CLASS XII INTEGRALS CHAPTER 7

EX. 7.6 SOLUTIONS

Question 1:

$x \sin x$

ANS :

Let $I = \int x \sin x \, dx$

Taking x as first function and $\sin x$ as second function and integrating by parts, we obtain

$$I = x \int \sin x \, dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin x \, dx \right\} dx$$
$$= x(-\cos x) - \int 1 \cdot (-\cos x) dx$$
$$= -x \cos x + \sin x + C$$

Question 2:

$x \sin 3x$

ANS :

Let $I = \int x \sin 3x \, dx$

Taking x as first function and $\sin 3x$ as second function and integrating by parts, we obtain

$$I = x \int \sin 3x \, dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin 3x \, dx \right\}$$
$$= x \left(\frac{-\cos 3x}{3} \right) - \int 1 \cdot \left(\frac{-\cos 3x}{3} \right) dx$$
$$= \frac{-x \cos 3x}{3} + \frac{1}{3} \int \cos 3x \, dx$$
$$= \frac{-x \cos 3x}{3} + \frac{1}{9} \sin 3x + C$$

Question 3:

$x^2 e^x$

Let $I=\int x^2 e^x dx$

Taking x^2 as first function and e^x as second function and integrating by parts, we obtain

$$I = x^2\int e^x\,dx-\int\left\{\left(\frac{d}{dx}x^2\right)\int e^x dx\right\}dx$$
$$= x^2e^x-\int 2x\cdot e^x dx$$
$$= x^2e^x-2\int x\cdot e^x dx$$

Again integrating by parts, we obtain

$$= x^2e^x-2\left[x\cdot\int e^x dx-\int\left\{\left(\frac{d}{dx}x\right)\cdot\int e^x dx\right\}dx\right]$$
$$= x^2e^x-2\left[xe^x-\int e^x dx\right]$$
$$= x^2e^x-2\left[xe^x-e^x\right]$$
$$= x^2e^x-2xe^x+2e^x+C$$
$$= e^x\left(x^2-2x+2\right)+C$$

Question 4:

$x\log x$

ANS :

Let $I=\int x\log x dx$

Taking $\log x$ as first function and x as second function and integrating by parts, we obtain

$$I=\log x\int x\,dx-\int\left\{\left(\frac{d}{dx}\log x\right)\int x\,dx\right\}dx$$
$$=\log x\cdot\frac{x^2}{2}-\int\frac{1}{x}\cdot\frac{x^2}{2}dx$$
$$=\frac{x^2\log x}{2}-\int\frac{x}{2}dx$$
$$=\frac{x^2\log x}{2}-\frac{x^2}{4}+C$$

Question 5:

$x\log 2x$

ANS :

Let $I = \int x \log 2x dx$

Taking $\log 2x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log 2x \int x\, dx - \int\left\{\left(\frac{d}{dx} 2\log x\right)\int x\, dx\right\}dx$$

$$= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + C$$

Question 6:

$x^2 \log x$

ANS :

Let $I = \int x^2 \log x\, dx$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$I = \log x \int x^2\, dx - \int\left\{\left(\frac{d}{dx} \log x\right)\int x^2 dx\right\}dx$$

$$= \log x\left(\frac{x^3}{3}\right) - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{x^3 \log x}{3} - \int \frac{x^2}{3} dx$$

$$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$$

Question 7:

$x \sin^{-1} x$

ANS :

Let $I = \int x \sin^{-1} x \, dx$

Taking $\sin^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \sin^{-1} x \int x \, dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1} x \right) \int x \, dx \right\} dx$$

$$= \sin^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \int \sqrt{1-x^2} dx - \int \frac{1}{\sqrt{1-x^2}} dx \right\}$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x - \sin^{-1} x \right\} + C$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1-x^2} + \frac{1}{4} \sin^{-1} x - \frac{1}{2} \sin^{-1} x + C$$

$$= \frac{1}{4} \left(2x^2 - 1\right) \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + C$$

Question 8:

$x \tan^{-1} x$

ANS :

