CLASS XII INTEGRALS CHAPTER 7

EX. 7.7 SOLUTIONS

Question 1:

$\sqrt{4-x^2}$

ANS :

$$\text{Let } I=\int\sqrt{4-x^2}\,dx=\int\sqrt{(2)^2-(x)^2}\,dx$$

$$\text{It is known that, } \int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}\frac{a^2}{2}\sin^{-1}\frac{x}{a}+\text{C}$$

$$\therefore I=\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}+\text{C}$$

$$=\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+\text{C}$$

Question 2:

$\sqrt{1-4x^2}$

ANS :

$$\text{Let } I=\int\sqrt{1-4x^2}\,dx=\int\sqrt{(1)^2-(2x)^2}\,dx$$

$$\text{Let } 2x=t \;\Rightarrow\; 2\,dx=dt$$

$$\therefore I=\frac{1}{2}\int\sqrt{(1)^2-(t)^2}\,dt$$

$$\text{It is known that, } \int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+\text{C}$$

$$\Rightarrow I=\frac{1}{2}\left[\frac{t}{2}\sqrt{1-t^2}+\frac{1}{2}\sin^{-1}t\right]+\text{C}$$

$$=\frac{t}{4}\sqrt{1-t^2}+\frac{1}{4}\sin^{-1}t+\text{C}$$

$$=\frac{2x}{4}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+\text{C}$$

$$=\frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+\text{C}$$

Question 3:

$\sqrt{x^2+4x+6}$

ANS :

$$\text{Let } I=\int\sqrt{x^2+4x+6}\;dx$$
$$=\int\sqrt{x^2+4x+4+2}\;dx$$
$$=\int\sqrt{\left(x^2+4x+4\right)+2}\;dx$$
$$=\int\sqrt{(x+2)^2+\left(\sqrt{2}\right)^2}\;dx$$

It is known that, $\int\sqrt{x^2+a^2}\,dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\therefore I=\frac{(x+2)}{2}\sqrt{x^2+4x+6}+\frac{2}{2}\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$$
$$=\frac{(x+2)}{2}\sqrt{x^2+4x+6}+\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$$

Question 4:

$\sqrt{x^2+4x+1}$

ANS :

$$\text{Let } I=\int\sqrt{x^2+4x+1}\,dx$$
$$=\int\sqrt{\left(x^2+4x+4\right)-3}\,dx$$
$$=\int\sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\;dx$$

It is known that, $\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I=\frac{(x+2)}{2}\sqrt{x^2+4x+1}-\frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right|+C$$

Question 5:

$\sqrt{1-4x-x^2}$

ANS :

$$\text{Let } I = \int\sqrt{1-4x-x^2}\,dx$$
$$= \int\sqrt{1-\left(x^2+4x+4-4\right)}\,dx$$
$$= \int\sqrt{1+4-(x+2)^2}\,dx$$
$$= \int\sqrt{\left(\sqrt{5}\right)^2-(x+2)^2}\,dx$$

It is known that, $\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{1-4x-x^2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$$

Question 6:

$\sqrt{x^2+4x-5}$

ANS :

$$\text{Let } I = \int\sqrt{x^2+4x-5}\,dx$$
$$= \int\sqrt{\left(x^2+4x+4\right)-9}\,dx$$
$$= \int\sqrt{(x+2)^2-(3)^2}\,dx$$

It is known that, $\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x-5}-\frac{9}{2}\log\left|(x+2)+\sqrt{x^2+4x-5}\right|+C$$

Question 7:

$\sqrt{1+3x-x^2}$

ANS :

Let $I = \int \sqrt{1+3x-x^2}\,dx$

$$= \int \sqrt{1-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)}\,dx$$

$$= \int \sqrt{\left(1+\frac{9}{4}\right)-\left(x-\frac{3}{2}\right)^2}\,dx$$

$$= \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}\,dx$$

It is known that, $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$$\therefore I = \frac{x-\frac{3}{2}}{2}\sqrt{1+3x-x^2}+\frac{13}{4\times 2}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}\right)+C$$

$$= \frac{2x-3}{4}\sqrt{1+3x-x^2}+\frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right)+C$$

Question 8:

$\sqrt{x^2+3x}$

ANS :

Let $I = \int \sqrt{x^2+3x}\,dx$

$$= \int \sqrt{x^2+3x+\frac{9}{4}-\frac{9}{4}}\,dx$$

$$= \int \sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$$

It is known that, $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I = \frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x}-\frac{\frac{9}{4}}{2}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

$$= \frac{(2x+3)}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

Question 9:

$\sqrt{1+\frac{x^2}{9}}$

ANS :

Let $I=\int\sqrt{1+\frac{x^2}{9}}dx=\frac{1}{3}\int\sqrt{9+x^2}\,dx=\frac{1}{3}\int\sqrt{(3)^2+x^2}\,dx$

It is known that, $\int\sqrt{x^2+a^2}dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\therefore I=\frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9}+\frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right]+C$$
$$=\frac{x}{6}\sqrt{x^2+9}+\frac{3}{2}\log\left|x+\sqrt{x^2+9}\right|+C$$

Question 10:

$\int\sqrt{1+x^2}\,dx$ is equal to

A. $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$

B. $\frac{2}{3}\left(1+x^2\right)^{\frac{2}{3}}+C$

C. $\frac{2}{3}x\left(1+x^2\right)^{\frac{3}{2}}+C$

D. $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right|+C$

ANS :

It is known that, $\int\sqrt{a^2+x^2}dx=\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\therefore\int\sqrt{1+x^2}dx=\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$$

Hence, the correct answer is A.

Question 11:

$\int \sqrt{x^2 - 8x + 7}\,dx$ is equal to

A. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

B. $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+C$

C. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

D. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

ANS :

$$\begin{aligned}
\text{Let } I &= \int \sqrt{x^2-8x+7}\,dx \\
&= \int \sqrt{\left(x^2-8x+16\right)-9}\,dx \\
&= \int \sqrt{(x-4)^2-(3)^2}\,dx
\end{aligned}$$

It is known that, $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I = \frac{(x-4)}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4)+\sqrt{x^2-8x+7}\right|+C$$

Hence, the correct answer is D.