it is given that $1^0 = .01745$

31) A man is walking at the rate of 4.5km/hr towards the foot of the tower 120m high. At

what rate is he approaching the top of the tower when he is 50m away from the tower?

32) Find the rate of change of the curved surface of a right circular cone of radius r and height h with respect to the change in radius.

33) Find the angle between the parabola $y^2=4ax$ and $x^2=4by$ at their point of intersection other than origin.

34) If y= alogx $+bx^2+x$ has its extreme values at x=-1 and x= 2, then find a & b.

Show that a local Minimum value of f(x)= $x+\frac{1}{x}, x \neq 0$ is greater than a local maximum value.

35) Find the Absolute maxima and Absolute minimum values of the function

f(x)= $(\frac{1}{2}-x\ )^2+x^3 on[-2,-25]$

36) Determine the Maximum and Minimum Values of the function

y=2Cos 2x-Cos 4x, $0\leq x\leq \pi$

37) Find the local minimum value of f(x) = $3+|x|$, $x\in R$

38) A given quantity of metal is to be cast into a solid half circular cylinder (i.e with rectangular base and semi circular ends). Show that in order that the total surface area may be minimum , the ratio of the length of the cylinder to the diameter of its circular ends is

39). A window has the shape of a rectangle surrounded by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

40). Show that the isosceles triangles of maximum area that can be inscribed in a given circle is an equilateral triangle

CHAPTER -7
INTEGRALS

Indefinite Integrals

1. Evaluate $\int x^5\sqrt{a^3+x^3}\,dx$
2. Evaluate $\int \frac{Sec2x\tan 2x}{\sqrt{e^{2\sec 2x}}}dx$
3. Evaluate $\int \frac{a^x}{\sqrt{1-a^{2x}}}dx$
4. Evaluate $\int 5^{5^{5^x}}5^{5^x}5^x dx$
5. Evaluate: $\int Sec^{-1}x\,dx$
6. Evaluate: $\int \sin(\log x)\,dx$
7. Evaluate: $\int \frac{x}{(x^2+1)(x^2+4)}dx$
8. Evaluate $\int \frac{1}{(\sin x)^{3/4}(\cos x)^{5/4}}dx$
9. Evaluate $I=\int \tan x\tan 2x\tan 3x dx$
10. Evaluate $I=\int \frac{\sin 2x}{(a+b\cos x)^2}dx$
11. Evaluate $\int \sqrt{\sec x-1}dx$
12. Evaluate $I=\int \frac{tanx+\tan^3 x}{2+3\tan^2 x}dx$

13. Evaluate $\int \frac{dx}{\sin x+\sqrt{3}\cos x}$

14. Evaluate: $\int e^{x}(\frac{1}{x}-\frac{2}{x^{2}}+\frac{2}{x^{3}})dx$

15. Evaluate: $\int \frac{1}{(\sin x-2\cos x)(2\sin x+\cos x)}dx$

16. Evaluate: $\int \frac{\sin x}{\sin 4x}dx$

17. Evaluate: $\int \sin^{-1}\sqrt{\frac{x}{a+x}}dx$

18. Evaluate: $\int \frac{dx}{1+x+x^{2}+x^{3}}$

19. Evaluate: $\int x(\tan^{-1}x)^{2}dx$

20. Evaluate $\int \frac{1}{\sin x+\sec x}dx$

21. Evaluate $I=\int \frac{x\cos\alpha+1}{\left(x^{2}-2x\cos\alpha+1\right)^{3/2}}dx$

22. Evaluate $\int \frac{\sqrt{1+x^{2}}}{1-x^{2}}dx$

23. Evaluate $\int \frac{1}{\sin^{4}x+\cos^{4}x}dx$

24. Evaluate $I=\int \frac{1}{(x+1)\left(x^{2}+2x+2\right)}dx$

25. Evaluate $\int \frac{x^{2}}{(x\sin x+\cos x)^{2}}dx$

26. Evaluate $\int \frac{x^{2}}{x^{4}+x^{2}+1}dx$

27. Evaluate $\int \sqrt{\tan x}\,dx$

28. Evaluate $\int \frac{1}{3+\sin 2x}dx$

29. Evaluate $I=\int \sqrt{\frac{\sin(x+\alpha)}{\sin(x-\alpha)}}dx$

30. Evaluate: $\int \frac{dx}{(e^{x}+1)^{3}}$

31. Evaluate: $\int \frac{\sqrt{x^{2}+1}\left[\log\left(x^{2}+1\right)-2\log x\right]}{x^{4}}dx$

32. Evaluate: $\int \frac{\sec x}{1+\operatorname{cosec} x} dx$

33. .Evaluate: $\int \frac{1}{\sin x+\sin 2x} dx$

34. Evaluate: $\int e^x \sin^2 x\, dx$

35. Evaluate: $\int x(\log x)^2\, dx$

36. Evaluate: $\int \frac{1}{\sin x+\tan x} dx$

Definite Integrals

37). Evaluate $\int_{-1}^{1} \frac{x^2}{1+x^2} dx$

38). Evaluate $\int_{0}^{2} x\sqrt{2-x}\; dx$

39) . Evaluate $\int_{0}^{1} \log\left(\frac{1}{x}-1\right) dx$

40). Evaluate $\int_{0}^{\frac{\pi}{2}} \frac{x}{\sin x+\cos x} dx$

41). Evaluate $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-\cos 2x} dx$

42). Evaluate $\int_{0}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+\sin x\cos x} dx$

43). Evaluate $\int_{\frac{1}{e}}^{e} \left|\log_e x\right| dx$

44).Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{3}{2}}} dx$

45).Evaluate $\int_{0}^{1.5} \left[x^2\right] dx$

46).Evaluate $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} |\sin x|\, dx$

