CLASS XII INTEGRALS CHAPTER 7

EX. 7.2 SOLUTIONS

1. $\dfrac{2x}{1+x^2}$ 2. $\dfrac{(\log x)^2}{x}$ 3. $\dfrac{1}{x+x\log x}$

ANS :

1.

Let $1+x^2=t$

$\therefore 2x\,dx=dt$

$\Rightarrow \int\frac{2x}{1+x^2}dx=\int\frac{1}{t}dt$

$=\log|t|+C$

$=\log|1+x^2|+C$

$=\log(1+x^2)+C$

2.

Let $\log|x|=t$

$\therefore \frac{1}{x}dx=dt$

$\Rightarrow \int\frac{(\log|x|)^2}{x}dx=\int t^2dt$

$=\frac{t^3}{3}+C$

$=\frac{(\log|x|)^3}{3}+C$

3.

$\frac{1}{x+x\log x}=\frac{1}{x(1+\log x)}$

Let $1+\log x=t$

$\therefore \frac{1}{x}dx=dt$

$\Rightarrow \int\frac{1}{x(1+\log x)}dx=\int\frac{1}{t}dt$

$=\log|t|+C$

$=\log|1+\log x|+C$

4. $\sin x \sin(\cos x)$ 5. $\sin(ax+b)\cos(ax+b)$

ANS :

4.

$\sin x \cdot \sin(\cos x)$

Let $\cos x = t$

$\therefore -\sin x\, dx = dt$

$$\Rightarrow \int \sin x \cdot \sin(\cos x)\, dx = -\int \sin t\, dt$$
$$= -[-\cos t] + C$$
$$= \cos t + C$$
$$= \cos(\cos x) + C$$

5.

$$\sin(ax+b)\cos(ax+b) = \frac{2\sin(ax+b)\cos(ax+b)}{2} = \frac{\sin 2(ax+b)}{2}$$

Let $2(ax+b) = t$

$\therefore 2a\,dx = dt$

$$\Rightarrow \int \frac{\sin 2(ax+b)}{2} dx = \frac{1}{2}\int \frac{\sin t\ dt}{2a}$$
$$= \frac{1}{4a}[-\cos t] + C$$
$$= \frac{-1}{4a}\cos 2(ax+b) + C$$

6. $\sqrt{ax+b}$ 7. $x\sqrt{x+2}$

ANS :

6.

Let $ax + b = t$

$\Rightarrow adx = dt$

$\therefore dx = \frac{1}{a}dt$

$\Rightarrow \int (ax+b)^{\frac{1}{2}} dx = \frac{1}{a}\int t^{\frac{1}{2}} dt$

$= \frac{1}{a}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{2}{3a}(ax+b)^{\frac{3}{2}} + C$

7.

Let $(x+2) = t$

$\therefore dx = dt$

$\Rightarrow \int x\sqrt{x+2}dx = \int (t-2)\sqrt{t}dt$

$= \int \left(t^{\frac{3}{2}} - 2t^{\frac{1}{2}}\right)dt$

$= \int t^{\frac{3}{2}}dt - 2\int t^{\frac{1}{2}}dt$

$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}} + C$

$= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

8. $x\sqrt{1+2x^2}$

ANS :

Let $1 + 2x^2 = t$

$\therefore 4x dx = dt$

$$\Rightarrow \int x\sqrt{1+2x^2}\,dx = \int \frac{\sqrt{t}\,dt}{4}$$

$$= \frac{1}{4}\int t^{\frac{1}{2}}\,dt$$

$$= \frac{1}{4}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}} + C$$

9. $(4x+2)\sqrt{x^2+x+1}$

ANS :

Let $x^2 + x + 1 = t$

$\therefore (2x + 1)dx = dt$

$$\int (4x+2)\sqrt{x^2+x+1}\;dx$$

$$= \int 2\sqrt{t}\;dt$$

$$= 2\int \sqrt{t}\;dt$$

$$= 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}} + C$$

10. $\frac{1}{x-\sqrt{x}}$

ANS :

$$\frac{1}{x-\sqrt{x}}=\frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}$$

