

CLASS XII

CHAPTER 2 INVERSE TRIGONOMETRY

NCERT MISC EX. SOLUTIONS

Question 1:

Find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

ANS:

We know that $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$, which is the principal value branch of $\cos^{-1}x$.

Here, $\frac{13\pi}{6} \notin [0, \pi]$.

Now, $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ can be written as:

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right] = \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right], \text{ where } \frac{\pi}{6} \in [0, \pi].$$

$$\therefore \cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right] = \frac{\pi}{6}$$

Question 2:

Find the value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

ANS:

We know that $\tan^{-1}(\tan x) = x$ if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which is the principal value branch of $\tan^{-1}x$.

Here, $\frac{7\pi}{6} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Now, $\tan^{-1}\left(\tan \frac{7\pi}{6}\right)$ can be written as:

$$\begin{aligned}\tan^{-1}\left(\tan \frac{7\pi}{6}\right) &= \tan^{-1}\left[\tan\left(2\pi - \frac{5\pi}{6}\right)\right] && [\tan(2\pi - x) = -\tan x] \\ &= \tan^{-1}\left[-\tan\left(\frac{5\pi}{6}\right)\right] = \tan^{-1}\left[\tan\left(-\frac{5\pi}{6}\right)\right] = \tan^{-1}\left[\tan\left(\pi - \frac{5\pi}{6}\right)\right] \\ &= \tan^{-1}\left[\tan\left(\frac{\pi}{6}\right)\right], \text{ where } \frac{\pi}{6} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\end{aligned}$$

$$\therefore \tan^{-1}\left(\tan \frac{7\pi}{6}\right) = \tan^{-1}\left(\tan \frac{\pi}{6}\right) = \frac{\pi}{6}$$

Question 3:

Prove $2 \sin^{-1} \frac{3}{5} = \tan^{-1} \frac{24}{7}$

ANS:

Let $\sin^{-1} \frac{3}{5} = x$. Then, $\sin x = \frac{3}{5}$.

$$\Rightarrow \cos x = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

$$\therefore \tan x = \frac{3}{4}$$

$$\therefore x = \tan^{-1} \frac{3}{4} \Rightarrow \sin^{-1} \frac{3}{5} = \tan^{-1} \frac{3}{4}$$

Now, we have:

$$\begin{aligned}\text{L.H.S.} &= 2 \sin^{-1} \frac{3}{5} = 2 \tan^{-1} \frac{3}{4} \\ &= \tan^{-1} \left(\frac{2 \times \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2} \right) && \left[2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2} \right] \\ &= \tan^{-1} \left(\frac{\frac{3}{2}}{\frac{16-9}{16}} \right) = \tan^{-1} \left(\frac{3}{2} \times \frac{16}{7} \right) \\ &= \tan^{-1} \frac{24}{7} = \text{R.H.S.}\end{aligned}$$

Question 4:

$$\text{Prove } \sin^{-1} \frac{8}{17} + \sin^{-1} \frac{3}{5} = \tan^{-1} \frac{77}{36}$$

ANS:

$$\text{Let } \sin^{-1} \frac{8}{17} = x. \text{ Then, } \sin x = \frac{8}{17} \Rightarrow \cos x = \sqrt{1 - \left(\frac{8}{17}\right)^2} = \sqrt{\frac{225}{289}} = \frac{15}{17}.$$

$$\therefore \tan x = \frac{8}{15} \Rightarrow x = \tan^{-1} \frac{8}{15}$$

$$\therefore \sin^{-1} \frac{8}{17} = \tan^{-1} \frac{8}{15} \quad \dots(1)$$

$$\text{Now, let } \sin^{-1} \frac{3}{5} = y. \text{ Then, } \sin y = \frac{3}{5} \Rightarrow \cos y = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25}} = \frac{4}{5}.$$

$$\therefore \tan y = \frac{3}{4} \Rightarrow y = \tan^{-1} \frac{3}{4}$$

$$\therefore \sin^{-1} \frac{3}{5} = \tan^{-1} \frac{3}{4} \quad \dots(2)$$

Now, we have:

$$\begin{aligned} \text{L.H.S.} &= \sin^{-1} \frac{8}{17} + \sin^{-1} \frac{3}{5} \\ &= \tan^{-1} \frac{8}{15} + \tan^{-1} \frac{3}{4} && \text{[Using (1) and (2)]} \\ &= \tan^{-1} \frac{\frac{8}{15} + \frac{3}{4}}{1 - \frac{8}{15} \times \frac{3}{4}} \\ &= \tan^{-1} \left(\frac{32 + 45}{60 - 24} \right) && \left[\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy} \right] \\ &= \tan^{-1} \frac{77}{36} = \text{R.H.S.} \end{aligned}$$