Let $I = \int x \tan^{-1} x \, dx$

Taking $\tan^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int x \, dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int x \, dx \right\} dx$$

$$= \tan^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{1+x^2} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{1+x^2} dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2} \right) dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2} \right) dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \left(x - \tan^{-1} x \right) + C$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{x}{2} + \frac{1}{2} \tan^{-1} x + C$$

Question 9:

$x\cos^{-1}x$

ANS :

Let $I=\int x\cos^{-1}x\,dx$

Taking $\cos^{-1}x$ as first function and x as second function and integrating by parts, we obtain

$$I=\cos^{-1}x\int x\,dx-\int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int x\,dx\right\}dx$$

$$=\cos^{-1}x\frac{x^2}{2}-\int\frac{-1}{\sqrt{1-x^2}}\cdot\frac{x^2}{2}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\left\{\sqrt{1-x^2}+\left(\frac{-1}{\sqrt{1-x^2}}\right)\right\}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\sqrt{1-x^2}dx-\frac{1}{2}\int\left(\frac{-1}{\sqrt{1-x^2}}\right)dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}I_1-\frac{1}{2}\cos^{-1}x \qquad ...(1)$$

where, $I_1=\int\sqrt{1-x^2}dx$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{d}{dx}\sqrt{1-x^2}\int x\,dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{-2x}{2\sqrt{1-x^2}}.x\,dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{-x^2}{\sqrt{1-x^2}}dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\left\{\int\sqrt{1-x^2}\,dx+\int\frac{-dx}{\sqrt{1-x^2}}\right\}$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\left\{I_1+\cos^{-1}x\right\}$$

$$\Rightarrow 2I_1=x\sqrt{1-x^2}-\cos^{-1}x$$

$$\therefore I_1=\frac{x}{2}\sqrt{1-x^2}-\frac{1}{2}\cos^{-1}x$$

Substituting in (1), we obtain

$$I=\frac{x\cos^{-1}x}{2}-\frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2}-\frac{1}{2}\cos^{-1}x\right)-\frac{1}{2}\cos^{-1}x$$

$$=\frac{\left(2x^2-1\right)}{4}\cos^{-1}x-\frac{x}{4}\sqrt{1-x^2}+C$$

Question 10:

$\left(\sin^{-1} x\right)^2$

ANS :

Let $I = \int \left(\sin^{-1} x\right)^2 \cdot 1\, dx$

Taking $\left(\sin^{-1} x\right)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I = \left(\sin^{-1} x\right) \int 1\, dx - \int \left\{ \frac{d}{dx} \left(\sin^{-1} x\right)^2 \cdot \int 1 \cdot dx \right\} dx$$

$$= \left(\sin^{-1} x\right)^2 \cdot x - \int \frac{2 \sin^{-1} x}{\sqrt{1-x^2}} \cdot x\, dx$$

$$= x\left(\sin^{-1} x\right)^2 + \int \sin^{-1} x \cdot \left(\frac{-2x}{\sqrt{1-x^2}} \right) dx$$

$$= x\left(\sin^{-1} x\right)^2 + \left[\sin^{-1} x \int \frac{-2x}{\sqrt{1-x^2}}\, dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1} x \right) \int \frac{-2x}{\sqrt{1-x^2}} dx \right\} dx \right]$$

$$= x\left(\sin^{-1} x\right)^2 + \left[\sin^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}\, dx \right]$$

$$= x\left(\sin^{-1} x\right)^2 + 2\sqrt{1-x^2} \sin^{-1} x - \int 2\, dx$$

$$= x\left(\sin^{-1} x\right)^2 + 2\sqrt{1-x^2} \sin^{-1} x - 2x + C$$

Question 11:

$\dfrac{x \cos^{-1} x}{\sqrt{1-x^2}}$

ANS :