47).Evaluate $\int_{-3}^{3} \frac{|x+2|}{x+2} dx$

48). Evaluate $\int_{-1}^{\frac{3}{2}} |x \sin \pi x|\, dx$

49). Evaluate $\int_{0}^{1} \frac{\log(1+x)}{1+x^2} dx$

50) Evaluate $\int_{1}^{4} \left(|x-1|+|x-2|+|x-3|\right) dx$

51).Evaluate as a limit of sum $\int_{-1}^{1} e^{-5x}\, dx$

52). Evaluate $\int_{0}^{a} \cot^{-1}\left(\frac{1-ax+x^2}{a}\right) dx$

53). Prove that $\int_{0}^{2\pi} \frac{x \sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx = \pi^2$

54).Evaluate $\int_{0}^{\infty} \log\left(x+\frac{1}{x}\right) \frac{1}{1+x^2} dx$

55). Evaluate $\int_{0}^{\pi} \frac{x}{1-\cos\alpha \sin x} dx$

CHAPTER - 8
APPLICATION OF INTEGRALS

1. Draw the graphs of the curves y = sin x and y = cos x, $0 \leq x \leq \frac{\pi}{2}$ .Find the common area between the above curves with the X – axis.
2. Find the area bounded by the lines x + 2 y = 2 ; y – x= 1 and 2x + y = 7
3. Find the area bounded by the line y = x and the curve $y = x^3$.
4. Find the area bounded by the lines $y = 1 + |1+x|$, x = -2 , x = 3 and y = 0.
5. Find the area enclosed between the curve $y = \sqrt{x}$ and the line y = x.
6. Find the area bounded by the curve $y = e^{|x|}$ and the line y = 3 with X- axis.
7. Find the area bounded by the curve $y = |\tan x|$ and the line $y = \sqrt{3}$.
8. Find the area included between the curve $y = x - [x]$ and the line x = 3 with X &Y axis.
9. Find the area enclosed between the curve $y = |\sin x|$ and the line $y = \frac{1}{2}$ within the interval $\left[\frac{5\pi}{6}, \frac{7\pi}{6}\right]$.
10. Find the common area between the curve $y = \sqrt{5-x^2}$ and the lines $y = |x - 1|$.

CHAPTER – 9
DIFFERENTIAL EQUATIONS

1. Solve $\frac{dy}{dx} + \cos x \cos y = 0$
2. Find the degree and order of the differential equation $(1+3\frac{dy}{dx})^{2/3} = (4\frac{d^3y}{dx^3})$
3. Find the differential equation of the family of curves given by $x^2 + y^2 = 2ax$
4. Find the integrating factor of the differential equation $x\frac{dy}{dx} - y - 2x^3 = 0$.
5. Verify that yx = c is a solution of the differential equation $\frac{ydx - xdy}{y} = 0$
6. Verify that $y = \sqrt{\frac{1+e^x}{1-e^x}}$ is solution of the differential equation $\frac{dy}{dx} = \frac{e^x}{(1-e^x)\sqrt{1-e^{2x}}}$
7. Find the equation of the family of curves whose x and y intercepts of the tangent at any point p are respectively double the x and y co-ordinates of the same point p respectively..
8. The line normal to a given curve at each point (x,y) on the curve passes through the point (2,0). If the curve contains the point (2, 3), find its equation. Prove that the curve with the property that all its normal pass through a constant point is a circle.
9. A population grows at the rate of 8% per year. How long does it takes for the population to double?
10. Solve: $(1+e^{2x})dy + (1+y^2)e^x dx = 0$, given that y =1 when x =0
11. Solve: $\left(1+y^2\right)dx = \left(\tan^{-1} y - x\right)dy$
13. Solve the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1, x \neq 0$
14 Prove that the solution of the differential equation

$$\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}} is\ y^3\sqrt{1-x^6}-x^3\sqrt{1-y^6}=cons\tan t$$

15 . Solve: x logx $\frac{dy}{dx}+y=\frac{2}{x}\log x$

16. Solve: (x+y+1) $\frac{dy}{dx}=1$

17. Solve: x $\frac{dy}{dx}=y(\log y-\log x+1)$

18. Solve: $\left(x\sqrt{x^2+y^2}-y^2\right)dx+xydy=0$

19. A bank pays interest by continuous compounding that is by treating the interest rate as the instantaneous rate of change of the principal. Suppose that in an account the interest at 8% per year compounded continuously. Calculate the percentage increase in such an account over one year. (Take $e^{0.08}$ = 1.08333 approximately)

20. Solve the differential equation $\frac{d^2x}{dy^2}=1+\sin y$, given that x = 0 and $\frac{dx}{dy}=0$ when y = 0.

21. Solve the differential equation $\frac{d^2y}{dx^2}=xe^x, given\ that\ y=0\ and\ \frac{dy}{dx}=0\ when\ x=0.$

22. Solve: $(xdx-ydx)\sin\left(\frac{y}{x}\right)=(ydx+xdy)x\cos\left(\frac{y}{x}\right)$.

23. Solve the differential equation $\sqrt{1-y^2}\,dx=\left(\sin^{-1}y-x\right)dy$.

24. Show that the differential equation

(x - y) $\frac{dy}{dx}$ = x + 2y is homogenous and solve it.

25. Find a particular solution of the differential equation

$\frac{dy}{dx}$ + y cotx = 4x cosecx (x≠0) given that y = 0 when x = $\frac{\pi}{2}$.

CHAPTER – 10
VECTORS

1. Find a unit vector parallel to XY - plane and perpendicular to the vector 4i - 3j + k

2. If $\left|\vec{a}\right| = \sqrt{26}, \left|\vec{b}\right| = 7 \; and \left|\vec{a} \times \vec{b}\right| = 35, find\; \vec{a}.\vec{b}$

3. Write number of unit vectors perpendicular to $\hat{i} + \hat{j}$ *and* $\hat{j} + \hat{k}$.

4. *If G is the centroid of the triangle ABC. Show that* $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \vec{0}$.

