PRACTICE PAPER

FIRST TERMINAL EXAMINATION 2011-12

MATHEMATICS

Class XII

Time: 3 Hours **Max. Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six mark each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. If $f: R \to R$ is given by $f(x) = \left(3 - x^3\right)^{1/3}$, then write the value of fof.

2. If $A = \{1, 2, 3\}$, then how many number of binary operations can be defined on A.

3. Write the value of $\cos\left[\tan^{-1}\left(\frac{3}{4}\right)\right]$.

4. What is the principal value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{5\pi}{6}\right)$.

5. If the matrix $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew-symmetric, write the values of a, b and c.

6. Write the value of the following determinant: $\begin{vmatrix} 2 & 2 & 2 \\ x & y & z \\ y+z & z+x & x+y \end{vmatrix}$.

7. If $A=\begin{bmatrix} x+4 & 2 \\ 3 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} x & 2x-1 \\ 2 & 5 \end{bmatrix}$ and $|AB|=12$, find the value(s) of x.

8. If $y=\sin^{-1}\left(\frac{x^3}{2+x^3}\right)+\sec^{-1}\left(\frac{2+x^3}{x^3}\right)$, write the value of $\frac{dy}{dx}$.

9. Find the point(s) on the curve $y=3x^2-12x+5$ at which the tangent is parallel to x-axis.

10. Write the value of the integral: $\int \frac{1-\sin x}{\cos^2 x}dx$.

SECTION – B

11. Define a binary operation $*$ on the set {0, 1, 2, 3, 4, 5} as

$$a*b=\begin{cases} a+b & \text{if } a+b<6 \\ a+b-6 & \text{if } a+b\geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element *a* of the set is invertible with 6 – *a* being the inverse of *a*.

OR

Let R be a relation defined on the set of integers Z as $R=\{(a,b)\in Z \Leftrightarrow |a-b|\leq 5\}$. Check whether the given relation is (i) reflexive (ii) symmetric (iii) transitive.

12. Solve the following equation: $\cot^{-1}x-\cot^{-1}(x+2)=\frac{\pi}{12}$; $x>0$

OR

Prove that: $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

13. If $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, find $f(A)$ where $f(x)=x^2-5x+7$. Hence find A^{-1}.

14. Verify Rolle's Theorem for the function $f(x)=x^2+x-6$ in the interval $[-3,2]$.

OR

Evaluate the following: $\int \frac{1}{1+\cot x}dx$

15. Show that the function $f(x)=|x+2|$ is continuous at every $x\in R$ but fails to be differentiable at $x=-2$.

16. If $\log\left(\sqrt{x^2+y^2}\right)=\tan^{-1}\frac{y}{x}$, prove that $\frac{dy}{dx}=\frac{x+y}{x-y}$.

17. Find $\frac{dy}{dx}$ if $y = x^{\cot x}+\left(\sin x\right)^x$.

18. If $x=2\cos\theta-\cos 2\theta$ and $y=2\sin\theta-\sin 2\theta$, find $\left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{2}}$.

19. Using differentials, find the approximate value of $\left(33.1\right)^{1/5}$

OR

Find equation(s) of the tangent to the curve $y=4x^3-3x+4$, which are perpendicular to $9y+x+3=0$.

20. Find the intervals on which the function $f(x)=5+36x+3x^2-2x^3$ is:

(i) Increasing (ii) Decreasing:

21. Find the absolute maximum value and absolute minimum value of the following function:

$f(x)=3x^4-8x^3+12x^2-48x+1$ in $[1,4]$

22. A manufacturer produces two types of steel trunks. He has two machines A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type of trunk requires 3 hours on machine A and 2 hours on machine B. Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of Rs 30 and Rs 25 per trunk on the first type and second type respectively. How many trunks of each type must he make each day to make the maximum profit? Formulate the problem as L.P.P.

SECTION – C

23. Let $f:N\to R$ be a function defined as $f(x)=4x^2+12x+15$. Show that $f:N\to S$, where, S is the range of *f*, is invertible. Find the inverse of *f*.

24. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$. Also differentiate the given expression with respect to $\log\left(1-x^2\right)$.

25. Use product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations:

$x-y+2z=1$, $2y-3z=1$, $3x-2y+4z=2$.

26. Using properties of determinants show that:

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc+bc+ca+ab$$

OR

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx).$$

27. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m.

28. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

OR

A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

29. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin content of one kg food is given below:

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs Rs 16 and one kg of food Y costs Rs 20. Find the least cost of the mixture which will produce the required diet? Express this as an LPP and solve it graphically.