CLASS XII MATRICES CHAPTER 3

MISC.EX. SOLUTIONS

Question 1:

Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n = a^n I + na^{n-1} bA$, where I is the identity matrix of order 2 and $n \in \mathbf{N}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1) : (aI + bA) = aI + ba^0 A = aI + bA$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

That is,

$$P(k) : (aI + bA)^k = a^k I + ka^{k-1} bA$$

Now, we prove that the result is true for $n = k + 1$.

Consider

$$\begin{aligned} (aI + bA)^{k+1} &= (aI + bA)^k (aI + bA) \\ &= \left(a^k I + ka^{k-1} bA\right)(aI + bA) \\ &= a^{k+1} I + ka^k bAI + a^k bIA + ka^{k-1} b^2 A^2 \\ &= a^{k+1} I + (k+1) a^k bA + ka^{k-1} b^2 A^2 \qquad \ldots(1) \end{aligned}$$

Now, $A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

From (1), we have:

$$(aI+bA)^{k+1} = a^{k+1}I + (k+1)a^k bA + O$$
$$= a^{k+1}I + (k+1)a^k bA$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have:

$(aI+bA)^n = a^n I + na^{n-1}bA$ where $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $n \in \mathbf{N}$

Question 2:

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$, $n \in \mathbf{N}$

It is given that $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

To show: $\mathrm{P}(n): A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, n \in \mathbf{N}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1): \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

That is $P(k):\ A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$

Now, we prove that the result is true for $n = k + 1$.

Now, $A^{k+1} = A \cdot A^k$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \\ 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \\ 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \end{bmatrix}$$

Therefore, the result is true for $n = k + 1$.

Thus by the principle of mathematical induction, we have:

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}, \ n \in \mathbf{N}$$

Question 3:

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ where n is any positive integer

It is given that $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$

To prove: $\mathrm{P}(n): A^n=\begin{bmatrix}1+2n & -4n\\ n & 1-2n\end{bmatrix}, n\in\mathbf{N}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1):\ A^1=\begin{bmatrix}1+2 & -4\\ 1 & 1-2\end{bmatrix}=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}=A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

That is,

$$P(k):\ A^k=\begin{bmatrix}1+2k & -4k\\ k & 1-2k\end{bmatrix}, n\in\mathbf{N}$$

Now, we prove that the result is true for $n = k + 1$.

Consider

$$A^{k+1}=A^k\cdot A$$

Consider

$A^{k+1} = A^k \cdot A$

$$= \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$
$$= \begin{bmatrix} 3(1+2k)-4k & -4(1+2k)+4k \\ 3k+1-2k & -4k-1(1-2k) \end{bmatrix}$$
$$= \begin{bmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{bmatrix}$$
$$= \begin{bmatrix} 3+2k & -4-4k \\ 1+k & -1-2k \end{bmatrix}$$
$$= \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ 1+k & 1-2(k+1) \end{bmatrix}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have:

$$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, n \in \mathbf{N}$$

Question 4:

If A and B are symmetric matrices, prove that $AB - BA$ is a skew symmetric matrix.

It is given that A and B are symmetric matrices. Therefore, we have:

$$A' = A \text{ and } B' = B \qquad ...(1)$$

$$\begin{aligned}\text{Now, } (AB-BA)' &= (AB)'-(BA)' && \left[(A-B)' = A'-B'\right] \\ &= B'A' - A'B' && \left[(AB)' = B'A'\right] \\ &= BA - AB && \left[\text{Using } (1)\right] \\ &= -(AB-BA)\end{aligned}$$

$$\therefore (AB-BA)' = -(AB-BA)$$

Thus, $(AB - BA)$ is a skew-symmetric matrix.

Question 5:

Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric.