5. If $\vec{a}$ is a non zero vector of magnitude a then find the value of λ if $\lambda \vec{a}$ is a unit Vector.

6. Show that the sum of three vectors determined by the medians of a triangle directed from the vertices is zero.

7. Prove that the lines joining the mid-points of two opposite sides and the mid-points of the diagonals of a quadrilateral form a parallelogram.

8. Show that the straight line joining the mid-points of non-parallel sides of a trapezium is parallel to the parallel sides and half of their sum.

9. Use the vector method to prove that the lines joining the vertices of a tetrahedron to the centroids of the opposite faces are concurrent.

10. Find all the values of λ such that (x, y, z) ≠ (o ,o, o) and $\left(\hat{i} + \hat{j} + 3\hat{k}\right)x + (3\hat{i} - 3\hat{j} + \hat{k})y + (-4\hat{i} + 5\hat{j})z = \lambda(x\hat{i} + y\hat{j} + z\hat{k})$

11. Prove that the middle point of the hypotenuse of a right angled triangle is equidistant from its vertices.

12. In a triangle AOB, angle AOB $= 90^0$ .If P and Q are the points of trisection of AB, show that $OP^2 + OQ^2 = \frac{5}{9}AB^2$.

13. For any vector $\vec{a}$, show that $\left|\vec{a}\times\hat{i}\right|^2+\left|\vec{a}\times\hat{j}\right|^2+\left|\vec{a}\times\hat{k}\right|^2=2\left|\vec{a}\right|^2$.

14. If A, B, C, D are four points such that

$\vec{AB}=m\left(2\vec{i}-6\vec{j}+2\vec{k}\right), \vec{BC}=\vec{i}-2\vec{j}\ and$

$\vec{CD}=n\left(-6\vec{i}+15\vec{j}-3\vec{k}\right).$

Find the conditions of the scalars m, n such that CD intersect AB at same point E. Also find the area of the triangle BCE.

15. If A, B, C, D be any four points in space prove that

$\left|\vec{AB}\times\vec{CD}+\vec{BC}\times\vec{AD}+\vec{CA}\times\vec{BD}\right|=4\left(Area\,\Delta ABC\right).$

16.Let $\vec{OA}=\vec{a}, \vec{OB}=10\vec{a}+2\vec{b}\ and\ \vec{OC}=\vec{b}\ where\ O\ is$ origin . Let p denote the area of the quadrilateral OABC and q denote the area of the parallelogram with OA and OC as adjacent sides .
Prove that p = 6q.

17. The lines joining the vertices of a tetrahedron to the centroids of opposite faces are concurrent

18. Points F and E are taken on the sides BC and CD of a parallelogram ABCD such that $\left|\vec{BF}\right|:\left|\vec{FC}\right|=\mu:1$. and $\left|\vec{DE}\right|:\left|\vec{EC}\right|=\lambda:1$. The straight lines FD and AE intersect at the point O .
Find the ratio of $\left|\vec{FO}\right|:\left|\vec{OD}\right|$.

19. ABCD is a quadrilateral such that $\vec{AB}=\vec{b}, \vec{AD}=\vec{d}, \vec{AC}=m\vec{b}+p\vec{d}$ show that the area of quadrilateral ABCDE is $\frac{1}{2}\left|m+p\right|\left|\vec{b}\times\vec{d}\right|$.

20. The vector $-\hat{i}+\hat{j}+\hat{k}$ bisects angle between the vectors $\vec{c}\ and\ 3\hat{i}+4\hat{j}$.
Determine unit vector along $\vec{c}$.

CHAPTER – 11
THREE DIMENSIONAL GEOMETRY

1) Find the direction of angles of the line joining points. (-1,-5,-10) and the point of intersection of the line

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$$

and the plane x-y+z=5 with x, y ,z axes.

2) Find the perpendicular distance of a vertex of a cube from its one of the diagonal, not passes through the vertex.

3) Find the distance of the point (-2,3,-4) from the line

$$\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$$

measured parallel to the plane 4x + 12y -3z + 1=0.

4) Separate the equation xy + yz = 0 into two planes and find out whether the plane are || or $\perp$ to each other.

5) If A(1,2,3) and B(3,6,11) are images to each other w.r.t. a plane. Find the vector equation of the plane mirror. Find the value of λ if the plane mirror is $\perp$ to 2x – 3y + λz – 5 = 0.

6) Find k, if the plane 2x – 4y + z – 7 = 0 contains the line

$$x-4=y-2=\frac{z-k}{2}$$

7) Find the point on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point (1,2,3).

8) Find the Direction Cosines of the line joining the images of the point (1,2,3) w.r.t. xy and yz planes.

9) A line makes the same angle θ with each of the X and Z axes. If the angle β , which it makes with Y axis such that $\sin^2\beta=3\sin^2\theta$, then find the value of θ

10) Prove that the two planes x-2y+2z=6 and 3x-6y+6z=2 are parallel. Also (i) find the distance between the planes. (ii) find the intercept on the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{-1}$ between the two planes.

11). What is the direction cosines of a line equally inclined to the axes.?

12). What is the equation of Y axis in vector and Cartesian form in three dimensional space?

13). If the projection of the line segment on X, Y, Z axes are respectively $4, \frac{3}{2}, 1$ find the length of the line segment.

14). Find the distance of the point (2,3,4) from the plane 3x+2y+2z+5=0 measured parallel to the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$

15). Find the equation of the line passing through the point (2,3,2) and parallel to the line $\vec{r} = -2\hat{i} + 3\hat{j} + \lambda(2\hat{i} - 3\hat{j} + 6\vec{k})$ and also find the distance between them.

16). Show that the equation of the plane which meets the axes in A,B and C and the centroid of triangle ABC is the point (u,v,w) is $\frac{x}{u} + \frac{y}{v} + \frac{z}{w} = 3$

17). Find the vector equation of plane which is at a distance of 5 units from the origin and which has -1,2, 2 as the direction ratios of a normal to it.