Let $I = \int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$

$$I = \frac{-1}{2}\int \frac{-2x}{\sqrt{1-x^2}} \cdot \cos^{-1} x dx$$

Taking $\cos^{-1} x$ as first function and $\left(\frac{-2x}{\sqrt{1-x^2}}\right)$ as second function and integrating by parts, we obtain

$$I = \frac{-1}{2}\left[\cos^{-1} x \int \frac{-2x}{\sqrt{1-x^2}}dx - \int\left\{\left(\frac{d}{dx}\cos^{-1} x\right)\int \frac{-2x}{\sqrt{1-x^2}}dx\right\}dx\right]$$
$$= \frac{-1}{2}\left[\cos^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}\, dx\right]$$
$$= \frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1} x + \int 2\, dx\right]$$
$$= \frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1} x + 2x\right] + C$$
$$= -\left[\sqrt{1-x^2}\cos^{-1} x + x\right] + C$$

Question 12:

$x\sec^2 x$

ANS :

Let $I = \int x\sec^2 x dx$

Taking x as first function and $\sec^2 x$ as second function and integrating by parts, we obtain

$$I = x\int \sec^2 x\, dx - \int\left\{\left\{\frac{d}{dx}x\right\}\int \sec^2 x\, dx\right\}dx$$
$$= x\tan x - \int 1 \cdot \tan x dx$$
$$= x\tan x + \log|\cos x| + C$$

Question 13:

$\tan^{-1} x$

ANS :

Let $I = \int 1 \cdot \tan^{-1} x dx$

Taking $\tan^{-1} x$ as first function and 1 as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int 1 dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int 1 \cdot dx \right\} dx$$

$$= \tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x \, dx$$

$$= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx$$

$$= x \tan^{-1} x - \frac{1}{2} \log \left| 1+x^2 \right| + \mathrm{C}$$

$$= x \tan^{-1} x - \frac{1}{2} \log \left(1+x^2 \right) + \mathrm{C}$$

Question 14:

$x(\log x)^2$

ANS :

$$I = \int x (\log x)^2 \, dx$$

Taking $(\log x)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I = (\log x)^2 \int x \, dx - \int \left[\left\{ \left(\frac{d}{dx} \log x \right)^2 \right\} \int x \, dx \right] dx$$

$$= \frac{x^2}{2} (\log x)^2 - \left[\int 2 \log x \cdot \frac{1}{x} \cdot \frac{x^2}{2} dx \right]$$

$$= \frac{x^2}{2} (\log x)^2 - \int x \log x \, dx$$

Again integrating by parts, we obtain

$$I = \frac{x^2}{2} (\log x)^2 - \left[\log x \int x \, dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x \, dx \right\} dx \right]$$

$$= \frac{x^2}{2} (\log x)^2 - \left[\frac{x^2}{2} - \log x - \int \frac{1}{x} \cdot \frac{x^2}{2} dx \right]$$

$$= \frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{1}{2} \int x \, dx$$

$$= \frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{x^2}{4} + \mathrm{C}$$

Question 15:

$\left(x^2+1\right)\log x$

ANS :

Let $I=\int\left(x^2+1\right)\log x\,dx=\int x^2\log x\,dx+\int\log x\,dx$

Let $I = I_1 + I_2$ … (1)

Where, $I_1=\int x^2\log x\,dx$ and $I_2=\int\log x\,dx$

$I_1=\int x^2\log x dx$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$I_1=\log x-\int x^2dx-\int\left\{\left(\frac{d}{dx}\log x\right)\int x^2dx\right\}dx$$
$$=\log x\cdot\frac{x^3}{3}-\int\frac{1}{x}\cdot\frac{x^3}{3}dx$$
$$=\frac{x^3}{3}\log x-\frac{1}{3}\left(\int x^2dx\right)$$
$$=\frac{x^3}{3}\log x-\frac{x^3}{9}+C_1 \qquad \ldots(2)$$

$I_2=\int\log x\,dx$

Taking $\log x$ as first function and 1 as second function and integrating by parts, we obtain

$$I_2=\log x\int 1\cdot dx-\int\left\{\left(\frac{d}{dx}\log x\right)\int 1\cdot dx\right\}$$
$$=\log x\cdot x-\int\frac{1}{x}\cdot x dx$$
$$=x\log x-\int 1dx$$
$$=x\log x-x+C_2 \qquad \ldots(3)$$