Let $\left(\sqrt{x}-1\right)=t$

$$\therefore \frac{1}{2\sqrt{x}}dx=dt$$

$$\Rightarrow \int\frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}dx=\int\frac{2}{t}dt$$

$$=2\log|t|+C$$
$$=2\log\left|\sqrt{x}-1\right|+C$$

11. $\frac{x}{\sqrt{x+4}}, x>0$

ANS :

Let $x+4=t$

$\therefore dx=dt$

$$\int\frac{x}{\sqrt{x+4}}dx=\int\frac{(t-4)}{\sqrt{t}}dt$$
$$=\int\left(\sqrt{t}-\frac{4}{\sqrt{t}}\right)dt$$
$$=\frac{t^{\frac{3}{2}}}{\frac{3}{2}}-4\left(\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right)+C$$
$$=\frac{2}{3}(t)^{\frac{3}{2}}-8(t)^{\frac{1}{2}}+C$$
$$=\frac{2}{3}t\cdot t^{\frac{1}{2}}-8t^{\frac{1}{2}}+C$$
$$=\frac{2}{3}t^{\frac{1}{2}}(t-12)+C$$
$$=\frac{2}{3}(x+4)^{\frac{1}{2}}(x+4-12)+C$$
$$=\frac{2}{3}\sqrt{x+4}\,(x-8)+C$$

12. $(x^3-1)^{\frac{1}{3}}x^5$

ANS :

Let $x^3-1=t$

$\therefore 3x^2dx=dt$

$$\Rightarrow \int(x^3-1)^{\frac{1}{3}}x^5dx=\int(x^3-1)^{\frac{1}{3}}x^3\cdot x^2dx$$

$$=\int t^{\frac{1}{3}}(t+1)\frac{dt}{3}$$

$$=\frac{1}{3}\int\left(t^{\frac{4}{3}}+t^{\frac{1}{3}}\right)dt$$

$$=\frac{1}{3}\left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}}+\frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right]+C$$

$$=\frac{1}{3}\left[\frac{3}{7}t^{\frac{7}{3}}+\frac{3}{4}t^{\frac{4}{3}}\right]+C$$

$$=\frac{1}{7}(x^3-1)^{\frac{7}{3}}+\frac{1}{4}(x^3-1)^{\frac{4}{3}}+C$$

13. $\dfrac{x^2}{(2+3x^3)^3}$

ANS :

Let $2+3x^3=t$

$\therefore 9x^2\,dx=dt$

$$\Rightarrow \int\frac{x^2}{(2+3x^3)^3}dx=\frac{1}{9}\int\frac{dt}{(t)^3}$$

$$=\frac{1}{9}\left[\frac{t^{-2}}{-2}\right]+C$$

$$=\frac{-1}{18}\left(\frac{1}{t^2}\right)+C$$

$$=\frac{-1}{18(2+3x^3)^2}+C$$

14. $\dfrac{1}{x(\log x)^m}, x > 0$

ANS :

Let $\log x = t$

$\therefore \dfrac{1}{x}dx = dt$

$\Rightarrow \int \dfrac{1}{x(\log x)^m}dx = \int \dfrac{dt}{(t)^m}$

$= \left(\dfrac{t^{-m+1}}{1-m}\right) + C$

$= \dfrac{(\log x)^{1-m}}{(1-m)} + C$

15. $\dfrac{x}{9-4x^2}$

ANS :

Let $9-4x^2 = t$

$\therefore -8x\,dx = dt$

$\Rightarrow \int \dfrac{x}{9-4x^2}dx = \dfrac{-1}{8}\int \dfrac{1}{t}dt$

$= \dfrac{-1}{8}\log|t| + C$

$= \dfrac{-1}{8}\log\left|9-4x^2\right| + C$

16. e^{2x+3}

ANS :

Let $2x+3 = t$

$\therefore 2dx = dt$

$\Rightarrow \int e^{2x+3}dx = \dfrac{1}{2}\int e^t dt$

$= \dfrac{1}{2}\left(e^t\right) + C$

$= \dfrac{1}{2}e^{(2x+3)} + C$

17. $\dfrac{x}{e^{x^2}}$

ANS :