18). A line makes angles α, β, γ *and* δ with the four diagonals of a cube prove that

$(i)\, Sin^2\alpha + Sin^2\beta + Sin^2\gamma + Sin^2\delta = \frac{8}{3}$

$(ii) \sum \cos 2\alpha = -\frac{2}{3}$

19). Show that the angle between any two diagonals of a cube is $\frac{\pi}{2} - Co\sec^{-1}(3)$

20). If a point A(1,2,3) move towards and reaches a line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ in shortest distance and the point A move towards and reaches a line $\frac{x}{0} = \frac{y-2}{-3} = \frac{z+3}{3}$ in shortest distance. Find the distance between the two new locations of A.

CHAPTER -12
LINEAR PROGRAMMING PROBLEMS

1). Find whether the maximum value of the objective function $Z = -x+2y$ exists or not , subject to the following constraints.

$x \geq 2$

$x + y \geq 5$

$x + 2y \geq 6 \; and \; y \geq 0$

2). Find whether the minimum values of the objective function $Z = -50x+20y$ exists or not , subject to the following constrains

$2x - y \geq -5$

$x + y \geq 3$

$2x - 3y \leq 12$

$x \geq 0, y \geq 0$

3) Maximize $Z= 2x+3y$

subject to the constraints

$x + y \geq 2$

$x + 2y \geq 3$

$x \geq 0, y \geq 0$

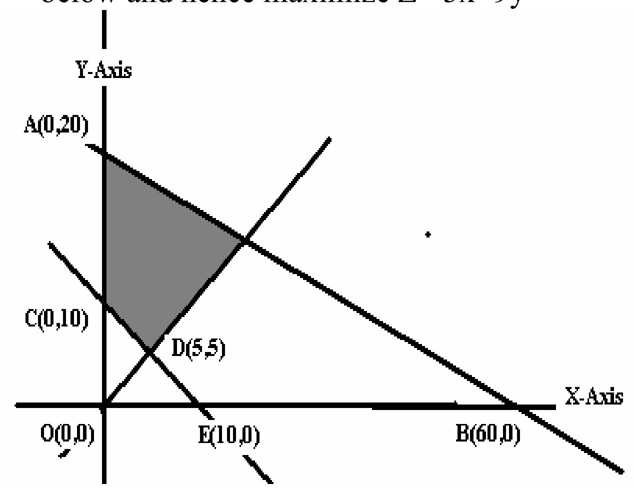
4). Kellogg is a new cereal formed of a mixture of bran and rice that contains at least 88 gms of protein and at least 36 mg of iron. Knowing that bran contains 80gms of protein and 40mg of iron per kg, and that rice contains 100gms of protein and 30 mg of iron per kg, find the minimum cost of producing this new cereal if bran costs Rs. 5/- per Kg and rice Costs Rs.4/- per Kg.

5). A brick manufacturer has two depots A and B with stock 30,000 and 20,000 bricks respectively. He receives orders from 3 buildings P,Q and R for 15,000, 20,000 and 15,000 bricks respectively. The costs of transporting 1,000 bricks to the building from the depot (in Rs.) are given below.

From/To	P	Q	R
A	40	20	30
B	20	60	40

How should the manufacturer to fulfill the orders so as to keep the cost of transportation minimum. Solve it graphically.

6). Find the constraints of the L.P.P if its graphical representation is given below and hence maximize Z= 3x+9y



7). A manufacturer produces two products A and B during a given period of time . These products require four different operations, viz. Grinding, Turning, Assembly and Testing. The requirement in hours per unit of manufacturing of the product is given below:

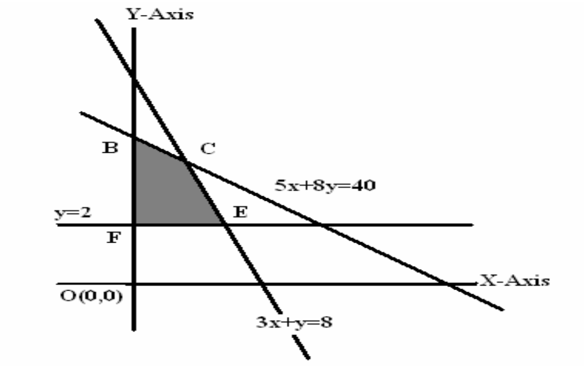
Operation	A	B
Grinding	1	2
Turning	3	1
Assembly	4	3
Testing	5	4

The available capacities of these operation in hours for the given time are:

Grinding	30	Turning	60
Assembly	200	Testing	200

Profit on each unit of A is Rs.3and Rs.2 for each unit of B. Formulate the problem as LPP.

8). Constrains of a L.P.P. represents the graph given below. Write the constrains and Minimize Z=6x+7y



CHAPTER -13
PROBABILITY

1) Find the minimum number of tosses of a pair of dice so that the probability of getting the sum of digits on the dice equal to 7 or at least one toss is greater than 0.95, given $\log_{10}^{2} = 0.3010$ & $\log_{10}^{3} = 0.4771$
2) The sum of mean and variance of a binomial distribution is 15 and their product is 54, find the distribution.
3) If A and B are events such that $p(A \cup B) = \frac{3}{4}$, $p(A \cap B) = \frac{1}{4}$, $p(\overline{A}) = \frac{2}{3}$, find $p(\overline{A} \cap B)$.

4) Two dice are rolled one after the other. Find the probability that the number on the first is smaller than the number on the second.

5). A man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of eleven steps, he is one step away from the starting point.

6).Three numbers are chosen at random without replacement 1,2,3,…,10. Find the probability that the minimum of the chosen numbering is 3 or their maximum is 7.

7). In a bolt factory three machines A,B and C, where A produces one-fourth, C produces two-fifth of the products. Production of defective products in % by A, B,C are respectively 5,4 and 2. An item is drawn at random and found to be difficult. What is the probability it was produced by either A or C.

8). Two persons A and B throw a pair of dice alternately beginning with A. If Cosα represents the probability that B gets a doublet and wins before A gets a total of 9 to win. Find α.