Using equations (2) and (3) in (1), we obtain

$$I=\frac{x^3}{3}\log x-\frac{x^3}{9}+C_1+x\log x-x+C_2$$
$$=\frac{x^3}{3}\log x-\frac{x^3}{9}+x\log x-x+\left(C_1+C_2\right)$$
$$=\left(\frac{x^3}{3}+x\right)\log x-\frac{x^3}{9}-x+C$$

Question 16:

$e^x(\sin x+\cos x)$

ANS :

Let $I=\int e^x(\sin x+\cos x)dx$

Let $f(x)=\sin x$

$\Rightarrow\ f'(x)=\cos x$

$\therefore\ I=\int e^x\{f(x)+f'(x)\}dx$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+C$

$\therefore I=e^x\sin x+C$

Question 17:

$\frac{xe^x}{(1+x)^2}$

ANS :

Let $I = \int \frac{xe^x}{(1+x)^2} dx = \int e^x \left\{ \frac{x}{(1+x)^2} \right\} dx$

$$= \int e^x \left\{ \frac{1+x-1}{(1+x)^2} \right\} dx$$

$$= \int e^x \left\{ \frac{1}{1+x} - \frac{1}{(1+x)^2} \right\} dx$$

Let $f(x) = \frac{1}{1+x} \Rightarrow f'(x) = \frac{-1}{(1+x)^2}$

$$\Rightarrow \int \frac{xe^x}{(1+x)^2} dx = \int e^x \{ f(x) + f'(x) \} dx$$

It is known that, $\int e^x \{ f(x) + f'(x) \} dx = e^x f(x) + C$

$$\therefore \int \frac{xe^x}{(1+x)^2} dx = \frac{e^x}{1+x} + C$$

Question 18:

$$e^x \left(\frac{1+\sin x}{1+\cos x} \right)$$

ANS :

$$e^x\left(\frac{1+\sin x}{1+\cos x}\right)$$

$$=e^x\left(\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$

$$=\frac{e^x\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$$

$$=\frac{1}{2}e^x\cdot\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}}\right)^2$$

$$=\frac{1}{2}e^x\left[\tan\frac{x}{2}+1\right]^2$$

$$=\frac{1}{2}e^2\left(1+\tan\frac{x}{2}\right)^2$$

$$=\frac{1}{2}e^x\left[1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$=\frac{1}{2}e^x\left[\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$\frac{e^x(1+\sin x)dx}{(1+\cos x)}=e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right] \qquad ...(1)$$

Let $\tan\frac{x}{2}=f(x) \Rightarrow f'(x)=\frac{1}{2}\sec^2\frac{x}{2}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+C$

From equation (1), we obtain

$$\int\frac{e^x(1+\sin x)}{(1+\cos x)}dx=e^x\tan\frac{x}{2}+C$$

Question 19:

$$e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$$

ANS ;

Let $I = \int e^x \left[\frac{1}{x} - \frac{1}{x^2}\right] dx$

Also, let $\frac{1}{x} = f(x) \Rightarrow f'(x) = \frac{-1}{x^2}$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$\therefore I = \frac{e^x}{x} + C$

Question 20:

$\frac{(x-3)e^x}{(x-1)^3}$

ANS :

$$\int e^x \left\{\frac{x-3}{(x-1)^3}\right\} dx = \int e^x \left\{\frac{x-1-2}{(x-1)^3}\right\} dx$$
$$= \int e^x \left\{\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right\} dx$$

Let $f(x) = \frac{1}{(x-1)^2} \Rightarrow f'(x) = \frac{-2}{(x-1)^3}$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$\therefore \int e^x \left\{\frac{(x-3)}{(x-1)^2}\right\} dx = \frac{e^x}{(x-1)^2} + C$

Question 21:

$e^{2x} \sin x$

ANS :

Let $I=\int e^{2x}\sin x\,dx$ ...(1)

