CLASS XII INTEGRALS CHAPTER 7

EX. 7.3 SOLUTIONS

1. $\sin^2(2x+5)$

ANS :

$$\sin^2(2x+5) = \frac{1-\cos 2(2x+5)}{2} = \frac{1-\cos(4x+10)}{2}$$

$$\Rightarrow \int \sin^2(2x+5)\,dx = \int \frac{1-\cos(4x+10)}{2}dx$$

$$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos(4x+10)\,dx$$

$$= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right) + C$$

$$= \frac{1}{2}x - \frac{1}{8}\sin(4x+10) + C$$

2. $\sin 3x \cos 4x$

ANS :

It is known that, $\sin A \cos B = \frac{1}{2}\{\sin(A+B)+\sin(A-B)\}$

$$\therefore \int \sin 3x \cos 4x\,dx = \frac{1}{2}\int \{\sin(3x+4x)+\sin(3x-4x)\}\,dx$$

$$= \frac{1}{2}\int \{\sin 7x + \sin(-x)\}\,dx$$

$$= \frac{1}{2}\int \{\sin 7x - \sin x\}\,dx$$

$$= \frac{1}{2}\int \sin 7x\,dx - \frac{1}{2}\int \sin x\,dx$$

$$= \frac{1}{2}\left(\frac{-\cos 7x}{7}\right) - \frac{1}{2}(-\cos x) + C$$

$$= \frac{-\cos 7x}{14} + \frac{\cos x}{2} + C$$

3. cos 2x cos 4x cos 6x

ANS :

It is known that, $\cos A\cos B=\frac{1}{2}\{\cos(A+B)+\cos(A-B)\}$

$$\therefore \int\cos 2x(\cos 4x\cos 6x)\,dx=\int\cos 2x\left[\frac{1}{2}\{\cos(4x+6x)+\cos(4x-6x)\}\right]dx$$

$$=\frac{1}{2}\int\{\cos 2x\cos 10x+\cos 2x\cos(-2x)\}\,dx$$

$$=\frac{1}{2}\int\{\cos 2x\cos 10x+\cos^2 2x\}\,dx$$

$$=\frac{1}{2}\int\left[\left\{\frac{1}{2}\cos(2x+10x)+\cos(2x-10x)\right\}+\left(\frac{1+\cos 4x}{2}\right)\right]dx$$

$$=\frac{1}{4}\int(\cos 12x+\cos 8x+1+\cos 4x)\,dx$$

$$=\frac{1}{4}\left[\frac{\sin 12x}{12}+\frac{\sin 8x}{8}+x+\frac{\sin 4x}{4}\right]+C$$

4. $\sin^3(2x+1)$

ANS :

Let $I=\int\sin^3(2x+1)$

$$\Rightarrow \int\sin^3(2x+1)\,dx=\int\sin^2(2x+1)\cdot\sin(2x+1)\,dx$$

$$=\int(1-\cos^2(2x+1))\sin(2x+1)\,dx$$

Let $\cos(2x+1)=t$

$$\Rightarrow -2\sin(2x+1)\,dx=dt$$

$$\Rightarrow \sin(2x+1)\,dx=\frac{-dt}{2}$$

$$\Rightarrow I=\frac{-1}{2}\int(1-t^2)\,dt$$

$$=\frac{-1}{2}\left\{t-\frac{t^3}{3}\right\}$$

$$=\frac{-1}{2}\left\{\cos(2x+1)-\frac{\cos^3(2x+1)}{3}\right\}$$

$$=\frac{-\cos(2x+1)}{2}+\frac{\cos^3(2x+1)}{6}+C$$

5. $\sin^3 x \cos^3 x$

ANS :

$$\text{Let } I = \int \sin^3 x \cos^3 x \cdot dx$$
$$= \int \cos^3 x \cdot \sin^2 x \cdot \sin x \cdot dx$$
$$= \int \cos^3 x \left(1 - \cos^2 x\right) \sin x \cdot dx$$

Let $\cos x = t$

$\Rightarrow -\sin x \cdot dx = dt$

$$\Rightarrow I = -\int t^3 \left(1 - t^2\right) dt$$
$$= -\int \left(t^3 - t^5\right) dt$$
$$= -\left\{\frac{t^4}{4} - \frac{t^6}{6}\right\} + C$$
$$= -\left\{\frac{\cos^4 x}{4} - \frac{\cos^6 x}{6}\right\} + C$$
$$= \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C$$