9).A bag contains 6 red and 5 blue balls and another bag contains 5 red and 8 blue balls. A ball is drawn at random from the first bag and without noticing its colour is put in the second bag. A ball is then drawn from the second bag. Find the probability that the ball drawn from the second bag is blue in colour.

10). A, B and C throw a die alternatively till one of them gets any number "more than 4" and wins the game. Find their respective probabilities of winning if A starts the game followed by B and C.

11). One letter has to come from "LONDON" or "CLIFTON" . Only ON is seen on the post mark, find the probability of this letter from LONDON.

12). Three stamps have been selected from 21 stamps which are marked from 1 to 21. Find the probability the number on selected stamps are in A.P.

13).A bag contains 3 red balls bearing one of the 1,2,3 (one number on one ball) and two black balls bearing the numbers 4 or 6. A ball is drawn and its number is noted and the ball is replaced in the bag. Then another ball is drawn and its number is noted. Find the probability of drawing:

(i) 2 on the first draw and 6 on the second draw.

(ii) a number ≤ 2 on the first draw and 4 on the second draw.

(iii) a total of 5.

14).In an examination, an examinee either guesses or copies or knows the answer of multiple choice questions with four choices. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6 .The probability that his answer is correct given that he copied it is 1/8. Find the probability that he knew the answer to the question, given that he correctly answered it.

15). In a class having 60% boys, 5% of the boys and 10% of the girls have an I.Q more than 150. A student is selected at random and found to have an I.Q of more than 150. Find the probability that the selected student is a boy.

16).Find the probability distribution of the number of kings drawn when 2 cards are drawn one by one without replacement from a pack of 52 playing cards

17).A bag contains 5 white, 7 red and 8 black balls. If 5 balls are thrown one by one with replacement. Find the probability distribution that exactly 5 red balls drawn.

ANSWERS/HINTS

CHAPTER - 1

1. f is not onto 2. e = -1 3. No inverse 6. 2 7. $f^{-1}(x)=\sqrt{4-x^2}$

10. $f^{-1}(x)=\frac{3x}{2-5x}$ 11. $f^{-1}(x)=\sqrt{x}$

CHAPTER - 2

2) $x = \frac{4}{3}, \frac{-3}{8}$ 4) x = -1 6) $x = \frac{\alpha+\beta}{1-\alpha\beta}$ 9) $\frac{6\pi}{7}$ 14) $\frac{\pi}{2}$ 15) 0.96

CHAPTER - 3

1. -3 2. 0 3. $\begin{pmatrix} 30 & -4 \\ -12 & 6 \end{pmatrix}$ 4. Identity matrix of order 2. 6. Null matrix.

11. $A^{-1} = -\frac{1}{5}\begin{bmatrix} 2 & 0 & -3 \\ 1 & -1 & 0 \\ -2 & -1 & 2 \end{bmatrix}$ 13. A+B 14.Zero 15. $B = \begin{bmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$

CHAPTER – 4

1). $A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$ 2). 1296 3). ± 8 4). $\theta = \frac{\pi}{6}$

5). Zero 6). 21 7). 20736 8). Zero 11).x = 0,3a

12). Zero 15). x = 2, y = - 1, z = -2. 20). Zero

CHAPTER – 5

4). Continuous at x=0 Derivable only at at x = 0   5) Continuous   6) a = 8

7) . a=3, b=2   8). Continuity at x=0   9). No point of discontinuity

11). Discontinuous   12) Continuous for all values of a

13) yes continuous for all $x \in R$ 14). Ans a=1, b=$\sqrt{\frac{3}{2}}$, $-\pi/6$ 15). $(0, \infty)$

16). **Continuous at R –{1,2}**   **17). $F^1(2)=0$**

18).

a) **x+1**

b) **$[\cos x^x\{x^x(1+\log x)\}]$**

c) **y/[x(1-xcosy)]**

d) $\frac{x-y}{x(1+\log x)}$

e) **$2x\, e^{x^2}$**

f) **$2\sin^{-1}x/\sqrt{1-x^2}$**

g) **$(2^{x+1}\log 2)/(1+4^x)$**

h) **$-\sqrt{(b^2-a^2)}/(b+a\cos x)$**

i) $$\frac{\dfrac{1}{\dfrac{a-y}{x^2+y^2}}}{Sec^2\dfrac{y}{b}-\dfrac{x}{x^2}+y^2}$$

j) **$1/2\sqrt{(1-x^2)}$**

k) **$[1/\sqrt{(1-x^2)}]+[2x/\sqrt{1-x^4}]$**

l) $Cos^x(x)^x[LogCosx^x - x\tan x^x\{x^x(1+\log x)\}]$

m) **$-e^{-ax^2\log\sin x}[ax2\cot x+2ax\log\sin x]$**

19). **$-\cos t/\sin t\,[(\cos^2 t-3\sin^2 t)/(3\cos^2 t-\sin^2 t)$**

21). $\dfrac{dy_1}{dy_2} = \dfrac{(Sinx)^x[x\cot x + \log Sinx]}{x^{\sin x}[\frac{Sinx}{x} + CosxLogx]}$

24). **1** 25). **-2cosx.e^{-cosx}** 29). **log$_7$ $\frac{e}{x}$ log$_e$ x**

30). $Cos\sqrt{Cos\sqrt{x}}.\dfrac{1}{2\sqrt{Cos\sqrt{x}}}.(\dfrac{-Sin\sqrt{x}}{2\sqrt{x}})$ 31). $\dfrac{-1}{4\sqrt{a+\sqrt{a+x}}.\sqrt{a+x}}$

32).**1/[2√x(1+x)]** 33). **Secx** 38). x = $\frac{7}{2} \in (3,4)$

CHAPTER - 6

1). 1.3838 3). 0.7 4). 1 % 5). 10% 6). **X = 3, 1/3**

7). 1/11 8). 1cm/sec 9) a>0 10) Decreasing

11) Increasing 12). Strictly increasing13) $\lambda \geq$ ½ **14). a>1**

15). a= -4, b= -316). K=9 17). (0,0) 18). Strictly Increasing

19) Yes 20) 90^o or $\pi/2$ **21) ab=1 22). -8 23). -1/3**

24). k$\in(-\infty,2)$ **25) (0, 1/e)** 26).**4m/min 27) 20 28).** $\dfrac{\frac{3}{8\pi}cm}{sec}$ **29). ½**