Question 5:

$$\text{Prove } \cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$$

ANS:

$$\text{Let } \cos^{-1} \frac{4}{5} = x. \text{ Then, } \cos x = \frac{4}{5} \Rightarrow \sin x = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}.$$

$$\therefore \tan x = \frac{3}{4} \Rightarrow x = \tan^{-1} \frac{3}{4}$$

$$\therefore \cos^{-1} \frac{4}{5} = \tan^{-1} \frac{3}{4} \quad \dots(1)$$

$$\text{Now, let } \cos^{-1} \frac{12}{13} = y. \text{ Then, } \cos y = \frac{12}{13} \Rightarrow \sin y = \frac{5}{13}.$$

$$\therefore \tan y = \frac{5}{12} \Rightarrow y = \tan^{-1} \frac{5}{12}$$

$$\therefore \cos^{-1} \frac{12}{13} = \tan^{-1} \frac{5}{12} \quad \dots(2)$$

$$\text{Let } \cos^{-1} \frac{33}{65} = z. \text{ Then, } \cos z = \frac{33}{65} \Rightarrow \sin z = \frac{56}{65}.$$

$$\therefore \tan z = \frac{56}{33} \Rightarrow z = \tan^{-1} \frac{56}{33}$$

$$\therefore \cos^{-1} \frac{33}{65} = \tan^{-1} \frac{56}{33} \quad \dots(3)$$

Now, we will prove that:

$$\text{L.H.S.} = \cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13}$$

$$= \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{5}{12} \quad [\text{Using (1) and (2)}]$$

$$= \tan^{-1} \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} \quad \left[\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy} \right]$$

$$= \tan^{-1} \frac{36+20}{48-15}$$

$$= \tan^{-1} \frac{56}{33}$$

$$= \tan^{-1} \frac{56}{33} \quad [\text{by (3)}]$$

$$= \text{R.H.S.}$$

Question 6:

$$\text{Prove } \cos^{-1} \frac{12}{13} + \sin^{-1} \frac{3}{5} = \sin^{-1} \frac{56}{65}$$

ANS:

$$\text{Let } \sin^{-1} \frac{3}{5} = x. \text{ Then, } \sin x = \frac{3}{5} \Rightarrow \cos x = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25}} = \frac{4}{5}.$$

$$\therefore \tan x = \frac{3}{4} \Rightarrow x = \tan^{-1} \frac{3}{4}$$

$$\therefore \sin^{-1} \frac{3}{5} = \tan^{-1} \frac{3}{4} \quad \dots(1)$$

$$\text{Now, let } \cos^{-1} \frac{12}{13} = y. \text{ Then, } \cos y = \frac{12}{13} \Rightarrow \sin y = \frac{5}{13}.$$

$$\therefore \tan y = \frac{5}{12} \Rightarrow y = \tan^{-1} \frac{5}{12}$$

$$\therefore \cos^{-1} \frac{12}{13} = \tan^{-1} \frac{5}{12} \quad \dots(2)$$

$$\text{Let } \sin^{-1} \frac{56}{65} = z. \text{ Then, } \sin z = \frac{56}{65} \Rightarrow \cos z = \frac{33}{65}.$$

$$\therefore \tan z = \frac{56}{33} \Rightarrow z = \tan^{-1} \frac{56}{33}$$

$$\therefore \sin^{-1} \frac{56}{65} = \tan^{-1} \frac{56}{33} \quad \dots(3)$$

Now, we have:

$$\begin{aligned} \text{L.H.S.} &= \cos^{-1} \frac{12}{13} + \sin^{-1} \frac{3}{5} \\ &= \tan^{-1} \frac{5}{12} + \tan^{-1} \frac{3}{4} \quad [\text{Using (1) and (2)}] \\ &= \tan^{-1} \frac{\frac{5}{12} + \frac{3}{4}}{1 - \frac{5}{12} \cdot \frac{3}{4}} \quad \left[\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy} \right] \\ &= \tan^{-1} \frac{20+36}{48-15} \\ &= \tan^{-1} \frac{56}{33} \\ &= \sin^{-1} \frac{56}{65} = \text{R.H.S.} \quad [\text{Using (3)}] \end{aligned}$$