6. $\sin x \sin 2x \sin 3x$

ANS :

It is known that, $\sin A \sin B = \frac{1}{2}\left\{\cos(A - B) - \cos(A + B)\right\}$

$$\therefore \int \sin x \sin 2x \sin 3x \; dx = \int \left[\sin x \cdot \frac{1}{2}\left\{\cos(2x - 3x) - \cos(2x + 3x)\right\}\right] dx$$
$$= \frac{1}{2}\int \left(\sin x \cos(-x) - \sin x \cos 5x\right) dx$$
$$= \frac{1}{2}\int \left(\sin x \cos x - \sin x \cos 5x\right) dx$$
$$= \frac{1}{2}\int \frac{\sin 2x}{2} \, dx - \frac{1}{2}\int \sin x \cos 5x \; dx$$
$$= \frac{1}{4}\left[\frac{-\cos 2x}{2}\right] - \frac{1}{2}\int \left\{\frac{1}{2}\sin(x + 5x) + \sin(x - 5x)\right\} dx$$
$$= \frac{-\cos 2x}{8} - \frac{1}{4}\int \left(\sin 6x + \sin(-4x)\right) dx$$
$$= \frac{-\cos 2x}{8} - \frac{1}{4}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{4}\right] + C$$
$$= \frac{-\cos 2x}{8} - \frac{1}{8}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{2}\right] + C$$
$$= \frac{1}{8}\left[\frac{\cos 6x}{3} - \frac{\cos 4x}{2} - \cos 2x\right] + C$$

7. $\sin 4x \sin 8x$

ANS :

It is known that, $\sin A \sin B = \frac{1}{2}\cos(A-B) - \cos(A+B)$

$$\therefore \int \sin 4x \sin 8x \, dx = \int \left\{ \frac{1}{2}\cos(4x-8x) - \cos(4x+8x) \right\} dx$$

$$= \frac{1}{2}\int (\cos(-4x) - \cos 12x) \, dx$$

$$= \frac{1}{2}\int (\cos 4x - \cos 12x) \, dx$$

$$= \frac{1}{2}\left[\frac{\sin 4x}{4} - \frac{\sin 12x}{12} \right]$$

8. $\dfrac{1-\cos x}{1+\cos x}$

ANS :

$$\frac{1-\cos x}{1+\cos x} = \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} \qquad \left[2\sin^2 \frac{x}{2} = 1 - \cos x \text{ and } 2\cos^2 \frac{x}{2} = 1 + \cos x \right]$$

$$= \tan^2 \frac{x}{2}$$

$$= \left(\sec^2 \frac{x}{2} - 1 \right)$$

$$\therefore \int \frac{1-\cos x}{1+\cos x} dx = \int \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

$$= \left[\frac{\tan \frac{x}{2}}{\frac{1}{2}} - x \right] + C$$

$$= 2\tan \frac{x}{2} - x + C$$

9. $\dfrac{\cos x}{1+\cos x}$

ANS :

$$\frac{\cos x}{1+\cos x}=\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[\cos x=\cos^2\frac{x}{2}-\sin^2\frac{x}{2} \text{ and } \cos x=2\cos^2\frac{x}{2}-1\right]$$

$$=\frac{1}{2}\left[1-\tan^2\frac{x}{2}\right]$$

$$\therefore \int\frac{\cos x}{1+\cos x}dx=\frac{1}{2}\int\left(1-\tan^2\frac{x}{2}\right)dx$$

$$=\frac{1}{2}\int\left(1-\sec^2\frac{x}{2}+1\right)dx$$

$$=\frac{1}{2}\int\left(2-\sec^2\frac{x}{2}\right)dx$$

$$=\frac{1}{2}\left[2x-\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]+C$$

$$=x-\tan\frac{x}{2}+C$$

10. $\sin^4 x$

ANS :