30). 1.03490 31) $\dfrac{-\frac{45}{26}km}{hrs}$

32) (ds/dr = ∏(2r^2+h^2)/√r^2+h^2) 33). $\theta = \tan-1(\dfrac{3(ab)^{1/3}}{a^{2/3}+b^{2/3}})$

34) **a=2, b= - ½ 35) Max value 178 at x=10 , Abs mini value 18 at x=6**

36). Max value 3/2 at $\pi/6, 5\pi/6$ **Min value -3 at** $\pi/2$ **37). minimum value 3** **38).** $\pi:(\pi+2)$ 39). $\dfrac{12}{6-\sqrt{3}}$ m & $\dfrac{18-6\sqrt{3}}{6-\sqrt{3}}m$

CHAPTER – 7

1). $\frac{2}{15}\left(a^3+x^3\right)^{5/2} - \frac{2}{9}a^3\left(a^3+x^3\right)^{3/2}+c$ 2). $-\frac{1}{2}e^{-\sec 2x}+c$ 3). $\frac{1}{\log a}\sin^{-1}a^x+c$

4). $\frac{1}{(\log 5)^3} 5^{5^{5^x}} + c$ 5). $x\sec^{-1} x - \log\left|x+\sqrt{x^2-1}\right| + C$ 6). $\frac{X}{2}(Sin(logx) - cos(logx)) + c$

7). $\frac{1}{6}log\left|\frac{x^2+1}{x^2+4}\right| + c$ 8) $4cot^{-\frac{1}{4}}x + c$ 9). $\frac{\log sec3x}{3} - \frac{logsec2x}{2} - logsecx + c$

10). $= \frac{-2}{b^2}\left(\log(a+b\cos x) + \frac{a}{a+b\cos x} + c\right)$ 11). $-log\left|cosx + \frac{1}{2} + \sqrt{cos2x + cosx}\right| + c$

12). $\frac{1}{6}log|2 + 3tan^2x| + c$ 13) $.\frac{1}{2}log\left|cosec\left(x+\frac{\pi}{3}\right) - cot\left(x+\frac{\pi}{3}\right)\right| + c$

14). $e^x\left(\frac{1}{x} - \frac{1}{x^2}\right) + C$ 15). $\frac{1}{5}\log\left|\frac{\tan x - 2}{2\tan x + 1}\right| + C$

16). $\frac{1}{4\sqrt{2}}\log\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| - \frac{1}{8}\log\left|\frac{1+\sin x}{1-\sin x}\right| + C$ 17). $a\left[\frac{x}{a}\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + \tan^{-1}\sqrt{\frac{x}{a}}\right] + C$

18). $\frac{1}{2}\left[\log|1+x| - \frac{1}{2}\log\left|1+x^2\right| + \tan^{-1} x\right] + C$

19). $\left(\tan^{-1} x\right)^2 . \frac{x^2}{2} - \tan^{-1} x.x + \log\left(\sqrt{1+x^2}\right) + \frac{\left(\tan^{-1} x\right)^2}{2} + C$

20). $= \frac{1}{2\sqrt{3}}\log\left|\frac{\sin x - \cos x + \sqrt{3}}{\cos x - \sin x + \sqrt{3}}\right| + \tan - 1(\sin x + \cos x) + c$

21).). $\frac{cosec^2\alpha}{\sqrt{x^2 - 2xcos\alpha + 1}}(x^2 - xcos^2 \propto -2cos \propto)$ +c

22). I = $x + 3\log|x-4| - 24\log|x-5| + 30\log|x-6| + C$

23). $tan^{-1}\left(\frac{tan^{-1}x - 1}{\sqrt{2}tanx}\right) + c$ 24) $log\left|\frac{x+1}{\sqrt{(x^2+2x+2)}}\right|$

25) $xsecx\left(\frac{-1}{xsinx + cosx}\right) - tanx + c$ 26) . $= \frac{1}{2}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{3}}\right) + \frac{1}{4}\log\left|\frac{x^2-x+1}{x^2+x+1}\right| + c$

27).

put $t - \frac{1}{t} = u$ $t + \frac{1}{t} = v$

$= \int \frac{du}{u^2+2} + \int \frac{dv}{v^2-2}$

$= \frac{1}{\sqrt{2}}\tan - 1\left(\frac{\tan x - 1}{\sqrt{2}\tan x}\right) + \frac{1}{2\sqrt{2}}\log\left|\frac{\tan x - \sqrt{2\tan x} + 1}{\tan x + \sqrt{2\tan x} + 1}\right| + c$

28). $= \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{3\tan x + 1}{2\sqrt{2}}\right) + c$ 29). $-sin^{-1}\left(\frac{cosx}{cos\alpha}\right) + log\left|sin2x + \sqrt{sin^2x - sin^2\alpha}\right| + c$

30). Let $t = e^x + 1$

$-\log t+\frac{1}{t}+\frac{1}{2t^2}+\log(t-1)+c$

$-\log(e^x+1)+\frac{1}{e^x+1}+\frac{1}{2(e^x+1)^2}+x+c$

31). $\frac{-1}{3}\log\left(1+\frac{1}{x^2}\right)\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}-\frac{4}{9}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}+C$ 32). Log(1+secx) +c

33). $\frac{1}{2}\log(\cos x+1)+\frac{1}{6}\log(\cos x-1)-\frac{1}{6}\log(2\cos x+1)+c$ 34). $\frac{2}{5}e^x(\sin 2x+\cos 2x)+c$