Question 7:

$$\text{Prove } \tan^{-1} \frac{63}{16} = \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5}$$

ANS:

$$\text{Let } \sin^{-1} \frac{5}{13} = x. \text{ Then, } \sin x = \frac{5}{13} \Rightarrow \cos x = \frac{12}{13}.$$

$$\therefore \tan x = \frac{5}{12} \Rightarrow x = \tan^{-1} \frac{5}{12}$$

$$\therefore \sin^{-1} \frac{5}{13} = \tan^{-1} \frac{5}{12} \quad \dots(1)$$

$$\text{Let } \cos^{-1} \frac{3}{5} = y. \text{ Then, } \cos y = \frac{3}{5} \Rightarrow \sin y = \frac{4}{5}.$$

$$\therefore \tan y = \frac{4}{3} \Rightarrow y = \tan^{-1} \frac{4}{3}$$

$$\therefore \cos^{-1} \frac{3}{5} = \tan^{-1} \frac{4}{3} \quad \dots(2)$$

Using (1) and (2), we have

$$\begin{aligned} \text{R.H.S.} &= \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5} \\ &= \tan^{-1} \frac{5}{12} + \tan^{-1} \frac{4}{3} \\ &= \tan^{-1} \left(\frac{\frac{5}{12} + \frac{4}{3}}{1 - \frac{5}{12} \times \frac{4}{3}} \right) \quad \left[\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy} \right] \\ &= \tan^{-1} \left(\frac{15+48}{36-20} \right) \\ &= \tan^{-1} \frac{63}{16} \\ &= \text{L.H.S.} \end{aligned}$$

Question 8:

$$\text{Prove } \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$$

ANS:

$$\begin{aligned}
\text{L.H.S.} &= \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{8} \\
&= \tan^{-1} \left(\frac{\frac{1}{5} + \frac{1}{7}}{1 - \frac{1}{5} \times \frac{1}{7}} \right) + \tan^{-1} \left(\frac{\frac{1}{3} + \frac{1}{8}}{1 - \frac{1}{3} \times \frac{1}{8}} \right) && \left[\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy} \right] \\
&= \tan^{-1} \left(\frac{7+5}{35-1} \right) + \tan^{-1} \left(\frac{8+3}{24-1} \right) \\
&= \tan^{-1} \frac{12}{34} + \tan^{-1} \frac{11}{23} \\
&= \tan^{-1} \frac{6}{17} + \tan^{-1} \frac{11}{23} \\
&= \tan^{-1} \left(\frac{\frac{6}{17} + \frac{11}{23}}{1 - \frac{6}{17} \times \frac{11}{23}} \right) \\
&= \tan^{-1} \left(\frac{138+187}{391-66} \right) \\
&= \tan^{-1} \left(\frac{325}{325} \right) = \tan^{-1} 1 \\
&= \frac{\pi}{4} = \text{R.H.S.}
\end{aligned}$$

Question 9:

Prove $\tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1} \left(\frac{1-x}{1+x} \right)$, $x \in [0, 1]$

ANS:

Let $x = \tan^2 \theta$. Then, $\sqrt{x} = \tan \theta \Rightarrow \theta = \tan^{-1} \sqrt{x}$.

$$\therefore \frac{1-x}{1+x} = \frac{1-\tan^2 \theta}{1+\tan^2 \theta} = \cos 2\theta$$

Now, we have:

$$\text{R.H.S.} = \frac{1}{2} \cos^{-1} \left(\frac{1-x}{1+x} \right) = \frac{1}{2} \cos^{-1} (\cos 2\theta) = \frac{1}{2} \times 2\theta = \theta = \tan^{-1} \sqrt{x} = \text{L.H.S.}$$

Question 10:

Prove $\cot^{-1} \left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right) = \frac{x}{2}$, $x \in \left(0, \frac{\pi}{4} \right)$