Let $x^2 = t$

$\therefore 2xdx = dt$

$\Rightarrow \int \frac{x}{e^{x^2}} dx = \frac{1}{2} \int \frac{1}{e^t} dt$

$= \frac{1}{2} \int e^{-t} dt$

$= \frac{1}{2}\left(\frac{e^{-t}}{-1}\right) + C$

$= -\frac{1}{2} e^{-x^2} + C$

$= \frac{-1}{2e^{x^2}} + C$

ANS :

18. $\dfrac{e^{\tan^{-1} x}}{1+x^2}$

ANS :

Let $\tan^{-1} x = t$

$\therefore \frac{1}{1+x^2} dx = dt$

$\Rightarrow \int \frac{e^{\tan^{-1} x}}{1+x^2} dx = \int e^t dt$

$= e^t + C$

$= e^{\tan^{-1} x} + C$

19. $\dfrac{e^{2x}-1}{e^{2x}+1}$

ANS :

$$\frac{e^{2x}-1}{e^{2x}+1}$$

Dividing numerator and denominator by e^x, we obtain

$$\frac{\frac{\left(e^{2x}-1\right)}{e^x}}{\frac{\left(e^{2x}+1\right)}{e^x}}=\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

Let $e^x+e^{-x}=t$

$$\therefore\ \left(e^x-e^{-x}\right)dx=dt$$

$$\Rightarrow \int\frac{e^{2x}-1}{e^{2x}+1}dx=\int\frac{e^x-e^{-x}}{e^x+e^{-x}}dx$$

$$=\int\frac{dt}{t}$$

$$=\log|t|+C$$

$$=\log\left|e^x+e^{-x}\right|+C$$

20. $\dfrac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

ANS :

Let $e^{2x}+e^{-2x}=t$

$$\therefore\ \left(2e^{2x}-2e^{-2x}\right)dx=dt$$

$$\Rightarrow 2\left(e^{2x}-e^{-2x}\right)dx=dt$$

$$\Rightarrow \int\left(\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\right)dx=\int\frac{dt}{2t}$$

$$=\frac{1}{2}\int\frac{1}{t}dt$$

$$=\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+C$$

21. $\tan^2(2x-3)$

ANS :

$\tan^2(2x-3)=\sec^2(2x-3)-1$

Let $2x-3=t$

$\therefore 2dx=dt$

$$\Rightarrow \int \tan^2(2x-3)\,dx=\int\left[\left(\sec^2(2x-3)\right)-1\right]dx$$
$$=\frac{1}{2}\int\left(\sec^2 t\right)dt-\int 1dx$$
$$=\frac{1}{2}\int \sec^2 t\,dt-\int 1dx$$
$$=\frac{1}{2}\tan t-x+C$$
$$=\frac{1}{2}\tan(2x-3)-x+C$$

22. $\sec^2(7-4x)$

ANS :

Let $7-4x=t$

$\therefore -4dx=dt$

$$\therefore \int \sec^2(7-4x)\,dx=\frac{-1}{4}\int \sec^2 t\,dt$$
$$=\frac{-1}{4}(\tan t)+C$$
$$=\frac{-1}{4}\tan(7-4x)+C$$

23. $\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$

ANS :

Let $\sin^{-1}x=t$

$$\therefore \frac{1}{\sqrt{1-x^2}}dx=dt$$

$$\Rightarrow \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx=\int t\,dt$$

$$=\frac{t^2}{2}+C$$
$$=\frac{\left(\sin^{-1}x\right)^2}{2}+C$$

24. $\dfrac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$

ANS :

$$\frac{2\cos x-3\sin x}{6\cos x+4\sin x}=\frac{2\cos x-3\sin x}{2(3\cos x+2\sin x)}$$

Let $3\cos x+2\sin x=t$

$\therefore\ (-3\sin x+2\cos x)dx=dt$

$$\int\frac{2\cos x-3\sin x}{6\cos x+4\sin x}\,dx=\int\frac{dt}{2t}$$

$$=\frac{1}{2}\int\frac{1}{t}dt$$

$$=\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log|2\sin x+3\cos x|+C$$