$$\sin^4 x=\sin^2 x\sin^2 x$$

$$=\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$$

$$=\frac{1}{4}(1-\cos 2x)^2$$

$$=\frac{1}{4}\left[1+\cos^2 2x-2\cos 2x\right]$$

$$=\frac{1}{4}\left[1+\left(\frac{1+\cos 4x}{2}\right)-2\cos 2x\right]$$

$$=\frac{1}{4}\left[1+\frac{1}{2}+\frac{1}{2}\cos 4x-2\cos 2x\right]$$

$$=\frac{1}{4}\left[\frac{3}{2}+\frac{1}{2}\cos 4x-2\cos 2x\right]$$

$$\therefore \int\sin^4 x\,dx=\frac{1}{4}\int\left[\frac{3}{2}+\frac{1}{2}\cos 4x-2\cos 2x\right]dx$$

$$=\frac{1}{4}\left[\frac{3}{2}x+\frac{1}{2}\left(\frac{\sin 4x}{4}\right)-\frac{2\sin 2x}{2}\right]+C$$

$$=\frac{1}{8}\left[3x+\frac{\sin 4x}{4}-2\sin 2x\right]+C$$

$$=\frac{3x}{8}-\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+C$$

11. $\cos^4 2x$

ANS :

$$\cos^4 2x = \left(\cos^2 2x\right)^2$$
$$= \left(\frac{1+\cos 4x}{2}\right)^2$$
$$= \frac{1}{4}\left[1+\cos^2 4x + 2\cos 4x\right]$$
$$= \frac{1}{4}\left[1+\left(\frac{1+\cos 8x}{2}\right)+2\cos 4x\right]$$
$$= \frac{1}{4}\left[1+\frac{1}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]$$
$$= \frac{1}{4}\left[\frac{3}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]$$
$$\therefore \int \cos^4 2x\, dx = \int\left(\frac{3}{8}+\frac{\cos 8x}{8}+\frac{\cos 4x}{2}\right)dx$$
$$= \frac{3}{8}x+\frac{\sin 8x}{64}+\frac{\sin 4x}{8}+C$$

12. $\dfrac{\sin^2 x}{1+\cos x}$

ANS :

$$\frac{\sin^2 x}{1+\cos x} = \frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}} \quad \left[\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2};\ \cos x = 2\cos^2\frac{x}{2}-1\right]$$
$$= \frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$
$$= 2\sin^2\frac{x}{2}$$
$$= 1-\cos x$$
$$\therefore \int \frac{\sin^2 x}{1+\cos x}dx = \int(1-\cos x)dx$$
$$= x-\sin x + C$$

13. $\dfrac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}$

ANS :

$$\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}=\frac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2\alpha}{2}}{-2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}} \qquad \left[\cos C-\cos D=-2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$=\frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=\frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$=2\left[\cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right]$$

$$=2\left[\cos(x)+\cos\alpha\right]$$

$$=2\cos x+2\cos\alpha$$

$$\therefore\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx=\int 2\cos x+2\cos\alpha$$

$$=2\left[\sin x+x\cos\alpha\right]+\mathrm{C}$$

14. $\dfrac{\cos x-\sin x}{1+\sin 2x}$

ANS :

$$\frac{\cos x-\sin x}{1+\sin 2x}=\frac{\cos x-\sin x}{\left(\sin^2 x+\cos^2 x\right)+2\sin x\cos x}$$

$$\left[\sin^2 x+\cos^2 x=1;\ \sin 2x=2\sin x\cos x\right]$$

$$=\frac{\cos x-\sin x}{(\sin x+\cos x)^2}$$

Let $\sin x+\cos x=t$

$\therefore(\cos x-\sin x)\,dx=dt$

$$\Rightarrow\int\frac{\cos x-\sin x}{1+\sin 2x}dx=\int\frac{\cos x-\sin x}{(\sin x+\cos x)^2}dx$$

$$=\int\frac{dt}{t^2}$$

$$=\int t^{-2}dt$$

$$=-t^{-1}+\mathrm{C}$$

$$=-\frac{1}{t}+\mathrm{C}$$

$$=\frac{-1}{\sin x+\cos x}+\mathrm{C}$$

15. $\tan^3 2x \sec 2x$

ANS :

$$\tan^3 2x\sec 2x = \tan^2 2x\tan 2x\sec 2x$$
$$= \left(\sec^2 2x - 1\right)\tan 2x\sec 2x$$
$$= \sec^2 2x\cdot\tan 2x\sec 2x - \tan 2x\sec 2x$$
$$\therefore \int\tan^3 2x\sec 2x\,dx = \int\sec^2 2x\tan 2x\sec 2x\,dx - \int\tan 2x\sec 2x\,dx$$
$$= \int\sec^2 2x\tan 2x\sec 2x\,dx - \frac{\sec 2x}{2} + C$$