We suppose that A is a symmetric matrix, then $A' = A$ … (1)

Consider

$$\begin{aligned}(B'AB)' &= \{B'(AB)\}' \\ &= (AB)'(B')' && \left[(AB)' = B'A'\right] \\ &= B'A'(B) && \left[(B')' = B\right] \\ &= B'(A'B) \\ &= B'(AB) && [\text{Using (1)}]\end{aligned}$$

$$\therefore (B'AB)' = B'AB$$

Thus, if A is a symmetric matrix, then $B'AB$ is a symmetric matrix.

Now, we suppose that A is a skew-symmetric matrix.

Then, $A' = -A$

Consider

$$\begin{aligned}(B'AB)' &= \left[B'(AB)\right]' = (AB)'(B')' \\ &= (B'A')B = B'(-A)B \\ &= -B'AB\end{aligned}$$

$$\therefore (B'AB)' = -B'AB$$

Thus, if A is a skew-symmetric matrix, then $B'AB$ is a skew-symmetric matrix.

Hence, if A is a symmetric or skew-symmetric matrix, then $B'AB$ is a symmetric or skew-symmetric matrix accordingly.

Question 6:

Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation $A'A = I$

It is given that $A=\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

$$\therefore A'=\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

Now, $A'A=I$

$$\Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-xz+zx & 2yz-yz-yz & z^2+z^2+z^2 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing the corresponding elements, we have:

$$2x^2=1 \Rightarrow x=\pm\frac{1}{\sqrt{2}}$$

$$6y^2=1 \Rightarrow y=\pm\frac{1}{\sqrt{6}}$$

$$3z^2=1 \Rightarrow z=\pm\frac{1}{\sqrt{3}}$$

Question 7:

For what values of x, $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$?

We have:

$$\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$$

$$\Rightarrow\begin{bmatrix}1+4+1 & 2+0+0 & 0+2+2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$$

$$\Rightarrow\begin{bmatrix}6 & 2 & 4\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix}=O$$

$$\Rightarrow\left[6(0)+2(2)+4(x)\right]=O$$

$$\Rightarrow\left[4+4x\right]=\left[0\right]$$

$\therefore 4+4x=0$

$\Rightarrow x=-1$

Thus, the required value of x is -1.

Question 8:

If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2-5A+7I=O$

It is given that $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{bmatrix}$$
$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore \text{L.H.S.} = A^2 - 5A + 7I$$
$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$
$$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$= O = \text{R.H.S.}$$

$$\therefore A^2 - 5A + 7I = O$$

Question 9:

Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

We have:

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x+0-2 & 0-10+0 & 2x-5-3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \left[x(x-2)-40+2x-8\right] = O$$

$$\Rightarrow \left[x^2-2x-40+2x-8\right] = [0]$$

$$\Rightarrow \left[x^2-48\right] = [0]$$

$$\therefore x^2-48=0$$

$$\Rightarrow x^2=48$$

$$\Rightarrow x=\pm 4\sqrt{3}$$

Question 10:

A manufacturer produces three products x, y, z which he sells in two markets.

Annual sales are indicated below:

Market	Products		
I	10000	2000	18000
II	6000	20000	8000

(a) If unit sale prices of x, y and z are Rs 2.50, Rs 1.50 and Rs 1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are Rs 2.00, Rs 1.00 and 50 paise respectively. Find the gross profit.

(a) The unit sale prices of x, y, and z are respectively given as Rs 2.50, Rs 1.50, and Rs 1.00.

Consequently, the total revenue in market **I** can be represented in the form of a matrix as:

$$\begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$
$$= 10000 \times 2.50 + 2000 \times 1.50 + 18000 \times 1.00$$
$$= 25000 + 3000 + 18000$$
$$= 46000$$

The total revenue in market **II** can be represented in the form of a matrix as:

$$\begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$
$$= 6000 \times 2.50 + 20000 \times 1.50 + 8000 \times 1.00$$
$$= 15000 + 30000 + 8000$$
$$= 53000$$

Therefore, the total revenue in market **I** isRs 46000 and the same in market **II** isRs 53000.

(b) The unit cost prices of x, y, and z are respectively given as Rs 2.00, Rs 1.00, and 50 paise.