25. $\dfrac{1}{\cos^2 x\,(1-\tan x)^2}$

ANS :

$$\frac{1}{\cos^2 x(1-\tan x)^2}=\frac{\sec^2 x}{(1-\tan x)^2}$$

Let $(1-\tan x)=t$

$\therefore\ -\sec^2 x dx=dt$

$$\Rightarrow\int\frac{\sec^2 x}{(1-\tan x)^2}dx=\int\frac{-dt}{t^2}$$

$$=-\int t^{-2}dt$$

$$=+\frac{1}{t}+C$$

$$=\frac{1}{(1-\tan x)}+C$$

26. $\frac{\cos\sqrt{x}}{\sqrt{x}}$

ANS :

Let $\sqrt{x} = t$

$$\therefore \frac{1}{2\sqrt{x}}dx = dt$$

$$\Rightarrow \int\frac{\cos\sqrt{x}}{\sqrt{x}}dx = 2\int\cos t\,dt$$

$$= 2\sin t + C$$

$$= 2\sin\sqrt{x} + C$$

27. $\sqrt{\sin 2x}\cos 2x$

ANS :

Let $\sin 2x = t$

$$\therefore\ 2\cos 2x\,dx = dt$$

$$\Rightarrow \int\sqrt{\sin 2x}\ \cos 2x\,dx = \frac{1}{2}\int\sqrt{t}\,dt$$

$$= \frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{1}{3}t^{\frac{3}{2}} + C$$

$$= \frac{1}{3}(\sin 2x)^{\frac{3}{2}} + C$$

28. $\dfrac{\cos x}{\sqrt{1+\sin x}}$

ANS :

Let $1+\sin x = t$

$\therefore \cos x\, dx = dt$

$$\Rightarrow \int \frac{\cos x}{\sqrt{1+\sin x}} dx = \int \frac{dt}{\sqrt{t}}$$

$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= 2\sqrt{t} + C$$

$$= 2\sqrt{1+\sin x} + C$$

29. $\cot x \log \sin x$

ANS :

Let $\log \sin x = t$

$$\Rightarrow \frac{1}{\sin x} \cdot \cos x \; dx = dt$$

$\therefore \cot x \; dx = dt$

$$\Rightarrow \int \cot x \; \log \sin x \; dx = \int t \; dt$$

$$= \frac{t^2}{2} + C$$

$$= \frac{1}{2}(\log \sin x)^2 + C$$

30. $\dfrac{\sin x}{1+\cos x}$

ANS :

Let $1 + \cos x = t$

$\therefore -\sin x\, dx = dt$

$$\Rightarrow \int \frac{\sin x}{1+\cos x} dx = \int -\frac{dt}{t}$$

$$= -\log|t| + C$$

$$= -\log|1+\cos x| + C$$

31. $\dfrac{\sin x}{(1+\cos x)^2}$

ANS :

Let $1 + \cos x = t$

$\therefore -\sin x\, dx = dt$

$$\Rightarrow \int \frac{\sin x}{(1+\cos x)^2}\, dx = \int -\frac{dt}{t^2}$$

$$= -\int t^{-2} dt$$

$$= \frac{1}{t} + C$$

$$= \frac{1}{1+\cos x} + C$$

32. $\dfrac{1}{1+\cot x}$

ANS :

$$\text{Let } I = \int \frac{1}{1+\cot x} dx$$

$$= \int \frac{1}{1+\frac{\cos x}{\sin x}} dx$$

$$= \int \frac{\sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int \frac{2\sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x + \cos x)} dx$$

$$= \frac{1}{2} \int 1\, dx + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

$$= \frac{1}{2}(x) + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

Let $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)\, dx = dt$

$$\therefore I = \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t}$$

$$= \frac{x}{2} - \frac{1}{2} \log|t| + C$$

$$= \frac{x}{2} - \frac{1}{2} \log|\sin x + \cos x| + C$$

33. $\dfrac{1}{1-\tan x}$

ANS :

$$\text{Let } I = \int \frac{1}{1-\tan x}dx$$
$$= \int \frac{1}{1-\frac{\sin x}{\cos x}}dx$$
$$= \int \frac{\cos x}{\cos x - \sin x}dx$$
$$= \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x}dx$$
$$= \frac{1}{2}\int \frac{(\cos x - \sin x)+(\cos x + \sin x)}{(\cos x - \sin x)}dx$$
$$= \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx$$
$$= \frac{x}{2} + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx$$