35). $(\log x)^2\frac{x^2}{2}-\log x.\frac{x^2}{2}+\frac{x^2}{4}+c$ 36). log (cosecx – cotx) $+\frac{1}{4}\log(\frac{1+\cos x}{1-\cos x})-\frac{1}{2(1+\cos x)}+C$

37). $2-\frac{\pi}{2}$ 38). $\frac{16\sqrt{2}}{15}$ 39). Zero 40). $\frac{\pi}{4\sqrt{2}}\log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$ 41). $\frac{\sqrt{3}}{18}\pi^2$ 42). $\frac{\pi}{3\sqrt{3}}$

43). $2-\frac{2}{e}$ 44). 1 45). $\sqrt{2}\,(\sqrt{2}-1)$ 46). 2 47), 4 48) $\frac{1}{\pi^2}+\frac{3}{\pi}$

49). $\frac{\pi}{2}\log 2$ 50). $\frac{19}{2}$ 51). $(1/5)(e^5-e^{-5})$ 52). $2a\tan^{-1}a-\log(1+a^2)$ 54). $\pi\log 2$

55). $\frac{\pi(\pi-\alpha)}{\sin\alpha}$

CHAPTER – 8

1). 2 - $\sqrt{2}$ 2). 6 3). ½ 4). 13.5 5) 1/6 6). $\frac{2\pi}{\sqrt{3}}-\log 4$ 8). 3/2 9). $\frac{\pi}{6}+(\sqrt{3}-2)$

10) $\frac{5}{2}\left(\sin^{-1}\frac{1}{5}+\sin^{-1}\frac{2}{5}\right)-\frac{1}{2}$

CHAPTER - 9

1) Sinx + log(siny) = c 2).Order = 3; Degree = 3. 3). $2xy\frac{dy}{dx}=y^2-x^2$ 4). $\frac{1}{x}$

5). $\frac{dy}{dx}=-\frac{y}{x}$ 6) $\frac{e^x}{\sqrt{(1-e^{2x})}\,(1-e^x)}$

7). Equation of the family of curve is xy=c 8).$(x-2)^2+y^2=9$

9). $\frac{25}{2}\log 2$ years 10). $\tan^{-1}y+\tan^{-1}(e^x)=\pi/2$. 11). $xe^{\tan^{-1}y}=e^{\tan^{-1}y}(\tan^{-1}y-1)+c$

12). $ye^{2\sqrt{x}}=\int\frac{e^{-2\sqrt{x}}}{\sqrt{x}}e^{2\sqrt{x}}dx+c=2\sqrt{x}+c$ 13). $(x-a)^2+(y-b)^2=2c$

14). $y^{3}\sqrt{1-x^6} - x^3\sqrt{1-y^6} = \sin 3c$

15). $= 2\left[\frac{-\log x}{x} - \frac{1}{x}\right] + c$

16). $x = ce^{y} - y - 2$

17). $y = a\tan^{-1}\left(\frac{x+y}{a}\right) + c$

18). $\sqrt{x^2+y^2}$ + xlogcx = 0

19). $\frac{p_1 - p_o}{p_o} X100 = 8.33\%$

20). $x = \frac{y^2}{2} - \sin y + y$

21). $y = xe^{x} - 2e^{e} + x + 2$

22). sec (y/x)=cxy

23). $x = \sin^{-1}y - 1 + ce^{\sin^{-1}y}$

25). $y\sin x = 2x^2 - \frac{\pi^2}{2}$

CHAPTER – 10

1). Vector parallel to XY- plane will be of the form $a\vec{i} + b\vec{j}$ .If it is perpendicular to $4\vec{i} - 3\vec{j} + \vec{k}$, then $\left(4\vec{i} - 3\vec{j} + \vec{k}\right).\left(a\vec{i} + b\vec{j}\right) = 0$

$\Rightarrow$ b = $\frac{4a}{3}$

$\therefore$ the vector is $a\vec{i} + \frac{4a}{3}\vec{j} = \frac{a}{3}\left(3\vec{i} + 4\vec{j}\right)$

$\therefore$ The unit vector = $\frac{\frac{a}{3}\left(3\vec{i}+4\vec{j}\right)}{\frac{a}{3}\left(\sqrt{3^2+4^2}\right)} = \pm\frac{1}{5}\left(3\vec{i}+4\vec{j}\right)$.

2). $\left|\vec{a}\times\vec{b}\right| = 35$ i.e $|a||b|\sin\theta = 35$

$\sqrt{26}\times 7\sin\theta = 35 \Rightarrow \sin\theta = \frac{5}{\sqrt{26}}$

$\therefore \cos\theta = \sqrt{1-\frac{25}{26}} = \sqrt{\frac{1}{26}}$

$\vec{a}.\vec{b} = ab\cos\theta = 7$

3). 2

4).

$$\vec{GA}+\vec{GB}+\vec{GC}=\vec{OA}-\vec{OG}+\vec{OB}-\vec{OG}+\vec{OC}-\vec{OG}$$
$$=\vec{OA}+\vec{OB}+\vec{OC}-3\vec{OG}$$
$$=\vec{a}+\vec{b}+\vec{c}-3\frac{\vec{a}+\vec{b}+\vec{c}}{3}=\vec{0}$$

5). $|\lambda\vec{a}|=1 \Rightarrow |\lambda||\vec{a}|=1 \Rightarrow \lambda=\pm\frac{1}{a}$

6).Let ABC be the given triangle. Let AD, BE, CF be the medians.
Required sum of vectors =

$$(\vec{AB}+\vec{BD})+(\vec{BC}+\vec{CE})+(\vec{CA}+\vec{AF})$$
$$=(\vec{AB}+\vec{BC}+\vec{CA})+(\vec{BD}+\vec{CE}+\vec{AF})$$
$$=(\vec{AC}+\vec{CA})+(\frac{1}{2}\vec{BC}+\frac{1}{2}\vec{CA}+\frac{1}{2}\vec{AB})$$
$$=\frac{1}{2}(\vec{AC}+\vec{CA})=\vec{O}$$

7). Let ABCD be any quadrilateral. Let P,R be the mid points of the sides AB, CD respectively Let Q,S be the mid points of the diagonals AC and BD respectively.