Consequently, the total cost prices of all the products in market **I** can be represented in the form of a matrix as:

$$\begin{bmatrix}10000 & 2000 & 18000\end{bmatrix}\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix}$$
$$=10000\times 2.00+2000\times 1.00+18000\times 0.50$$
$$=20000+2000+9000$$
$$=31000$$

Since the total revenue in market **I** isRs 46000, the gross profit in this marketis (Rs 46000 − Rs 31000) Rs 15000.

The total cost prices of all the products in market **II** can be represented in the form of a matrix as:

$$\begin{bmatrix}6000 & 20000 & 8000\end{bmatrix}\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix}$$
$$=6000\times 2.00+20000\times 1.00+8000\times 0.50$$
$$=12000+20000+4000$$
$$=\text{Rs } 36000$$

Since the total revenue in market **II** isRs 53000, the gross profit in this market is (Rs 53000 − Rs 36000) Rs 17000.

Question 11:

Find the matrix X so that $X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$

It is given that:

$$X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a 2 × 3 matrix and the one given on the L.H.S. of the equation is a 2 × 3 matrix. Therefore, X has to be a 2 × 2 matrix.

Now, let $X=\begin{bmatrix}a & c\\ b & d\end{bmatrix}$

Therefore, we have:

$$\begin{bmatrix}a & c\\ b & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}a+4c & 2a+5c & 3a+6c\\ b+4d & 2b+5d & 3b+6d\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$$a+4c=-7,\quad 2a+5c=-8,\quad 3a+6c=-9$$
$$b+4d=2,\quad 2b+5d=4,\quad 3b+6d=6$$

Now, $a+4c=-7 \Rightarrow a=-7-4c$

$\therefore 2a+5c=-8 \Rightarrow -14-8c+5c=-8$

$\Rightarrow -3c=6$

$\Rightarrow c=-2$

$\therefore a=-7-4(-2)=-7+8=1$

Now, $b+4d=2 \Rightarrow b=2-4d$

$\therefore 2b+5d=4 \Rightarrow 4-8d+5d=4$

$\Rightarrow -3d=0$

$\Rightarrow d=0$

$\therefore b=2-4(0)=2$

Thus, $a=1$, $b=2$, $c=-2$, $d=0$

Hence, the required matrix X is $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$.

Question 12:

If A and B are square matrices of the same order such that $AB=BA$, then prove by induction that $AB^n=B^nA$. Further, prove that $(AB)^n=A^nB^n$ for all $n \in \mathbf{N}$

A and B are square matrices of the same order such that $AB = BA$.

To prove: $\quad P(n): AB^n = B^n A,\ n \in \mathbf{N}$

For $n = 1$, we have:

$$P(1): AB = BA \qquad [\text{Given}]$$
$$\Rightarrow AB^1 = B^1 A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$P(k): AB^k = B^k A \qquad ...(1)$$

Now, we prove that the result is true for $n = k + 1$.

$$\begin{aligned} AB^{k+1} &= AB^k \cdot B \\ &= \left(B^k A\right)B && [\text{By } (1)] \\ &= B^k (AB) && [\text{Associative law}] \\ &= B^k (BA) && [AB = BA \ (\text{Given})] \\ &= \left(B^k B\right)A && [\text{Associative law}] \\ &= B^{k+1} A \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have $AB^n = B^n A,\ n \in \mathbf{N}$.

Now, we prove that $(AB)^n = A^n B^n$ for all $n \in \mathbf{N}$

For $n = 1$, we have:

$(AB)^1 = A^1 B^1 = AB$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$(AB)^k = A^k B^k$ ...(2)

Now, we prove that the result is true for $n = k + 1$.

$$
\begin{aligned}
(AB)^{k+1} &= (AB)^k \cdot (AB) \\
&= \left(A^k B^k\right) \cdot (AB) && [\text{By } (2)] \\
&= A^k \left(B^k A\right) B && [\text{Associative law}] \\
&= A^k \left(AB^k\right) B && \left[AB^n = B^n A \text{ for all } n \in \mathbf{N}\right] \\
&= \left(A^k A\right) \cdot \left(B^k B\right) && [\text{Associative law}] \\
&= A^{k+1} B^{k+1}
\end{aligned}
$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have $(AB)^n = A^n B^n$, for all natural numbers.