Put $\cos x - \sin x = t \Rightarrow (-\sin x - \cos x)\,dx = dt$

$$\therefore I = \frac{x}{2} + \frac{1}{2}\int \frac{-(dt)}{t}$$
$$= \frac{x}{2} - \frac{1}{2}\log|t| + C$$
$$= \frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$$

34. $\dfrac{\sqrt{\tan x}}{\sin x \cos x}$

ANS :

$$\text{Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cos x}dx$$
$$= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x}dx$$
$$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x}dx$$
$$= \int \frac{\sec^2 x\,dx}{\sqrt{\tan x}}$$

Let $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$$\therefore I = \int \frac{dt}{\sqrt{t}}$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{\tan x} + C$$

35. $\dfrac{(1+\log x)^2}{x}$

ANS :

Let $1+\log x=t$

$\therefore \dfrac{1}{x}dx=dt$

$$\Rightarrow \int \frac{(1+\log x)^2}{x}\,dx=\int t^2\,dt$$

$$=\frac{t^3}{3}+C$$

$$=\frac{(1+\log x)^3}{3}+C$$

36. $\dfrac{(x+1)(x+\log x)^2}{x}$

ANS :

$$\frac{(x+1)(x+\log x)^2}{x}=\left(\frac{x+1}{x}\right)(x+\log x)^2=\left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let $(x+\log x)=t$

$\therefore \left(1+\dfrac{1}{x}\right)dx=dt$

$$\Rightarrow \int\left(1+\frac{1}{x}\right)(x+\log x)^2\,dx=\int t^2\,dt$$

$$=\frac{t^3}{3}+C$$

$$=\frac{1}{3}(x+\log x)^3+C$$

37. $\dfrac{x^3\sin\left(\tan^{-1}x^4\right)}{1+x^8}$

ANS :

Let $x^4 = t$

$\therefore 4x^3\,dx = dt$

$$\Rightarrow \int \frac{x^3\sin\left(\tan^{-1}x^4\right)}{1+x^8}dx = \frac{1}{4}\int \frac{\sin\left(\tan^{-1}t\right)}{1+t^2}dt \qquad ...(1)$$

Let $\tan^{-1}t = u$

$\therefore \dfrac{1}{1+t^2}dt = du$

From (1), we obtain

$$\int \frac{x^3\sin\left(\tan^{-1}x^4\right)dx}{1+x^8} = \frac{1}{4}\int \sin u\,du$$
$$= \frac{1}{4}(-\cos u) + C$$

$$= \frac{-1}{4}\cos\left(\tan^{-1}t\right) + C$$
$$= \frac{-1}{4}\cos\left(\tan^{-1}x^4\right) + C$$

38. $\displaystyle\int \frac{10x^9 + 10^x \log_{e^{10}} dx}{x^{10} + 10^x}$ equals

(A) $10^x - x^{10} + C$

(B) $10^x + x^{10} + C$

(C) $(10^x - x^{10})^{-1} + C$

(D) $\log(10^x + x^{10}) + C$

ANS :

Let $x^{10} + 10^x = t$

$\therefore \left(10x^9 + 10^x \log_e 10\right)dx = dt$

$$\Rightarrow \int \frac{10x^9 + 10^x\log_e 10}{x^{10} + 10^x}dx = \int \frac{dt}{t}$$

$$= \log t + C$$
$$= \log\left(10^x + x^{10}\right) + C$$

Hence, the correct answer is D.

39. $\int \frac{dx}{\sin^2 x \cos^2 x}$ equals

(A) $\tan x + \cot x + C$

(B) $\tan x - \cot x + C$

(C) $\tan x \cot x + C$

(D) $\tan x - \cot 2x + C$

ANS :

Let $I = \int \frac{dx}{\sin^2 x \cos^2 x}$

$$= \int \frac{1}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \sec^2 x dx + \int \text{cosec}^2 x dx$$

$$= \tan x - \cot x + C$$

Hence, the correct answer is B.