Integrating by parts, we obtain

$$I=\sin x\int e^{2x}dx-\int\left\{\left(\frac{d}{dx}\sin x\right)\int e^{2x}dx\right\}dx$$

$$\Rightarrow I=\sin x\cdot\frac{e^{2x}}{2}-\int\cos x\cdot\frac{e^{2x}}{2}dx$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\int e^{2x}\cos x\,dx$$

Again integrating by parts, we obtain

$$I=\frac{e^{2x}\cdot\sin x}{2}-\frac{1}{2}\left[\cos x\int e^{2x}dx-\int\left\{\left(\frac{d}{dx}\cos x\right)\int e^{2x}dx\right\}dx\right]$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\left[\cos x\cdot\frac{e^{2x}}{2}-\int(-\sin x)\frac{e^{2x}}{2}dx\right]$$

$$\Rightarrow I=\frac{e^{2x}\cdot\sin x}{2}-\frac{1}{2}\left[\frac{e^{2x}\cos x}{2}+\frac{1}{2}\int e^{2x}\sin x dx\right]$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}-\frac{1}{4}I \qquad \text{[From (1)]}$$

$$\Rightarrow I+\frac{1}{4}I=\frac{e^{2x}\cdot\sin x}{2}-\frac{e^{2x}\cos x}{4}$$

$$\Rightarrow \frac{5}{4}I=\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}$$

$$\Rightarrow I=\frac{4}{5}\left[\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}\right]+C$$

$$\Rightarrow I=\frac{e^{2x}}{5}\left[2\sin x-\cos x\right]+C$$

Question 22:

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

ANS :

Let $x=\tan\theta \Rightarrow dx=\sec^2\theta\, d\theta$

$$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right)=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)=\sin^{-1}(\sin 2\theta)=2\theta$$

$$\Rightarrow \int\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx=\int 2\theta\cdot\sec^2\theta\, d\theta=2\int\theta\cdot\sec^2\theta\, d\theta$$

Integrating by parts, we obtain

$$2\left[\theta\cdot\int\sec^2\theta d\theta-\int\left\{\left(\frac{d}{d\theta}\theta\right)\int\sec^2\theta d\theta\right\}d\theta\right]$$
$$=2\left[\theta\cdot\tan\theta-\int\tan\theta d\theta\right]$$
$$=2\left[\theta\tan\theta+\log|\cos\theta|\right]+C$$
$$=2\left[x\tan^{-1}x+\log\left|\frac{1}{\sqrt{1+x^2}}\right|\right]+C$$
$$=2x\tan^{-1}x+2\log(1+x^2)^{-\frac{1}{2}}+C$$
$$=2x\tan^{-1}x+2\left[-\frac{1}{2}\log(1+x^2)\right]+C$$
$$=2x\tan^{-1}x-\log(1+x^2)+C$$

Question 23:

$\int x^2e^{x^3}dx$ equals

(A) $\frac{1}{3}e^{x^3}+C$ (B) $\frac{1}{3}e^{x^2}+C$

(C) $\frac{1}{2}e^{x^3}+C$ (D) $\frac{1}{3}e^{x^2}+C$

ANS :

Let $I=\int x^2 e^{x^3}\,dx$

Also, let $x^3=t \Rightarrow 3x^2dx=dt$

$$\Rightarrow I=\frac{1}{3}\int e^t dt$$
$$=\frac{1}{3}\left(e^t\right)+C$$
$$=\frac{1}{3}e^{x^3}+C$$

Hence, the correct answer is A.

Question 24:

$\int e^x \sec x(1+\tan x)\,dx$ equals

(A) $e^x\cos x+C$ (B) $e^x\sec x+C$

(C) $e^x\sin x+C$ (D) $e^x\tan x+C$

ANS :

$$\int e^x \sec x(1+\tan x)\,dx$$

Let $I=\int e^x\sec x(1+\tan x)\,dx=\int e^x(\sec x+\sec x\tan x)\,dx$

Also, let $\sec x=f(x) \Rightarrow \sec x\tan x=f'(x)$

It is known that, $\int e^x\{f(x)+f'(x)\}\,dx=e^x f(x)+C$

$\therefore I=e^x\sec x+C$

Hence, the correct answer is B.