Let $\sec 2x = t$

$\therefore 2\sec 2x\tan 2x\,dx = dt$

$$\therefore \int\tan^3 2x\sec 2x\,dx = \frac{1}{2}\int t^2 dt - \frac{\sec 2x}{2} + C$$
$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$
$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$

16. $\tan^4 x$

Ans :

$$\tan^4 x$$
$$= \tan^2 x\cdot\tan^2 x$$
$$= \left(\sec^2 x - 1\right)\tan^2 x$$
$$= \sec^2 x\tan^2 x - \tan^2 x$$
$$= \sec^2 x\tan^2 x - \left(\sec^2 x - 1\right)$$
$$= \sec^2 x\tan^2 x - \sec^2 x + 1$$

$$\therefore \int\tan^4 x\,dx = \int\sec^2 x\tan^2 x\,dx - \int\sec^2 x\,dx + \int 1\cdot dx$$
$$= \int\sec^2 x\tan^2 x\,dx - \tan x + x + C \qquad \text{...(1)}$$

Consider $\int\sec^2 x\tan^2 x\,dx$

Let $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$\Rightarrow \int\sec^2 x\tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

From equation (1), we obtain

$$\int\tan^4 x\,dx = \frac{1}{3}\tan^3 x - \tan x + x + C$$

17. $\dfrac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

Ans :

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$

$$= \tan x \sec x + \cot x \operatorname{cosec} x$$

$$\therefore \quad \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}\, dx = \int (\tan x \sec x + \cot x \operatorname{cosec} x)\, dx$$

$$= \sec x - \operatorname{cosec} x + C$$

18. $\dfrac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

Ans :

$$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$$

$$= \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x} \qquad \left[\cos 2x = 1 - 2\sin^2 x\right]$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\therefore \quad \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\, dx = \int \sec^2 x\, dx = \tan x + C$$

19. $\dfrac{1}{\sin x \cos^3 x}$

ANS :

$$\frac{1}{\sin x\cos^3 x}=\frac{\sin^2 x+\cos^2 x}{\sin x\cos^3 x}$$
$$=\frac{\sin x}{\cos^3 x}+\frac{1}{\sin x\cos x}$$
$$=\tan x\sec^2 x+\frac{\frac{1\cos^2 x}{\sin x\cos x}}{\cos^2 x}$$
$$=\tan x\sec^2 x+\frac{\sec^2 x}{\tan x}$$

$$\therefore\ \int\frac{1}{\sin x\cos^3 x}dx=\int\tan x\sec^2 x\ dx+\int\frac{\sec^2 x}{\tan x}\ dx$$

Let $\tan x=t\Rightarrow\sec^2 x\ dx=dt$

$$\Rightarrow\int\frac{1}{\sin x\cos^3 x}dx=\int t dt+\int\frac{1}{t}dt$$
$$=\frac{t^2}{2}+\log|t|+C$$
$$=\frac{1}{2}\tan^2 x+\log|\tan x|+C$$

20. $\dfrac{\cos 2x}{(\cos x+\sin x)^2}$

ANS :

$$\frac{\cos 2x}{(\cos x+\sin x)^2}=\frac{\cos 2x}{\cos^2 x+\sin^2 x+2\sin x\cos x}=\frac{\cos 2x}{1+\sin 2x}$$

$$\therefore\ \int\frac{\cos 2x}{(\cos x+\sin x)^2}\ dx=\int\frac{\cos 2x}{(1+\sin 2x)}\ dx$$

Let $1+\sin 2x=t$

$\Rightarrow 2\cos 2x\ dx=dt$

$$\therefore\int\frac{\cos 2x}{(\cos x+\sin x)^2}dx=\frac{1}{2}\int\frac{1}{t}dt$$
$$=\frac{1}{2}\log|t|+C$$
$$=\frac{1}{2}\log|1+\sin 2x|+C$$
$$=\frac{1}{2}\log\left|(\sin x+\cos x)^2\right|+C$$
$$=\log|\sin x+\cos x|+C$$