Let $\vec{a},\vec{b},\vec{c},\vec{d}$ be the position vectors of A, B, C,D respectively

$$P.v.of\ P=\frac{\vec{a}+\vec{b}}{2},\vec{OQ}=\frac{\vec{a}+\vec{c}}{2},\vec{OR}=\frac{\vec{c}+\vec{d}}{2},\vec{OS}=\frac{\vec{b}+\vec{d}}{2}\ ,\ \ \vec{PQ}=\frac{\vec{c}-\vec{b}}{2},\vec{SR}=\frac{\vec{c}-\vec{b}}{2}$$

8). $\vec{OA}=\vec{a},\vec{OB}=\vec{b}\ \vec{OC}=\vec{C}\ \vec{OD}=\vec{d}$
$AB \parallel CD$,
$AB=m(\vec{c}-\vec{d}),\vec{OE}=\frac{\vec{a}+\vec{d}}{2},\vec{OF}=\frac{\vec{c}+\vec{b}}{2},\vec{EF}=\frac{m+1}{2}\vec{DC}$

9).let ABCD be the tetrahedron . Let $\vec{a},\vec{b},\vec{c},\vec{d}$ be the position vectors of the vertices A, B, C, D respectively. Let G_1,G_2,G_3,G_4 be the centroid of the

$$\therefore OG_1=\frac{\vec{b}+\vec{c}+\vec{d}}{3},OG_2=\frac{\vec{c}+\vec{d}+\vec{a}}{3},OG_3=\frac{\vec{d}+\vec{a}+\vec{b}}{3},\ OG_4=\frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

$$\therefore P.V.of\ G=\frac{3\left(\frac{\vec{b}+\vec{c}+\vec{d}}{3}\right)+1(\vec{a})}{3+1}=\frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{4}$$

The symmetry of P.V. of G shows that G also divide the lines BG_2, CG_3, DG_4 in the ratio 3 : 1 internally

10). $\begin{vmatrix} 1-\lambda & 3 & -4 \\ 1 & -(3+\lambda) & 5 \\ 3 & 1 & -\lambda \end{vmatrix}=0$

14).Let $\vec{EB}=p\,\vec{AB},\vec{CE}=q\,\vec{CD}$

$\because \vec{EB}+\vec{BD}+\vec{CD}=\vec{0} \Rightarrow p=\frac{1}{2m}, q=\frac{1}{3n}$

$Then\ area\square BCE=\frac{1}{2}\left|\vec{EB}\times\vec{BC}\right|=\frac{1}{2}\sqrt{6}.$

20) $C=-\frac{11}{15}i-\frac{10}{15}j-\frac{2}{15}k$

CHAPTER – 11

1). $Cos^{-1}\left(\frac{3}{13}\right), Cos^{-1}\left(\frac{4}{13}\right), Cos^{-1}\left(\frac{12}{13}\right)$ 2). $a\sqrt{\frac{2}{3}} where\ 'a'\ be\ the\ side$,

3). $\frac{\sqrt{4580}}{15}$ 4). y=0,x+z=0 and they are perpendicular to each other., 5). x+2y+4z-38=0 & λ=1,

6). k=7, 7). $\left(\frac{56}{17},\frac{43}{17},\frac{111}{17}\right)$ 8). $D.C.'s\ are \frac{1}{\sqrt{10}}, 0, \frac{-3}{\sqrt{10}}$ 9). $COS^{-1}\left(\sqrt{\frac{3}{5}}\right)$

10). $(i)\frac{16}{9}(ii)\frac{\sqrt{14}}{9}$ 11). $\left(\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}}\right)$ 12). Equation of Y-axis $\vec{r}=\lambda\hat{j}$ is vector form & $\frac{x}{0}=\frac{y}{1}=\frac{z}{0}=\lambda$ is Cartesian form 13). $\sqrt{19.25}$ 14). 7 Units.

15). $\vec{r}=2\hat{i}+3\hat{j}+2\vec{k}+\mu(2\hat{i}-3\hat{j}+6\vec{k})$ & $\frac{\sqrt{580}}{7}$, 17). ,$\vec{r}.(-\hat{i}+2\hat{j}+2\hat{k})=15$ (20) $4\sqrt{6}$

CHAPTER - 12

1) Maximum value does not exist.
2) Minimum value does not exist.
3) The objective function can be made as large as possible as we please. So the problem has unbounded solutions.
4) Minimum cost of cereal is Rs.4 & 60 paise.
5) Minimum transportation cost is Rs.1,200 when o,20,000,10,000 bricks are transported from the depot A and 15,000, 0, 5,000 bricks are transported from the depot B to the building P, Q and R respectively.
6) Constrains are

   $x+y\geq 10$

   $x+3y\leq 60$

   $x-y\leq 0$

   $x, y\geq 0$

   Maximum value of Z = 180 when x=y=15..

7). Maximize Z=3x+2y subject to x+2y ≤ 30;
3x+y≤60;4x+3y≤200;5x+4y≤200
x,y ≥ 0.

8). Minimum Value: 14.

CHAPTER – 13

(1) 17 (2) $\left(\frac{2}{3}+\frac{1}{3}\right)^{27}$ (3) 5/12 (4).5/12 (5). $462\left(\frac{6}{25}\right)^{5}$ (6)11/40. (7) 41/69 (8) $\cos^{-1}(4/7)$ (9)93/154 (10) 9/19,6/19,4/19 (11)12/17 (12) 10/133 (13) (i) 1/25 (ii) 2/25 (iii) 4/25. (14) 24/29 (15) 3/7.

(16) Probability distribution is

x	0	1	2
P(x)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

17). $p(x=5)={}^{5}c_{5}\left(\frac{7}{20}\right)^{5}\left(\frac{13}{20}\right)^{0}=\left(\frac{7}{20}\right)^{5}$