Question 13:

Choose the correct answer in the following questions:

If $A=\begin{bmatrix}\alpha & \beta \\ \gamma & -\alpha\end{bmatrix}$ is such that $A^2=I$ then

A. $1+\alpha^2+\beta\gamma=0$

B. $1-\alpha^2+\beta\gamma=0$

C. $1-\alpha^2-\beta\gamma=0$

D. $1+\alpha^2-\beta\gamma=0$

Answer: C

$$A=\begin{bmatrix}\alpha & \beta \\ \gamma & -\alpha\end{bmatrix}$$

$$\begin{aligned}\therefore A^2=A\cdot A&=\begin{bmatrix}\alpha & \beta \\ \gamma & -\alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta \\ \gamma & -\alpha\end{bmatrix}\\ &=\begin{bmatrix}\alpha^2+\beta\gamma & \alpha\beta-\alpha\beta \\ \alpha\gamma-\alpha\gamma & \beta\gamma+\alpha^2\end{bmatrix}\\ &=\begin{bmatrix}\alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2\end{bmatrix}\end{aligned}$$

Now, $A^2=I\Rightarrow\begin{bmatrix}\alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2\end{bmatrix}=\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$

On comparing the corresponding elements, we have:

$$\begin{aligned}&\alpha^2+\beta\gamma=1\\ &\Rightarrow\alpha^2+\beta\gamma-1=0\\ &\Rightarrow 1-\alpha^2-\beta\gamma=0\end{aligned}$$

Question 14:

If the matrix A is both symmetric and skew symmetric, then

A. A is a diagonal matrix

B. A is a zero matrix

C. A is a square matrix

D. None of these

Answer: B

If A is both symmetric and skew-symmetric matrix, then we should have

$$A' = A \text{ and } A' = -A$$
$$\Rightarrow A = -A$$
$$\Rightarrow A + A = O$$
$$\Rightarrow 2A = O$$
$$\Rightarrow A = O$$

Therefore, A is a zero matrix.

Question 15:

If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

A. A **B.** $I - A$ **C.** I **D.** $3A$

Answer: C

$$
\begin{aligned}
(I + A)^3 - 7A &= I^3 + A^3 + 3I^2A + 3A^2I - 7A \\
&= I + A^3 + 3A + 3A^2 - 7A \\
&= I + A^2 \cdot A + 3A + 3A - 7A \qquad \left[A^2 = A\right] \\
&= I + A \cdot A - A \\
&= I + A^2 - A \\
&= I + A - A \\
&= I
\end{aligned}
$$

$\therefore (I + A)^3 - 7A = I$

Thus, by the principle of mathematical induction, we have $AB^n = B^n A,\ n \in \mathbf{N}$.

Now, we prove that $(AB)^n = A^n B^n$ for all $n \in \mathbf{N}$

For $n = 1$, we have:

$(AB)^1 = A^1 B^1 = AB$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$(AB)^k = A^k B^k \qquad \ldots(2)$

Now, we prove that the result is true for $n = k + 1$.

$$\begin{aligned}
(AB)^{k+1} &= (AB)^k \cdot (AB) \\
&= (A^k B^k) \cdot (AB) && [\text{By } (2)] \\
&= A^k (B^k A) B && [\text{Associative law}] \\
&= A^k (AB^k) B && [AB^n = B^n A \text{ for all } n \in \mathbf{N}] \\
&= (A^k A) \cdot (B^k B) && [\text{Associative law}] \\
&= A^{k+1} B^{k+1}
\end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have $(AB)^n = A^n B^n$, for all natural numbers.