21. $\sin^{-1}(\cos x)$

ANS :

$\sin^{-1}(\cos x)$

Let $\cos x = t$

Then, $\sin x = \sqrt{1-t^2}$

$\Rightarrow (-\sin x)dx = dt$

$dx = \dfrac{-dt}{\sin x}$

$dx = \dfrac{-dt}{\sqrt{1-t^2}}$

$$\therefore \int \sin^{-1}(\cos x)dx = \int \sin^{-1} t\left(\frac{-dt}{\sqrt{1-t^2}}\right)$$

$$= -\int \frac{\sin^{-1} t}{\sqrt{1-t^2}}dt$$

Let $\sin^{-1} t = u$

$\Rightarrow \dfrac{1}{\sqrt{1-t^2}}dt = du$

$$\therefore \int \sin^{-1}(\cos x)dx = \int 4du$$

$$= -\frac{u^2}{2} + C$$

$$= \frac{-(\sin^1 t)^2}{2} + C$$

$$= \frac{-\left[\sin^{-1}(\cos x)\right]^2}{2} + C \qquad ...(1)$$

It is known that,

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\therefore \sin^{-1}(\cos x) = \frac{\pi}{2} - \cos^{-1}(\cos x) = \left(\frac{\pi}{2} - x\right)$$

Substituting in equation (1), we obtain

$$\int \sin^{-1}(\cos x)\,dx = \frac{-\left[\frac{\pi}{2} - x\right]^2}{2} + C$$

$$= -\frac{1}{2}\left(\frac{\pi^2}{2} + x^2 - \pi x\right) + C$$

$$= -\frac{\pi^2}{8} - \frac{x^2}{2} + \frac{1}{2}\pi x + C$$

$$= \frac{\pi x}{2} - \frac{x^2}{2} + \left(C - \frac{\pi^2}{8}\right)$$

$$= \frac{\pi x}{2} - \frac{x^2}{2} + C_1$$

22. $\dfrac{1}{\cos(x-a)\cos(x-b)}$

ANS :

$$\frac{1}{\cos(x-a)\cos(x-b)}=\frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$
$$=\frac{1}{\sin(a-b)}\left[\frac{\sin\left[(x-b)-(x-a)\right]}{\cos(x-a)\cos(x-b)}\right]$$
$$=\frac{1}{\sin(a-b)}\frac{\left[\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)\right]}{\cos(x-a)\cos(x-b)}$$
$$=\frac{1}{\sin(a-b)}\left[\tan(x-b)-\tan(x-a)\right]$$

$$\Rightarrow\int\frac{1}{\cos(x-a)\cos(x-b)}dx=\frac{1}{\sin(a-b)}\int\left[\tan(x-b)-\tan(x-a)\right]dx$$
$$=\frac{1}{\sin(a-b)}\left[-\log\left|\cos(x-b)\right|+\log\left|\cos(x-a)\right|\right]$$
$$=\frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C$$

23. $\int\dfrac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx$ is equal to

(A) $\tan x+\cot x+C$

(B) $\tan x+\operatorname{cosec} x+C$

(C) $-\tan x+\cot x+C$

(D) $\tan x+\sec x+C$

ANS :

$$\int\frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx=\int\left(\frac{\sin^2 x}{\sin^2 x\cos^2 x}-\frac{\cos^2 x}{\sin^2 x\cos^2 x}\right)dx$$
$$=\int\left(\sec^2 x-\operatorname{cosec}^2 x\right)dx$$
$$=\tan x+\cot x+C$$

Hence, the correct answer is A.

24. $\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$ equals

(A) $-\cot(ex^x) + C$

(B) $\tan(xe^x) + C$

(C) $\tan(e^x) + C$

(D) $\cot(e^x) + C$

ANS :

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$$

Let $ex^x = t$

$$\Rightarrow \left(e^x \cdot x + e^x \cdot 1\right) dx = dt$$

$$e^x(x+1) dx = dt$$

$$\therefore \int \frac{e^x(1+x)}{\cos^2(e^x x)} dx = \int \frac{dt}{\cos^2 t}$$

$$= \int \sec^2 t \, dt$$

$$= \tan t + C$$

$$= \tan\left(e^x \cdot x\right) + C$$

Hence, the correct answer is B.