ANS:

$$\begin{aligned}
& \text{Consider } \frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \\
&= \frac{(\sqrt{1+\sin x} + \sqrt{1-\sin x})^2}{(\sqrt{1+\sin x})^2 - (\sqrt{1-\sin x})^2} \quad (\text{by rationalizing}) \\
&= \frac{(1+\sin x) + (1-\sin x) + 2\sqrt{(1+\sin x)(1-\sin x)}}{1+\sin x - 1 + \sin x} \\
&= \frac{2(1 + \sqrt{1-\sin^2 x})}{2\sin x} = \frac{1 + \cos x}{\sin x} = \frac{2\cos^2 \frac{x}{2}}{2\sin \frac{x}{2} \cos \frac{x}{2}} \\
&= \cot \frac{x}{2} \\
\therefore \text{L.H.S.} &= \cot^{-1} \left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right) = \cot^{-1} \left(\cot \frac{x}{2} \right) = \frac{x}{2} = \text{R.H.S.}
\end{aligned}$$

Question 11:

$$\text{Prove } \tan^{-1} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x, \quad -\frac{1}{\sqrt{2}} \leq x \leq 1 \quad [\text{Hint: put } x = \cos 2\theta]$$

ANS:

Put $x = \cos 2\theta$ so that $\theta = \frac{1}{2} \cos^{-1} x$. Then, we have:

$$\begin{aligned}
\text{L.H.S.} &= \tan^{-1} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) \\
&= \tan^{-1} \left(\frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}} \right) \\
&= \tan^{-1} \left(\frac{\sqrt{2\cos^2 \theta} - \sqrt{2\sin^2 \theta}}{\sqrt{2\cos^2 \theta} + \sqrt{2\sin^2 \theta}} \right) \\
&= \tan^{-1} \left(\frac{\sqrt{2} \cos \theta - \sqrt{2} \sin \theta}{\sqrt{2} \cos \theta + \sqrt{2} \sin \theta} \right) \\
&= \tan^{-1} \left(\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \right) = \tan^{-1} \left(\frac{1 - \tan \theta}{1 + \tan \theta} \right) \\
&= \tan^{-1} 1 - \tan^{-1} (\tan \theta) \qquad \left[\tan^{-1} \left(\frac{x-y}{1+xy} \right) = \tan^{-1} x - \tan^{-1} y \right] \\
&= \frac{\pi}{4} - \theta = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x = \text{R.H.S.}
\end{aligned}$$

Question 12:

$$\text{Prove } \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} = \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3}$$

ANS:

$$\begin{aligned} \text{L.H.S.} &= \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} \\ &= \frac{9}{4} \left(\frac{\pi}{2} - \sin^{-1} \frac{1}{3} \right) \\ &= \frac{9}{4} \left(\cos^{-1} \frac{1}{3} \right) \quad \dots(1) \quad \left[\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right] \end{aligned}$$

$$\text{Now, let } \cos^{-1} \frac{1}{3} = x. \text{ Then, } \cos x = \frac{1}{3} \Rightarrow \sin x = \sqrt{1 - \left(\frac{1}{3}\right)^2} = \frac{2\sqrt{2}}{3}.$$

$$\therefore x = \sin^{-1} \frac{2\sqrt{2}}{3} \Rightarrow \cos^{-1} \frac{1}{3} = \sin^{-1} \frac{2\sqrt{2}}{3}$$

$$\therefore \text{L.H.S.} = \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} = \text{R.H.S.}$$

Question 13:

$$\text{Solve } 2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$$

ANS:

$$\begin{aligned} 2 \tan^{-1}(\cos x) &= \tan^{-1}(2 \operatorname{cosec} x) \\ \Rightarrow \tan^{-1} \left(\frac{2 \cos x}{1 - \cos^2 x} \right) &= \tan^{-1}(2 \operatorname{cosec} x) \quad \left[2 \tan^{-1} x = \tan^{-1} \frac{2x}{1 - x^2} \right] \end{aligned}$$

$$\Rightarrow \frac{2 \cos x}{1 - \cos^2 x} = 2 \operatorname{cosec} x$$

$$\Rightarrow \frac{2 \cos x}{\sin^2 x} = \frac{2}{\sin x}$$

$$\Rightarrow \cos x = \sin x$$

$$\Rightarrow \tan x = 1$$

$$\therefore x = \frac{\pi}{4}$$

Question 14:

$$\text{Solve } \tan^{-1} \frac{1-x}{1+x} = \frac{1}{2} \tan^{-1} x, (x > 0)$$

ANS:

$$\tan^{-1} \frac{1-x}{1+x} = \frac{1}{2} \tan^{-1} x$$

$$\Rightarrow \tan^{-1} 1 - \tan^{-1} x = \frac{1}{2} \tan^{-1} x \quad \left[\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy} \right]$$

$$\Rightarrow \frac{\pi}{4} = \frac{3}{2} \tan^{-1} x$$

$$\Rightarrow \tan^{-1} x = \frac{\pi}{6}$$

$$\Rightarrow x = \tan \frac{\pi}{6}$$

$$\therefore x = \frac{1}{\sqrt{3}}$$

Question 15:

Solve $\sin(\tan^{-1} x)$, $|x| < 1$ is equal to

(A) $\frac{x}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\frac{x}{\sqrt{1+x^2}}$

ANS:

$$\text{Let } \tan^{-1} x = y. \text{ Then, } \tan y = x \Rightarrow \sin y = \frac{x}{\sqrt{1+x^2}}.$$

$$\therefore y = \sin^{-1} \left(\frac{x}{\sqrt{1+x^2}} \right) \Rightarrow \tan^{-1} x = \sin^{-1} \left(\frac{x}{\sqrt{1+x^2}} \right)$$

$$\therefore \sin(\tan^{-1} x) = \sin \left(\sin^{-1} \frac{x}{\sqrt{1+x^2}} \right) = \frac{x}{\sqrt{1+x^2}}$$

The correct answer is D.

Question 16:

Solve $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, then x is equal to

- (A) $0, \frac{1}{2}$ (B) $1, \frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$

ANS:

$$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow -2\sin^{-1}x = \frac{\pi}{2} - \sin^{-1}(1-x)$$

$$\Rightarrow -2\sin^{-1}x = \cos^{-1}(1-x) \quad \dots(1)$$

$$\text{Let } \sin^{-1}x = \theta \Rightarrow \sin\theta = x \Rightarrow \cos\theta = \sqrt{1-x^2}.$$

$$\therefore \theta = \cos^{-1}(\sqrt{1-x^2})$$

$$\therefore \sin^{-1}x = \cos^{-1}(\sqrt{1-x^2})$$

Therefore, from equation (1), we have

$$-2\cos^{-1}(\sqrt{1-x^2}) = \cos^{-1}(1-x)$$

Put $x = \sin y$. Then, we have:

$$-2\cos^{-1}(\sqrt{1-\sin^2 y}) = \cos^{-1}(1-\sin y)$$

$$\Rightarrow -2\cos^{-1}(\cos y) = \cos^{-1}(1-\sin y)$$

$$\Rightarrow -2y = \cos^{-1}(1-\sin y)$$

$$\Rightarrow 1-\sin y = \cos(-2y) = \cos 2y$$

$$\Rightarrow 1-\sin y = 1-2\sin^2 y$$

$$\Rightarrow 2\sin^2 y - \sin y = 0$$

$$\Rightarrow \sin y(2\sin y - 1) = 0$$

$$\Rightarrow \sin y = 0 \text{ or } \frac{1}{2}$$

$$\therefore x = 0 \text{ or } x = \frac{1}{2}$$

But, when $x = \frac{1}{2}$, it can be observed that:

$$\begin{aligned}\text{L.H.S.} &= \sin^{-1}\left(1 - \frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2} \\ &= \sin^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2} \\ &= -\sin^{-1}\frac{1}{2} \\ &= -\frac{\pi}{6} \neq \frac{\pi}{2} \neq \text{R.H.S.}\end{aligned}$$

$\therefore x = \frac{1}{2}$ is not the solution of the given equation.

Thus, $x = 0$.

Hence, the correct answer is C.

Question 17:

Solve $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y}$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{-3\pi}{4}$

ANS:

$$\begin{aligned}&\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y} \\ &= \tan^{-1}\left[\frac{\frac{x}{y} - \frac{x-y}{x+y}}{1 + \left(\frac{x}{y}\right)\left(\frac{x-y}{x+y}\right)}\right] \qquad \left[\tan^{-1}y - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}\right] \\ &= \tan^{-1}\left[\frac{\frac{x(x+y) - y(x-y)}{y(x+y)}}{\frac{y(x+y) + x(x-y)}{y(x+y)}}\right] \\ &= \tan^{-1}\left(\frac{x^2 + xy - xy + y^2}{xy + y^2 + x^2 - xy}\right) \\ &= \tan^{-1}\left(\frac{x^2 + y^2}{x^2 + y^2}\right) = \tan^{-1}1 = \frac{\pi}{4}\end{aligned}$$

Hence, the correct answer is C.

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